Applied and Computational Mathematics
2023; 12(4): 82-91
http://www.sciencepublishinggroup.com/j/acm
doi: 10.11648/j.acm.20231204.11
ISSN: 2328-5605 (Print); ISSN: 2328-5613 (Online)

A Fiedler's Approach to LINEX Intuitionistic Fuzzy C-means Clustering Induced Spectral Initialization for Data Analysis

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To cite this article:

M. Nithya, K. Bhuvaneswari, S. Senthil. A Fiedler's Approach to LINEX Intuitionistic Fuzzy C-means Clustering Induced Spectral Initialization for Data Analysis. *Applied and Computational Mathematics.* Vol. 12, No. 4, 2023, pp. 82-91.
doi: 10.11648/j.acm.20231204.11

Received: May 19, 2023; **Accepted**: June 12, 2023; **Published**: July 21, 2023

Abstract: Clustering is a common technique for statistical data analysis. The clustering method based on intuitionistic fuzzy set has attracted more and more scholar's attention nowadays. This paper discusses the intuitionistic fuzzy C-means clustering algorithm. There are a number of clustering techniques developed in the past using different distance/similarity measure. In researchers have used various distance measure like Hamming distance, Euclidean distance etc., to solve the clustering problems. In this paper, we proposed a novel LINEX for intuitionistic fuzzy c means clustering based on minimal spanning tree using Fiedler's approach initialization method. Our main motives of using the LINEX methods consist inducing a class of robust non-Euclidean distance measures for the original data space to derive new objective functions and thus clustering the integration of datasets, enhancing robustness of the original clustering algorithms to noise and outliers, and still retaining computational simplicity. The proposed Fiedler's approach LINEX IFCM, which requires the determination of the eigenvector belonging to the second Eigen value of the Laplacian matrix. Finally, evaluation is illustrated by the intuitionistic fuzzy C-means clustering method and the method is compared with the fuzzy C-means clustering method as well.

Keywords: Intuitionistic Fuzzy C-means, Fiedler Value, Eigenvalue, Eigenvector, Minimal Spanning Tree, LINEX Function

1. Introduction

Data mining techniques have been recognized in different fields for discovering useful patterns and extracting information from the pool of available abundant data. Data mining provides automated tools for the process of knowledge discovery by analyzing available data. The purpose of clustering is to divide the data set into groups, so that similar data points fall into same cluster. Clustering is an unsupervised process, where the data is not supported with labeled information, so its aim is to infer the expected structure existing within a set of data points. Secondly, Unsupervised clustering techniques process totally unlabelled data, therefore suffer from the problem of defining number of clusters, prior random initialization of the cluster centers, problem of local traps and finally binding every data point to a class.

The notion of Intuitionistic Fuzzy Set (IFS) coined by Atanassov [1], for fuzzy set generalizations has interesting and useful applications in different domains. But few study on clustering is reported in the previous study on intuitionistic fuzzy sets. Zhang et al. [2], suggested a clustering approach, where an intuitionistic fuzzy similarity matrix is transformed to interval valued fuzzy matrix. Recently intuitionistic theory has been widely used with classical clustering algorithms to deal with the problem of uncertainty present in the real world unlabelled data. Different experiments have proved [3-5] that intuitionistic fuzzy set based method helps to better handle the problem of uncertainty as compared to fuzzy set. Intuitionistic fuzzy set (IFS) is a higher order extension of fuzzy set. Intuitionistic fuzzy sets are elaborated set, consisting of hesitation degree along with membership and non-membership degree. The hesitation degree helps to deal with the problem of uncertainty present in the unlabelled data. In literature, few researchers have used IFS effectively in different applications. Among these works, Xu and Wu [6] defined IFCM which presents the clustering of Intuitionistic fuzzy set (IFS). Pelekis et al. [7] demonstrated the process of clustering based on the intuitionistic fuzzy knowledge of data and recommended that the intuitionistic fuzzy clustering acquires the qualitative information, which may be estimated as per feature vector.

