Applied and Computational Mathematics
2022; 11(3): 74-80
http://www.sciencepublishinggroup.com/j/acm
doi: 10.11648/j.acm.20221103.13
ISSN: 2328-5605 (Print); ISSN: 2328-5613 (Online)

Analytical and Numerical Solutions for the (3+1)-dimensional Extended Quantum Zakharov-Kuznetsov Equation

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To cite this article:
Cheng Zhang. Analytical and Numerical Solutions for the (3+1)-dimensional Extended Quantum Zakharov-Kuznetsov Equation. *Applied and Computational Mathematics*. Vol. 11, No. 3, 2022, pp. 74-80. doi: 10.11648/j.acm.20221103.13

Received: April 26, 2022; **Accepted**: May 18, 2022; **Published**: May 31, 2022

Abstract: The Zakharov-Kuznetsov equation is an important model to describes the nonlinear pulse propagation in plasma physics, which guides the characteristic of weakly nonlinear ion-acoustic waves in plasma composed of cold ions and hot isothermal electrons in a uniform magnetic field. In the current study, we investigate the generalized trigonometric solutions and new travelling wave solutions of the (3+1)-dimensional extended quantum Zakharov- Kuznetsov equation through the (G'/G)-expansion method and the Sech-Tanh expansion method. Before applying these, we imply the traveling wave transformation to convert the (3+1)-dimensional extended quantum Zakharov- Kuznetsov equation to a nonlinear differential equation (NLODE). By the aid of Mathematics software, the dynamical images such as three-dimensional (3D) graphs, two-dimensional (2D) graphs and contour surfaces of local solutions are plotted by choosing the appropriate parameters. The obtained solutions show the simplicity and efficiency of the two approaches that can be applied for nonlinear equations as well as linear ones. Furthermore, the accuracy of the solutions obtained by the two different methods is verified by the Adomain decomposition method (ADM) and showed in tables respectively. The study of ADM method in this paper indivates it is an effective mathematical tool to calculate the numerical solutions and to verify the accuracy of the solutions.

Keywords: The (3+1)-dimensional Extended Quantum Zakharov–Kuznetsov Equation, The (G'/G)-Expansion Method, The Sech-Tanh Expansion Method, The ADM, The Analytical and Numerical Solutions

1. Introduction

Since the last century, nonlinear evolution equations have become of interest to many scholars. By applying them in the simulation of complex nonlinear phenomena, some of the difficulties in various areas of nonlinear science have been solved. A few decades ago, Zakharov and Kuznetsov introduced a kind of equation to describe ion-acoustic waves in a magnetized plasma containing cold ions and hot isothermal electrons [1]. Besides, Moslem et al. derived the quantum of the Zakharov-Kuznetsov equation [2],

$$\frac{\partial Q}{\partial t}+AQ+\frac{\partial Q}{\partial z}+k\frac{\partial^3 Q}{\partial z^3}+h\frac{\partial}{\partial z}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)Q=0$$

The nonlinear extended quantum Zakharov-Kuznetsov (NLEQZK) equation was first introduced by Sabry et al. They applied the reductive perturbation method to the quantum hydrodynamical equation and the Poisson equation [3].

Many researchers have investigated the NLEQZK by various computational techniques such as Hirota method [5], the auxiliary equation mapping method [7], the Lie symmetry method [6], etc. By these methods, many types of solutions have been obtained, such as shock solutions, periodic wave solutions and singular solutions.

The objective of this research is to perform the (G' /G)-expansion method [10-13], the Sech-Tanh expansion method [14-16] to find new solutions of NLEQZK equation, and to apply the Adomain decomposition method [17-20] to verify the accuracy of the solutions. Here, the form of NLEQZK is as follows [8, 9]:

$$\frac{\partial Q}{\partial t}+(AQ+BQ^2)\frac{\partial Q}{\partial z}+k\frac{\partial^3 Q}{\partial z^3}+h\frac{\partial}{\partial z}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)Q=0, \quad (1)$$

A, B, k, and h are constants that are not zero. x, y, z, t are the stretched space-time coordinates.

For purpose of converting (1) to a nonlinear differential equation (NLODE), we imply the traveling wave transformation:

$$Q(x,y,z,t) = q(\eta), \eta = \alpha x + \beta y + \gamma z - vt. \quad (2)$$

Inserting (2) into (1), we get the following NODE:

$$-vq' + A\gamma qq'' + B\gamma^2 q^2 q' + (k\gamma^3 + h\gamma(\alpha^2+\beta^2))q''' = 0. \quad (3)$$

Intergrating (3) for η, we get

$$-vq + \frac{1}{2}A\gamma q^2 + \frac{1}{3}B\gamma q^3 + (k\gamma^3 + h\gamma(\alpha^2+\beta^2))q'' + e_1 = 0, \quad (4)$$

where e_1 is a constant of integration.