Recently Chaira and Tamalika proposed novel IFS c-means for edge detection and segmenting medical images using Yager-type IF membership function [8, 9]. Atanassov's IF membership function [10] has been recently used to determine the optimal threshold value for grey-level image segmentation. Ananthi et al. [11] proposed grey-scale image segmentation using multiple membership functions, interval-valued IF for brain tumour segmentation [12] and Sugeno, fuzzy generator-based intuitionistic FCM clustering for crop images [13]. Verma used a new fuzzy factor to consider spatial context and Dubey [14] used complement function for hesitance membership function to segment MR brain image [15, 16]. The Gaussian kernel based FCM (GKFCM) for medical image segmentation is proposed by Yang and Tsai [17]. The hyper tangent FCM (HTFCM) based image segmentation for breast images is proposed by Karman et al. [18].

These algorithms utilized only intensity of pixel as the only feature for the segmentation of images and failed to classify noisy pixels accurately. The pixels in an image are exceedingly associated, i.e. the every pixel in the prompt vicinity have about the equivalent feature information unless there is some curve or contour. Subsequently, integrating spatial information along with the membership value results in more homogenous regions as compared to other methods. However, these techniques use the arbitrary initialization of cluster centers which give inaccurate outcomes and more time for optimization.

In this paper, to overcome the defects of the algorithms as mentioned, we proposed Fiedler's approach–LINEX IFCM, which requires the determination of the eigenvector belonging to the second Eigen value of the Laplacian matrix, named the MST using LINEX_IFCM. The intuitionistic fuzziness is embedded into the calculation process of similarity between the pixel and cluster centers, which achieves more accurate segmentation in the organization boundary. The algorithm is minimal spanning tree using Spectral initialization method by a given Fiedler value using LINEX_IFCM, which helps to speed up the convergence of the algorithm.

2. Minimal Spanning Tree Algorithm

In this section, we proposed Fiedler's approach LINEX measures for construct the minimal spanning tree.

2.1. LINEX Function MST

Given the grayscale point set D, the hierarchical methods starts by constructing a minimal spanning tree (MST) from the points in D. In $x=(x_1,x_2,.......x_n)^T$ and $y=(y_1,y_2,.......y_n)^T$ are two points of a MST and $e(x,y)$ is an edge between x and y then LINEX distance function between x and y is denoted by $d(x,y)$ and calculated using equation (1).

$$d_{LINEX}(x,y)=\exp\left(\frac{\|x-y\|}{\sigma}\right)-\left(\frac{\|x-y\|}{\sigma}\right)-1 \qquad (1)$$

2.2. Fiedler Method

The Fiedler Method is an easy way to partition a graph. The Fiedler Method is named after Miroslav Fiedler, a Czech mathematician, who worked in graph theory and linear algebra. This method was presented by him in 1973. Fiedler's beautiful results to the Laplacian matrix of the graph, this method partitions the data set D into two sets D1 and D2 based on the Eigen vector V corresponding to the second smallest Eigen value of Laplacian matrix. Finding the Laplacian matrix requires construction of A adjacency matrix and D degree matrix, so the Laplacian matrix l is formed as:

$$L=D-A \qquad (2)$$

If G is a simple connected graph with n vertices and if L is the Laplacian matrix for G then L has n real Eigenvalues satisfying $0=\lambda_1<\lambda_2\le\lambda_3\le..........\le\lambda_n$. The Fiedler Value or the algebraic connectivity of a graph is the second smallest Eeigenvalue of its Laplacian matrix L.

2.3. Algorithm for Determining the Initial Cluster Centers

Algorithm: LINEX MST

Input: Data points

Output: optimal number of cluster centers

Let e1 be an edge in the LINEX measure MST constructed from Data points

Let S_T be the set of disjoint subtrees of LINEX measure MST.

1. Create a node v, for each data points.
2. Compute the edge weight using equation (1)
3. Construct an LINEX measure MST from 2.
4. $S_T=\varphi$, $n_c=1$, $C=\varphi$.
5. To find the adjacency matrix and Degree matrix from 3.
6. To find Laplacian matrix L.
7. Based on a symmetric matrix L, we search for the eigenvector v_2 is used to recursively partition the graph by separating the components into negative and positive values.
8. For each $e1\in MST$.
9. To remove inconsistent edge from MST using 7.
10. $S_T=S_T\cup\{T'\}$ // T' is new disjoint subtrees (regions).
11. $n_c=n_c+1$.
12. Compute the center c_i of T_i using average of points.
13. $C=\bigcup_{T_i}\in S_T\{c_i\}$.
14. Update the clusters points, Repeat Step 6 to 13.

Finally we obtain the cluster centers.

3. Formulation of Proposed Method LINEX Intuitionistic Fuzzy C means Clustering

3.1. Intuitionistic Fuzzy C-means (IFCM) Algorithm

Intuitionistic fuzzy set given by Atanassov [1] considers both membership $\mu(x),\ x\in X$ and non-membership $\nu(x),\ x\in X$. An intuitionistic fuzzy set A in X, is written as

$$A=\{\ x,\mu_A(x),\ \nu_A(x)\backslash x\in X\ \}$$

where $\mu_A(x)\to[0,1]$, $\nu_A(x)\to[0,1]$ are the membership and non-membership degrees of an element in the set A with the condition $0\le\mu_A(x)+\nu_A(x)\le 1$ when $\nu_A(x)=1-\mu_A(x)$ for every x in the set A, then the set A becomes a fuzzy set. Also indicated a hesitation degree, $\pi_A(x)$ which arises due to lack of knowledge in defining the membership degree of each element x in the set A and is given by

$$\pi_A(x)=1-\mu_A(x)-\nu_A(x),\ 0\le\pi_A(x)\le 1.$$

In [1] intuitionistic fuzzy c-means, minimizes the objective function as:

$$J_{IFCM}=\sum_{i=1}^{C}\sum_{k=1}^{N}u_{ik}^{*m}\,\|x_i-c_k\|^2+\sum_{i-1}^{C}\pi_i^*\,e^{1-\pi_i^*}\qquad 1<m<\infty \tag{3}$$

$u_{ik}^*=u_{ik}+\pi_{ik}$, where u_{ik}^* denotes the intuitionistic fuzzy membership and

$$u_{ik}=\frac{1}{\sum_{j=1}^{c}\left[\dfrac{\|x_i-c_k\|}{\|x_i-c_j\|}\right]^{\frac{2}{m-1}}} \tag{4}$$

$$\pi_{ik}=1-u_{ik}-\left(\frac{1-u_{ik}}{1+\lambda u_{ik}}\right),\lambda>0 \tag{5}$$

$$c_k=\frac{\sum_{i=1}^{N}u_{ik}^{m}\,x_i}{\sum_{i=1}^{N}u_{ik}^{m}} \tag{6}$$

$$\pi_i^*=\frac{1}{N}\sum_{k=1}^{N}\pi_{ik} \tag{7}$$

This iteration will stop when

$$\max_{ij}\left\{\ \left|u_{ik}^{*k+1}-u_{ik}^{*k}\right|\ \right\}<\in,$$

where $\in$ is a termination criterion between 0 and 1, where as k is the iteration steps. This procedure converges to a local minimum or a saddle point of J_{IFCM}.