2. The (G'/G)-Expansion Method

2.1. Basic Concepts of the (G'/G)-Expansion Method

This section introduces the detailed description of the (G'/G)-expansion method to find traveling wave solutions of nonlinear partial differential equation (PDE).

Step 1 For a given partial differential equation (PDE):

$$P(u, u_x, u_y, u_z, u_t, u_{xx}, u_{xt}, u_{tt}, \dots) = 0 \quad (5)$$

where u = u(x, y, z, t). We first apply the wave transformation

$$u(x,y,z,t) = U(\eta), \eta = \alpha x + \beta y + \gamma z - vt \quad (6)$$

where α, β, γ and ν are constants to be determined. Then (5) is changed to an ordinary differential equation (ODE):

$$Q(U, U', U'', \dots) = 0 \quad (7)$$

Step 2 Assume the solutions of this ODE can be expressed as a polynomial in G'/G as follows:

$$U(\eta) = \sum c_i \left(\frac{G'}{G}\right)^i \quad (8)$$

c_i (i = 1, 2,..., N) are unknown real constants, and by the homogeneous balance between the highest order derivatives and the nonlinear terms in ODE, the positive integer i can be determined.

The function G(η) satisfies the following second order linear ODE

$$G'' + \lambda G' + \mu G = 0, \quad (9)$$

where λ and μ are constants that can be solved later.

Step 3 Substitute the solutions of (8) and (9) into (7), and an algebraic equation with powers of (G'/G) is obtained. Then a series of algebraic equations of c_i is given by equating the coefficients of each power of (G'/G) to zero.

Step 4 Solve the system of algebraic equations for c_i. Since we are familiar with the general solutions of (9), then substitute the general solutions

$$\frac{G'}{G} = \begin{cases} -\frac{\lambda}{2} + \frac{D}{C+D\eta}, \ \lambda^2 - 4\delta = 0 \\ -\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\delta}}{2}\left(\frac{C\cos h\left(\frac{\pi}{2}\eta\right) + D\sin h\left(\frac{-\lambda}{2}\eta\right)}{C\sin h\left(\frac{\sqrt{\lambda^2-4\delta}}{2}\eta\right) + D\cos h\left(\frac{\sqrt{\lambda^2-4\delta}}{2}\eta\right)}\right), \\ \lambda^2 - 4\delta > 0 \\ -\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\delta}}{2}\left(\frac{D\cos\left(\frac{\sqrt{40-\lambda^2}}{2}\eta\right) - C\sin\left(\frac{\sqrt{40-\lambda^2}}{2}\eta\right)}{D\sin\left(\frac{\sqrt{4\delta-\lambda^2}}{2}\eta\right) + D\cos h\left(\frac{\sqrt{4\delta-\lambda^2}}{2}\eta\right)}\right), \\ \lambda^2 - 4\delta < 0 \end{cases}$$

into (8), where C, D are unknown constants. We have more traveling wave solutions of the nonlinear evolution.

2.2. Implementation of the (G'/G)-Expansion Method

In the subsection, we imply the (G'/G)-expansion method to the (3+1)- dimensional NLEQZK equation.

Using the balance principle on

$$-vq + \frac{1}{2}A\gamma q^2 + \frac{1}{3}B\gamma q^3 + (K\gamma^3 + h\gamma(\alpha^2+\beta^2))q'' + e_1 = 0, \quad (10)$$

$$q(\eta) = c_0 + c_1 \frac{G'(\eta)}{G(\eta)} \quad (11)$$

where $c_1 \neq 0$, c_0 are constants. Then,

$$q'(\eta) = -\frac{c_1 G'(\eta)^2}{G(\eta)^2} + \frac{c_1 G''(\eta)}{G(\eta)} \quad (12)$$

and

$$q''(\eta) = \frac{2c_1 G'(\eta)^3}{G(\eta)^3} - \frac{3c_1 G'(\eta)G''(\eta)}{G(\eta)^2} + \frac{c_1 G^3(\eta)}{G(\eta)} \quad (13)$$

Substituting (11), (12) and (13) into (10) yields an algebraic equation about $\left(\frac{G'}{G}\right)^i$ $(i = 0, 1, 2, 3)$. B adding the coefficients about $\left(\frac{G'}{G}\right)^i$ with the same powers and setting every item to zero, we get a series of algebraic equations about c_0, c_1, v as follows:

$$\left(\frac{G'(\eta)}{G(\eta)}\right)^0 : -vc_0 + \frac{1}{2}(A\gamma c_0^2) + \frac{1}{3}(B\gamma c_0^3) + h\gamma\alpha^2\mu\lambda c_1 + h\gamma\beta^2\mu\lambda c_1 + e_1 = 0$$

$$\left(\frac{G'(\eta)}{G(\eta)}\right)^1 : -vc_1 + 2k\gamma^3\mu c_1 + 2h\gamma\alpha^2\mu c_1 + 2h\gamma\beta^2\mu c_1 + k\gamma^3\lambda^2 c_1 + h\gamma\beta^2\lambda^2 c_1 + A\gamma c_0 c_1 + B\gamma c_0^2 c_1 = 0,$$

$$\left(\frac{G'(\eta)}{G(\eta)}\right)^2 : 3k\gamma^3\lambda c_1 + 3h\gamma\alpha^2\lambda c_1 + 3h\gamma\beta^2\lambda c_1 + B\gamma^2 c_0 c_1 = 0$$

$$\left(\frac{G'(\eta)}{G(\eta)}\right)^3 : 2k\gamma^3 c_1 + 2h\gamma\alpha^2 c_1 + 2h\gamma\beta^2 c_1 = 0$$

Solve the above algebraic equations, we obtain

$$c_1 = \pm \frac{\sqrt{6}\sqrt{-k\gamma^3 - h\gamma(\alpha^2+\beta^2)}}{\sqrt{B\gamma}}$$

$$c_0 = \frac{-A\gamma + \sqrt{6}\sqrt{B\gamma}\sqrt{-k\gamma^3 - h\gamma(\alpha^2+\beta^2)}}{2B\gamma}$$

$$v = -\frac{A\gamma^2}{4B\gamma} + \frac{1}{2}\left(k\gamma^3 + h\gamma(\alpha^2+\beta^2)\right)(4\delta - \lambda^2)$$

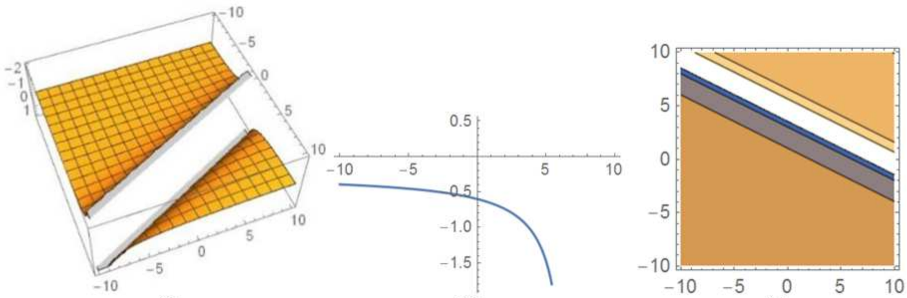
where λ and δ are arbitrary. Hence by (11) we have

$$q(\eta) = \frac{-A\gamma + \sqrt{6}\sqrt{B\gamma}\sqrt{-k\gamma^3 - h\gamma(\alpha^2+\beta^2)}}{2B\gamma} \pm \frac{\sqrt{6}\sqrt{-k\gamma^3 - h\gamma(\alpha^2+\beta^2)}}{\sqrt{B\gamma}}\left(\frac{G'(\eta)}{G(\eta)}\right) \tag{14}$$

Next, by substituting the general solutions of (9) in Step 4 into (14) respectively, we can obtain the following closed form solutions of (10):

Case 1: When $\lambda_2 - 4\delta = 0$,

$$q_{11}(\eta) = \frac{-A\gamma + \sqrt{6}\sqrt{B\gamma}\sqrt{-k\gamma^3 - h\gamma(\alpha^2+\beta^2)}}{2B\gamma} \pm \frac{\sqrt{6}\sqrt{-k\gamma^3 - h\gamma(\alpha^2+\beta^2)}}{\sqrt{B\gamma}}\left(-\frac{\lambda}{2} + \frac{D}{C+D\eta}\right) \tag{15}$$



(a) (b) (c)

Figure 1. *The 3D gragh, 2D graph and contour surface of (15), where α= 2, β= 0.4, γ= 0.2, B = 2, A = 0.1, h = 2, k = 5.*