3.2. LINEX Measure Based Intuitionistic Fuzzy C-means (IFCM) Algorithm

In this section, we want to use the LINEX measure in Intuitionistic fuzzy c-means (IFCM) algorithm when the over estimating and the under estimating are not of the same importance. The procedures are the same as a Intuitionistic fuzzy c-means algorithm. All the entities are assigned to their nearest centroid from MST, using a LINEX loss function as the dissimilarity distance. The procedure continues until there is no change in clusters. Now consider the following optimization problem,

Objective function:

$$J_{IFCM}(U,C)=\sum_{i=1}^{c}\ \sum_{j=1}^{n}u^{*m}{}_{ij}\,L_{LINEX}(x_j,c_i)+\sum_{i=1}^{c}\pi_i^*\,e^{1-\pi_i^*}$$
$$1<m<\infty \tag{8}$$

where $L_{LINEX}(x_{kj},v_i)=\exp\left(\dfrac{\|x-y\|}{\sigma}\right)-\left(\dfrac{\|x-y\|}{\sigma}\right)-1$

3.3. Updating Membership

To obtain equation for calculating membership we minimizing the objective function $J_{LINEX}(U,V)$ with constraint conditions.

$$0\le u_{ik}\le 1\ \ \forall\ i=1,2,\ldots\ldots\ldots\ldots c,\ k=1,2\ldots\ldots\ldots\ldots n.$$

$$\sum_{i=1}^{c}u_{ik}=1\ \forall\ k=1,2\ldots\ldots\ldots\ldots n$$

$$0\le\sum_{k=1}^{n}u_{ik}\le n\ \forall\ i=1,2,\ldots\ldots\ldots c$$

Can be solved by using the Lagrangian multiplication as follows:

To find min $J_{LINEX}(U,V)$ it is sufficient to minimize the following inner sum for fixed k:

put $L_{ik}=L_{LINEX}(u_{ik},v_{ij})$

Let $B=\left\{u_k=(u_{1k},u_{2k},\ldots\ldots.u_{ck})\in R^c/\sum_{i=1}^{c}u_{ik}=1,\ 0\le u_{ik}\le 1\right\}$

and $g(u_k)=\sum_{i=1}^{c}u^{m}{}_{ik}\,L_{ik}-\lambda\left(\sum_{i=1}^{c}u_{ik}-1\right)$

(λ,u_k) is stationary for F only if $\Delta_{\lambda,u_k}F(\lambda,u_k)=0,\ \ 0\in R^c$ that yields to

$$\frac{\partial F}{\partial \lambda}(\lambda,u_k)=\sum_{i=1}^{c} u_{ik}-1=0 \tag{9}$$

$$\frac{\partial F}{\partial u_{ik}}(\lambda,u_k)=m\,{u^{m-1}}_{ik}\;L_{ik}-\lambda=0 \tag{10}$$

From (9)

$$m\,{u^{m-1}}_{ik}\;L_{ik}-\lambda=0$$

$$m\,{u^{m-1}}_{ik}\;L_{ik}=\lambda$$

$${u^{m-1}}_{ik}=\frac{\lambda}{m\,L_{ik}}$$

$$u_{ik}=\left[\frac{\lambda}{m\,L_{ik}}\right]^{\frac{1}{m-1}}$$

Then

$$u_{ik}=\left[\frac{\lambda}{m}\right]^{\frac{1}{m-1}}\left[\frac{1}{L_{ik}}\right]^{\frac{1}{m-1}} \tag{11}$$

From (9)

$$\sum_{i=1}^{c} u_{lk}=\sum_{l=1}^{c}\left[\frac{\lambda}{m}\right]^{\frac{1}{m-1}}\left[\frac{1}{L_{ik}}\right]^{\frac{1}{m-1}}=1$$

$$\Rightarrow\left[\frac{\lambda}{m}\right]^{\frac{1}{m-1}}=\frac{1}{\sum_{l=1}^{c}\left[\frac{1}{L_{ik}}\right]^{\frac{1}{m-1}}} \tag{12}$$