Case 2: When $\lambda_2 - 4\delta > 0$,

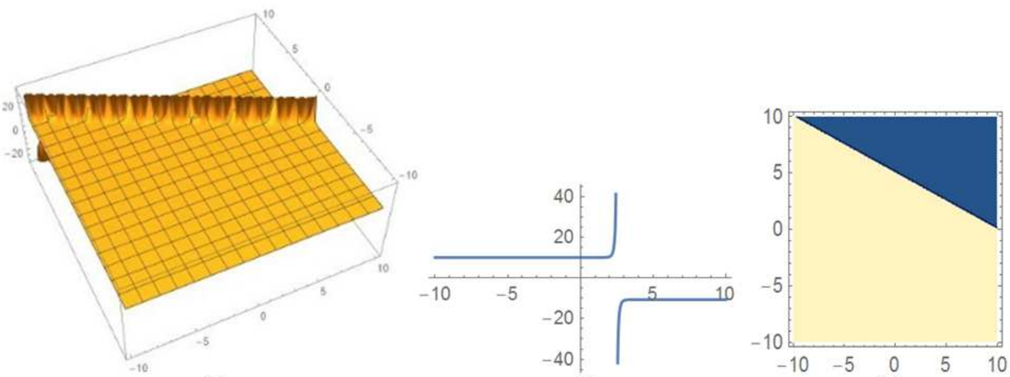
$$q(\eta) = \frac{-A\gamma + \sqrt{6}\sqrt{B\gamma}\sqrt{-k\gamma^3 - h\gamma(\alpha^2+\beta^2)}}{2B\gamma} \pm \frac{\sqrt{6}\sqrt{-k\gamma^3 - h\gamma(\alpha^2+\beta^2)}}{\sqrt{B\gamma}}\left(-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\delta}}{2}\cdot\frac{C\cosh\left(\frac{\sqrt{\lambda^2-4\delta}}{2}\eta\right) + D\sinh\left(\frac{\sqrt{\lambda^2-4\delta}}{2}\eta\right)}{C\sinh\left(\frac{\sqrt{\lambda^2-4\delta}}{2}\eta\right) + D\cosh\left(\frac{\sqrt{\lambda^2-4\delta}}{2}\eta\right)}\right)$$

i. If C = 0 and D≠0, then

$$q_{21}(\eta) = \frac{-A\gamma + \sqrt{6}\sqrt{B\gamma}\sqrt{-k\gamma^3 - h\gamma(\alpha^2+\beta^2)}\sqrt{-4\delta+\lambda^2}\tanh\left(\frac{1}{2}\eta\sqrt{-4\delta+\lambda^2}\right)}{2B\gamma} \tag{16}$$

ii. If *C*≠0 and *D* = 0, then

$$q_{22}(\eta) = \frac{-A\gamma + \sqrt{6}\sqrt{B\gamma}\sqrt{-k\gamma^3 - h\gamma(\alpha^2+\beta^2)}\sqrt{-4\delta+\lambda^2}\coth\left(\frac{1}{2}\eta\sqrt{-4\delta+\lambda^2}\right)}{2B\gamma} \tag{17}$$



(a) (b) (c)

Figure 2. *The 3D graph, 2D graph and contour surface of (17), where A = 1, B = 2, α = 1, β = 2, γ = 1, h = −1, k = −4, υ = 3.*

Case 3: When $\lambda_2 - 4\delta < 0$,

$$q(\eta) = \frac{-A\gamma+\sqrt{6}\sqrt{B\gamma}\sqrt{-k\gamma^3-h\gamma(\alpha^2+\beta^2)}}{2B\gamma} \pm \frac{\sqrt{6}\sqrt{-k\gamma^3-h\gamma(\alpha^2+\beta^2)}}{\sqrt{B\gamma}}\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\delta}}{2}\cdot\frac{D\cos\left(\frac{\sqrt{4\delta-\lambda^2}}{2}\eta\right)-C\sin\left(\frac{\sqrt{4\delta-\lambda^2}}{2}\eta\right)}{D\sin\left(\frac{\sqrt{4\delta-\lambda^2}}{2}\eta\right)+D\cosh\left(\frac{\sqrt{4\delta-\lambda^2}}{2}\eta\right)}\right)$$

i. If C = 0 and D≠ 0, then

$$q_{31}(\eta) = \frac{-A\gamma+\sqrt{6}\sqrt{B\gamma}\sqrt{-k\gamma^3-h\gamma(\alpha^2+\beta^2)}\sqrt{4\delta-\lambda^2}\cot\left(\frac{1}{2}\eta\sqrt{4\delta-\lambda^2}\right)}{2B\gamma} \tag{18}$$

ii. If $C \neq 0$ and $D = 0$, then

$$q_{32}(\eta) = \frac{A\gamma-\sqrt{6}\sqrt{B\gamma}\sqrt{-k\gamma^3-h\gamma(\alpha^2+\beta^2)}\sqrt{4\delta-\lambda^2}\tan\left(\frac{1}{2}\eta\sqrt{4\delta-\lambda^2}\right)}{2B\gamma} \tag{19}$$

where $\eta = \alpha x + \beta y + \gamma z - \upsilon t$.

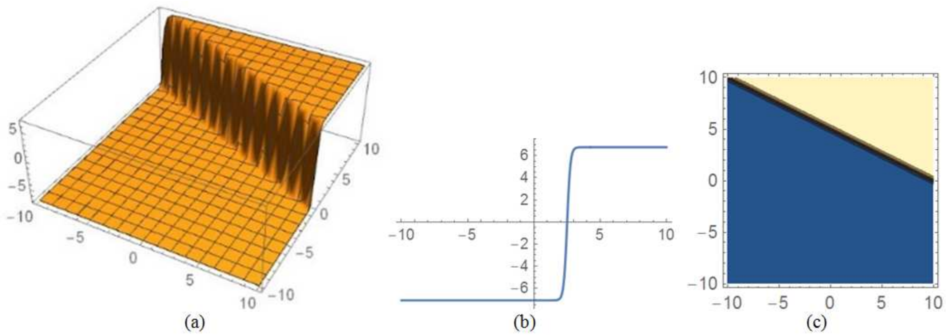


Figure 3. *The 3D graph, 2D graph and contour surface of (2.15), where A = 2, B = 6, γ = 1, k = −2, α = 1, h = −2, k = 2, λ = 6, δ = 5.*

3. The Sech-Tanh Expansion Method

3.1. Basic Concepts of the Sech–Tanh Expansion Method

We suppose that:

$$Q(x,y,z,t) = q(\eta), \eta = \alpha x + \beta y + \gamma z - vt,$$

has the travelling wave solution as follows:

$$q(\eta) = \sum \text{sech}^{i-1}(\eta)(a_i \text{sech}(\eta) + b_i \tanh(\eta)) + a_0 \tag{20}$$

where a_i, b_i, (i = 1, 2,...) are unknown constants, then we will figure them out.

Step 1 Setting the highest-order nonlinear term equal to the highest-order linear partial derivative in ODE then the value of i (i = 1, 2,...) is determined.

Step 2 Setting the coefficients of ($\text{sech}^j(\eta)\tanh^i(\eta)$) for i = 0, 1 and j = 1, 2,... to zero, we have a series of overdetermined equations about the a_0, a_i, b_i (i = 1, 2,...).

Step 3 Using mathematical programming software, we can solve the algebraic equations in Step 2.

3.2. Implementation of the Sech–Tanh Expansion Method

Utilizing the balance principle on

$$-vq + \frac{1}{2}A\gamma q2 + \frac{1}{3}B\gamma q^3 + \left(K\gamma^3 + h\gamma(\alpha^2+\beta^2)\right)q'' + e_1 = 0 \tag{21}$$

taking the homogeneous balance between nonlinear term q^3 and the highest derivative q'', we have i = 1. With i = 1, (20) has the form

$$q(\eta) = a_0 + a_1 \text{sech}(\eta) + b_1 \tanh(\eta). \tag{22}$$

Then,

$$q''(\eta) = -a_1 \text{sech}^3(\eta) - 2b_1 \text{sech}^2(\eta)\tan\text{h}(\eta) + a_1 \text{sech}(\eta)\tanh^2(\eta) \tag{23}$$

substituting (22), (23) into (21) yields an algebraic equation about Sechj Tanhi (η)sech j tanh i.

$$\text{Sech}(\eta): -va_1 + A\gamma a_0 a_1 = 0,$$

$$\text{Sech}^2(\eta): \frac{1}{2}A\gamma a_1^2 + B\gamma a_0 a_1^2 = 0,$$

$$\mathrm{Sech}^3(\eta): -k\gamma^3 a_1 + \frac{1}{3}B\gamma a_1{}^3 = 0,$$

$$\mathrm{Tanh}^0\,\mathrm{Sech}^0(\eta): -\nu a_0 + \frac{1}{2}A\gamma a_0^2 + \frac{1}{3}B\gamma a_0^3 = 0,$$

$$\mathrm{TanhSech}(\eta): A\gamma a_1 b_1 + 2B\gamma a_0 a_1 b_1 = 0,$$

$$\mathrm{TanhSech}^2(\eta): -2k\gamma^3 b_1 + B\gamma a_1{}^2 b_1 - 2\alpha^2\gamma b_1 h_0 - 2\beta^2\gamma b_1 h_0 = 0.$$

Solving the above algebraic equations, we get

$$b_1 = 0, a_1 = \pm\frac{\sqrt{3k\gamma^3+3(\alpha^2+\beta^2)\gamma h}}{B\gamma}$$

$$a_0 = -\frac{A\gamma}{2B\gamma}, \nu = -\frac{A^2\gamma^2}{4B\gamma}$$

Thus by (22) we have

$$q_{21}(\eta) = -\frac{A\gamma}{2B\gamma} + \frac{\sqrt{3k\gamma^3+3(\alpha^2+\beta^2)\gamma h}}{B\gamma}\mathrm{sech}(\eta) \tag{24}$$

$$q_{22}(\eta) = -\frac{A\gamma}{2B\gamma} - \frac{\sqrt{3k\gamma^3+3(\alpha^2+\beta^2)\gamma h}}{B\gamma}\mathrm{sech}(\eta) \tag{25}$$

where $\eta = \alpha x + \beta y + \gamma z - \upsilon t$.