Substitute (12) in (11). we obtain

$$u_{ik}=\frac{1}{\sum_{l=1}^{c}\left[\frac{1}{L_{ik}}\right]^{\frac{1}{m-1}}}\left[\frac{1}{L_{ik}}\right]^{\frac{1}{m-1}}$$

$$=\frac{1}{\sum_{l=1}^{c}\left[\frac{L_{ik}}{L_{lk}}\right]^{\frac{1}{m-1}}}$$

$$=\left[\sum_{l=1}^{c}\left[\frac{L_{ik}}{L_{lk}}\right]^{\frac{1}{m-1}}\right]^{-1}$$

$$u_{ik}=\left[\sum_{l=1}^{c}\left[\frac{L_{LINEX}\,(x_k,v_i)}{L_{LINEX}\,(x_k,v_l)}\right]^{\frac{1}{m-1}}\right]^{-1} \tag{13}$$

The general equation is used to obtain membership ranks for objects in data for finding meaningful groups.

3.4. Obtaining Cluster Prototype Updating

To find $\min J_{LINEX}\,(U,V)$ it is sufficient to minimize the following inner sum for fixed i:

$$\sum_{k=1}^{n} {u^m}_{ik}\left[\exp\left(\frac{\|x-y\|}{\sigma}\right)-\left(\frac{\|x-y\|}{\sigma}\right)-1\right]$$

Taking the partial derivative of objective function with respect to v_{ij} and setting the result to zero, we have the general form of updating center as

$$\frac{\partial\sum_{k=1}^{n}{u^m}_{ik}\,e^{\frac{\|x_{kj}-v_{ij}\|}{\sigma}}-\frac{1}{\sigma}\sum_{k=1}^{n}{u^m}_{ik}\,x_{kj}+\frac{1}{\sigma}\sum_{i=1}^{n}{u^m}_{ik}\,v_{ij}-\sum_{i=1}^{n}{u^m}_{ik}}{\partial v_{ij}}=0$$

$$\Rightarrow -\frac{1}{\sigma}\sum_{k=1}^{n}{u^m}_{ik}\,e^{\left(\frac{x_{kj}-v_{ij}}{\sigma}\right)}+\frac{1}{\sigma}\sum_{k=1}^{n}{u^m}_{ik}=0$$

$$\Rightarrow -\frac{1}{\sigma}\sum_{k=1}^{n}{u^m}_{ik}\,e^{\left(\frac{x_{kj}-v_{ij}}{\sigma}\right)}=-\frac{1}{\sigma}\sum_{k=1}^{n}{u^m}_{ik}$$

$$\Rightarrow e^{-\frac{v_{ij}}{\sigma}}=\frac{\sum_{k=1}^{n}{u^m}_{ik}}{\sum_{k=1}^{n}{u^m}_{ik}\,e^{\frac{x_{kj}}{\sigma}}}$$

$$\therefore\quad v_{ij}=\sigma\;\log\left[\frac{\sum_{k=1}^{n}{u^m}_{ik}\,e^{\frac{x_{kj}}{\sigma}}}{\sum_{k=1}^{n}{u^m}_{ik}}\right] \tag{14}$$

where ${u_{ik}}^{*}=u_{ik}+\pi_{ik}$, where ${u_{ik}}^{*}$ denotes the intuitionistic fuzzy membership and

$$\pi_{ik}=1-u_{ik}-\left(\frac{1-u_{ik}}{1+\lambda u_{ik}}\right),\lambda>0 \tag{15}$$

$$\pi_i^* = \frac{1}{N}\sum_{k=1}^{N} \pi_{ik} \tag{16}$$

The LINEX measure is suitable for clustering in which it can actually induce the necessary conditions.

This iteration will stop when $\max_{ij}\left\{ \left| u_{ik}^{*k+1} - u_{ik}^{*k} \right| \right\} < \in$

The MST based LINEX_IFCM algorithm iteratively optimizes $J_{LINEX\ IFCM}$ by continuous updating u^*_{ik} and v_{ij} until the difference in successive u^*_{ik} values is very small $\leq \in$, where $\in$ is a small positive value between 0 and 1.