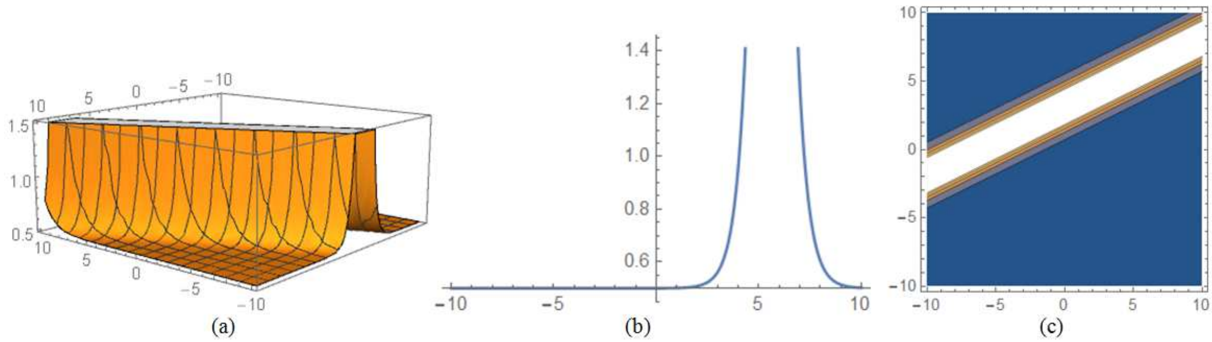


Figure 4. *The 3D graph, 2D graph and contour surfaces of (3.5) using A = −1, B = 1, γ = 1, k = 2, α = 1, β = −2, k = 2.*

4. Verification of the Accuracy of the Solutions

4.1. Basic Concepts of the Adomian Decomposition Method

The evaluation of the approximate solutions is achieved by employing the Adomian Decomposition Method (ADM) whose values are provided by the obtained analytical solutions. The method starts at the equation Fu(t) = g(t) in which F represents the general nonlinear ordinary differential operator including both linear and nonlinear terms, and the linear term is further decomposed into L+R where L is invertible and R stands for the remainder of the linear operator. In order to facilitate the assessment, L can be used as the highest derivative, thus avoiding complex integrals when complex Green's function is involved. Therefore, the equation may be stated as follows:

$$Lu + Ru + Nu = g,$$

where Nu represents the nonlinear terms. To solve Lu, we transform this equation into

$$Lu = g - Ru - Nu.$$

Because L is a reversible operation, we apply L^{-1} to both sides of this equation

$$L^{-1}Lu = L^{-1}g + L^{-1}Ru - L^{-1}Nu. \tag{26}$$

If this equation corresponds to an initial-value problem, the integral operator L^{-1} may be regarded as definite integrals from t_0 to t. If L is a second-order operator, L^{-1} is a twofold integration operator and $L^{-1}Lu = u - u(t_0) - (t - t_0)u'(t_0)$. When boundary value problems are involved, we use indefinite integrations and evaluate the constans according the given conditions. Solving (26) for u yields

$$u = A + Bt + L^{-1}g - L^{-1}Ru - L^{-1}Nu. \tag{27}$$

The nonlinear term Nu will be equated to the special polynomials $\sum A_n$ which will be discussed later, and u will be decomposed into $\sum u_n$. When the variable η in the analytical solutions is equal to zero, u_0 can be obtained.

$$\sum u_n = u_0 - L^{-1}R\sum u_n - L^{-1}\sum A_n$$

Consequently, we can write

$$u_1 = -L^{-1}Ru_0 - L^{-1}A_0$$
$$u_2 = -L^{-1}Ru_1 - L^{-1}A_1$$
$$u_3 = -L^{-1}Ru_2 - L^{-1}A_2$$
$$\cdots$$

A_0 depends only on u_0, A_1 depends only on u_0 and u_1, A_2 depends on u_0, u_1 and u_2, etc, because the polynomials A_n are generated for each nonlinear term.

They are defined by

$$A_0 = f(u_0)$$
$$A_1 = u_1(d/du_0)f(u_0)$$
$$A_2 = u_2(d/du_0)f(u_0) + (u_1^2/2!)f(u_0)$$
$$\cdots$$

It follows that the semianalytical solution of (27) can be written as $u = \sum u = u_0 + u_1 + u_2 + \cdots$. For further details about the ADM, see [17-20].