4. Efficient LINEX Measure Induced IFCM Based MST [LINEX_IFCM]

4.1. Efficient LINEX_FCM Algorithm

Stage 1: Set the cluster centroids $\left\{ c_j \right\}_{j=1}^{c}$ by using LINEX measure MST initialization method.

Stage 2: Compute the membership function using (13)

Stage 3: Update the cluster centroids using (14)

Stage 4: Go to stage (3)-(5), repeat until convergence.

Stage 7: Image segmentation after defuzzification and then a region labeling procedure is proposed.

Stage 8: The termination criterion is as follows

$\left| J_m - J_{m-1} \right| \prec \in$, where m is the iteration count,

$\in$ is a small number that can be set by the user.

The proposed efficient LINEX MST obtained cluster centers; the LINEX_IFCM algorithm continues iteratively updates, membership and centroids with these values. When this improved, Efficient LINEX_IFCM algorithm has converged, another defuzzification process takes place in order to convert the fuzzy partition matrix to a crisp partition matrix that is segmented.

4.2. Validation Function Based on Feature Structures

Two representative functions for the fuzzy partition namely; Partition coefficient V_{pc} and Validation function V_p are used to evaluate the validity of clustering [19, 20].

$$V_{pc} = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{c} u^{*\,2}_{ij} \tag{17}$$

$$V_p = \frac{\sum_{i=1}^{c}\sum_{j=1}^{n} u^m_{ij}\, L_{LINEX}\,(x_j, c_i) + \sum_{i=1}^{c} \pi_i^*\, e^{1-\pi_i^*}}{N \times \min\left\{ \left\| c_i - c_j \right\|^2 \right\}} \tag{18}$$

The proposed efficient MST obtained cluster centers; the MHMGIFCM algorithm continues iteratively updates, membership and centroids with these values. When this improved, Efficient MHIFCM algorithm has converged, another defuzzification process takes place in order to convert the fuzzy partition matrix to a crisp partition matrix that is segmented.