4.2. Implementation of the Adomian Decomposition Method

Combine equation (16) with the following conditions: $\alpha = 1, \beta = 2,\ h = -1, k = 4, \gamma = 1, A = 4, B = 1$, we obtain:

$$q_0 = -2,$$
$$q_1 = \frac{14}{3}\eta^2,$$
$$q_2 = -\frac{7}{3}\eta^4,$$
$$q_3 = \frac{7}{15}\eta^6,$$
$$\cdots$$

Consequently, the semianalytical solution of equation (7) can be written as

$$q(\eta) = \sum\ q_n = q_0 + q_1 + q_2 + q_3 + \cdots = \frac{7\eta^6}{15} - \frac{7\eta^4}{3} + \frac{14\eta^2}{3} - 2 + \cdots \tag{28}$$

By the same method, with respect to (3.5) and appropriate conditions, we obtain:

$$q_0 = 1,$$
$$q_1 = -\frac{3}{8}\eta^2,$$
$$q_2 = \frac{3}{32}\eta^4,$$
$$q_3 = \frac{-39}{1280}\eta^6,$$
$$\cdots$$

and the semianalytical solution of equation (7) is written in the following form:

$$q(\eta) = \sum\ q_n = q_0 + q_1 + q_2 + q_3 + \cdots = -\frac{39\eta^6}{1280} + \frac{3}{32}\eta^4 - \frac{3}{8}\eta^2 + 1 + \cdots \tag{29}$$

Next, we substitute $\eta = 0.001, \eta = 0.002, \cdots, \eta = 0.01$ into (16) and (24), the exact solutions of equation (7) according to the analytical solutions via the (G'/G)-expansion method and Sech-Tanh expansion method are obtained. Then substitute $\eta = 0.001, \eta = 0.002, \cdots, \eta = 0.01$ into (28) and (29), the approximate solutions of (7) can be obtained by applying the ADM. The results are shown in the following tables more intuitively.

Table 1. *Exact solutions, approximate solutions and absolute values of error with different values of η by ADM.*

Value of η	Exact	Approximate	Error
0.001	-1.999995	-1.99633	-3.66957×10^{-3}
0.002	-1.99998	-1.99265	-7.32979×10^{-3}
0.003	-1.99996	-1.98898	-1.09807×10^{-2}
0.004	-1.99993	-1.9853	-1.46222×10^{-2}
0.005	-1.99988	-1.98163	-1.82543×10^{-2}
0.006	-1.99983	-1.97795	-2.1877×10^{-2}
0.007	-1.99977	-1.97428	-2.54904×10^{-2}
0.008	-1.9997	-1.97061	-2.90943×10^{-2}
0.009	-1.99962	-1.96693	-3.26888×10^{-2}
0.01	-1.99953	-1.96326	-3.62739×10^{-2}

Value of η	Exact	Approximate	Error
0.001	0.999999	0.999999	-4.16667×10^{-8}
0.002	0.999998	0.999998	-1.66667×10^{-7}
0.003	0.999997	0.999997	-3.75004×10^{-7}
0.004	0.999994	0.999995	-6.66678×10^{-7}
0.005	0.999991	0.999992	-1.04169×10^{-6}
0.006	0.999987	0.999988	-1.50006×10^{-6}
0.007	0.999982	0.999984	-2.04178×10^{-6}
0.008	0.999976	0.999979	-2.66685×10^{-6}
0.009	0.99997	0.999973	-3.3753×10^{-6}
0.01	0.999963	0.99668	3.282×10^{-3}

According to Table 1, we see that the solutions of (7) by Sech-Tanh expanion method and (G'/G)-expansion method are both accurate, the latter has a faster convergence rate and higher accuracy.

5. Conclusion

In the present study, a new technique is employed to find the solutions of the (3+1)-dimensional extended quantum Zakharov-Kuznetsov equation by (G'/G)-expansion and the Sech-Tanh expansion method. Through implementing the traveling wave transform, the governing equation has been converted into a nonlinear differential equation. As for the proposed model, various analytical solutions have been constructed. The numerical solutions and the accuracy of all the solutions can be verified by the Adomain decomposition and we show these in tables. Furthermore, the physical structure of the results has been analyzed graphically.

References

[1] E. A. Kuznetsov, V. E. Zakharov (1974), On three dimensional solitons, Sov. Phys.—JETP, 39, 285–286.