5. Results and Discussion

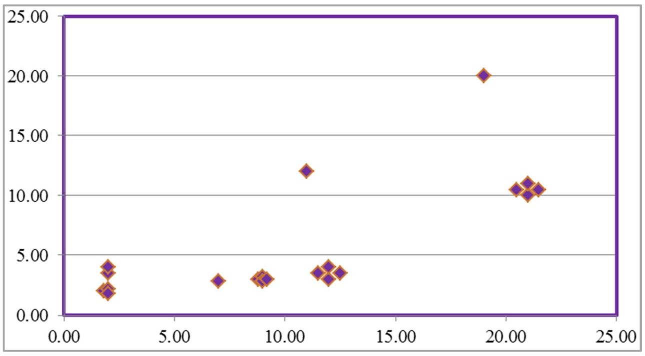

Figure 1. *Scatter diagram for random dataset.*

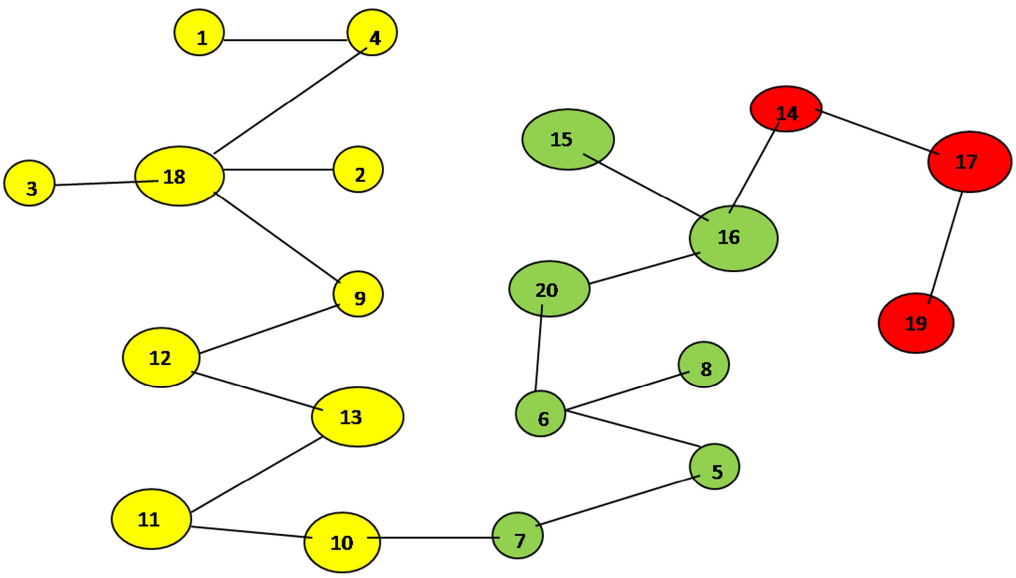

Figure 2. *LINEX distance based Minimal spanning tree connected through points.*

Table 1. *Random Data.*

Data			Intensity	Data			Intensity
S. No	**X**	**Y**	**I(v)**	**S. No**	**X**	**Y**	**I(v)**
1	1.80	2.00	0.50	11	12.00	4.00	0.80
2	2.00	2.20	0.91	12	11.50	3.50	0.45
3	2.00	1.80	0.12	13	12.50	3.50	0.55
4	2.00	3.50	0.40	14	21.00	10.00	0.65
5	8.80	3.00	0.50	15	21.00	11.00	0.25
6	9.00	3.20	0.38	16	20.50	10.50	0.35
7	9.00	2.80	0.60	17	21.50	10.50	0.75
8	9.20	3.00	0.12	18	2.00	4.00	0.70
9	7.00	2.80	0.80	19	19.00	20.00	0.60
10	12.00	3.00	0.90	20	11.00	12.00	0.40

Table 2. *Dissimilarity matrix.*

S.	Co-ordinate		intensity											
No	**x**	**y**	**I(v)**	**vertex**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	1.80	2.00	0.50	1	0.0000	0.5074	0.4270	0.3320	11.2008	12.7079	12.2028	15.3318	5.1123	49.3249
2	2.00	2.20	0.91	2		0.0000	2.5878	1.1597	11.8730	14.6006	11.9487	20.7327	3.9904	40.8562
3	2.00	1.80	0.12	3			0.0000	0.6665	12.0401	12.5096	13.9505	12.5354	7.7815	59.7461
4	2.00	3.50	0.40	4				0.0000	9.9285	10.7263	11.3774	12.7963	5.0748	47.0369
5	8.80	3.00	0.50	5					0.0000	0.0421	0.0312	0.4340	0.6075	1.8983
6	9.00	3.20	0.38	6						0.0000	0.1446	0.1869	1.0616	2.2628
7	9.00	2.80	0.60	7							0.0000	0.7319	0.5136	1.3817
8	9.20	3.00	0.12	8								0.0000	2.5527	4.0437
9	7.00	2.80	0.80	9									0.0000	3.8757
10	12.00	3.00	0.90	10										0.0000

Table 3. *LINEX measure based minimal spanning tree edges.*

S. No	Edges	LINEX measure	S. No	Edges	LINEX measure
1	(1,4)	0.3320	11	(7,5)	0.0312
2	(4,18)	0.2801	12	(5,6)	0.0421
3	(18,2)	0.6026	13	(6,8)	0.1869
4	(18,3)	2.2364	14	(6,20)	53.8331
5	(18,9)	4.3247	15	(20,16)	34.9406
6	(9,12)	3.7182	16	(16,15)	0.0744
7	(12,13)	0.1093	17	(16,14)	0.3060
8	(13,11)	0.2219	18	(14,17)	0.0744
9	(11,10)	0.1491	19	(17,19)	75.3462
10	(10,7)	1.3817			

Our LINEX distance based minimal spanning tree algorithm constructs LINEX_MST from the dissimilarity matrix is shown figure 2.