[2] W. M. Moslem, S. Ali, P. K. Shukla, X. Y. Tang, G Rowlands (2007), Solitary, explosive, and periodic solutions of the quantum Zakharov-Kuznetsov equation and its transverse instability, Phys. Plasmas, 14, 082308. https://doi.org/10.1063/1.2757612

[3] R. Sabry, W. M. Moslem, F. Haas, S. Ali, P. K. Shukla (2008), Explosive, soliton, and shock in a quantum electron-positron-ion magnetoplasma, Phys. Plasmas, 15, 122308. https://doi.org/10.1063/1.3037265

[4] K. K. Ali, R. Yilmazer, A. Yokus, et al (2020). Analytical solutions for the (3+1)-dimensional nonlinear extended quantum Zakharov–Kuznetsov equation in plasma physics, Physica A, 548, 124327. https://doi.org/10.1016/j.physa.2020.124327

[5] H. L. Zhen, B. Tian, Y. F. Wang, W. R. Sun, L. C. Liu (2014), Soliton solutions and chaotic motion of the extended Zakharov-Kuznetsov equations in a magnetized two-ion-temperature dustyplasma, Phys. Plasmas, 21, 073709. https://doi.org/10.1063/1.4885380

[6] S. Kumar, D. Kumar (2019), Solitary wave solutions of (3+1)-dimensional extended Zakharov–Kuznetsov equation by Lie symmetry approach, Comput, Math. Appl, 77, 2096–2113. https://doi.org/10.1016/j.camwa.2018.12.009

[7] A. R. Seadawy, Lu Dianchen (2016), Ion acoustic solitary wave solutions of three-dimensional nonlinear extended Zakharov–Kuznetsov dynamical equation in a magnetized two-ion-temperature dusty plasma, Results Phys, 6, 590–593.

[8] E. M. E. Zayed, R. M. A. Shohib, A. Al-Nowehy (2019), On solving the (3+1)-dimensional NLEQZK equation and the (3+1)-dimensional NLmZK equation using the extended simplest equation method, Comput. Math. Appl. https://doi.org/10.1016/j.camwa.2019.05.007

[9] Z. Yan (2009), Periodic solitary and rational wave solutions of the 3D extended quantum Zakharov–Kuznetsov equation in dense quantum plasmas, Phys. Lett. A, 373, 2432–2437. https://doi.org/10.1016/j.physleta.2009.04.018

[10] Younis, M,. Zafar, A., Ul-Haq, K., Rahman, M. (2013), Travelling wave solutions of fractional order coupled Burgers' equations by-expansion method, Am. J. Comput. Appl. Math, 3, 81-85.

[11] Biswas, A., Bhrawy, A. H., Abdelkawy, M. A., Alshaery, A. A., Hilal, E. M. (2014), Symbolic computation of some nonlinear fractional differential equations, Rom. J. Phys, 59, 433-442.

[12] Khan, K., Akbar, M. A., Koppelaar, H. (2015), Study of coupled nonlinear partial differential equations for finding exact analytical solutions, R. Soc. Open Sci, 2, 140406.

[13] Khan, H., Shoaib, Baleanu, D., Kumam, P., Al-Zaidy, J. F. (2019), Families of travelling waves solutions for fractional-order extended shallow water wave equations, using an innovative analytical method. IEEE Access, 7, 107523-107532. https://doi.org/10.1098/rsos.140406

[14] R. Attia, D. Lu, and M. Khater (2019), Chaos and relativistic energy momentum of the nonlinear time fractional Duffing equation, Mathematical and Computational Applications, vol. 24 (2019), 1-10. https://doi.org/10.3390/mca24010010

[15] H. M. Baskonus, D. A. Koc¸ (2016), and H. Bulut, Dark and new travelling wave solutions to the nonlinear evolution equation, Optik, 127, 8043–8055. https://doi.org/10.1016/j.ijleo.2016.05.1321

[16] F. Dusunceli (2019), New exponential and complex traveling wave solutions to the Konopelchenko-Dubrovsky model, Advances in Mathematical Physics, 7801247. https://doi.org/10.1155/2019/7801247

[17] Z. Odiba (2020) t, An optimized decomposition method for nonlinear ordinary and partial differential equations, Physica A: Statistical Mechanics and Its Applications, 541, 123323. https:// doi.org/10.1016/j. physa. 2019.123323

[18] H. O. Bakodah, A. A. Al Qarni, M. A. Banaja, Q. Zhou, S. P. Moshokoa, and A. Biswas (2017), Bright and dark irring optical solitons with improved Adomian decomposition method, Optik, 130, 1115—1123. https://doi.org/10.1016/j.ijleo.2016.11.123

[19] G. ADOMIAN (1986), Nonlinear Stochastic Operator Equation, Orlando, FL: Academic Press.

[20] G. ADOMIAN (1988), Applications of Nonlinear Stochastic Systems Theory to Physics Reidel, Dordrecht: Kluwer Academic Publisher.