Adjacency Matrix:

The adjacency matrix of a finite graph G of a 'n' vertices is the $n \times n$ matrix where the non-diagonal entry a_{ij} is the number of edges from vertex 'i' to vertex 'j' and the diagonal entry a_{ii} is either once or twice the number of edges from vertex 'i' to itself.

Table 4. *Adjacency Matrix for minimal spanning tree.*

Adjacency-Matrix																				
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1
7	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0
10	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0

Adjacency-Matrix																				
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
13	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1
17	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0
18	0	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
20	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0

Degree Matrix:

In the mathematical field of graph theory the degree matrix is a diagonal matrix which contains information about the degree of each vertex. That is the count of edges connecting a vertex v. If $i \neq j$ then replace the cell value with 0 otherwise degree of the vertex v_i .

Table 5. *Degree Matrix for minimal spanning tree.*

Degree Matrix																				
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2

Laplacian Matrix:

Given a simple graph G with n vertices, its Laplacian matrix is defined as:

L(i,j)= degree of vertex v_i if $i = j$, if $i \neq j$ and v_i is not adjacent to v_j and in all other case fill it with 0.

Table 6. *Laplacian Matrix for minimal spanning tree.*

Laplacian Adjacency-Matrix																				
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	1	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0
3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0
4	-1	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0
5	0	0	0	0	2	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	-1	3	0	-1	0	0	0	0	0	0	0	0	0	0	0	-1
7	0	0	0	0	-1	0	2	0	0	-1	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	2	0	0	-1	0	0	0	0	0	-1	0	0
10	0	0	0	0	0	0	-1	0	0	2	-1	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	-1	2	0	-1	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	-1	0	0	2	-1	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	-1	-1	2	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	-1	-1	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	3	0	0	0	-1
17	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	2	0	-1	0
18	0	-1	-1	-1	0	0	0	0	-1	0	0	0	0	0	0	0	0	4	0	0

Laplacian Adjacency-Matrix																				
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	0
20	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	-1	0	0	0	2

To Find Eigenvalue and Eigenvector in SPSS 16.0 Syntax:

matrix. /* computing eigenvalues and eigenvectors

get x /variables var00001, var00002, var00003, var00004, var00005, var00006, var00007, var00008, var00009, var00010, var00011, var00012, var00013, var00014, var00015, var00016, var00017, var00018, var00019, var00020. /* this creates a matrix with the n rows and p=20 columns

compute xtx=transpos(x)*x. /* compute x'x which is a symmetric matrix;

note: "transpos" could be shortened to just "t"

print xtx.

call eigen(xtx,eigvec,eigval). /*compute eigenvalues and eigenvectors of x'x

print eigval.

print eigvec.

/* the original matrix x'x can be represented approximately using the "spectral decomposition" of eigenvalues

and eigenvectors

compute approx1=eigval(1)*eigvec(:,1)*t(eigvec(:,1)).

print approx1.

compute approx2=eigval(1)*eigvec(:,1)*t(eigvec(:,1))+eigval(2)*eigvec(:,2)*t(eigvec(:,2)).

print approx2.

/* the approximation with only the largest eigenvalue is not bad, but with both it is perfect

end matrix.

Iteration: 1

Eigenvalue:

Second smallest Eigenvalue $\lambda_2 = 0.00085$

Eigenvector:

v_2 is used to recursively partition the graph by separating the components into negative and positive values.

$$v_2 = \left\{ \overset{-}{1}, \overset{-}{2}, \overset{-}{3}, \overset{-}{4}, \overset{+}{5}, \overset{+}{6}, \overset{+}{7}, \overset{+}{8}, \overset{-}{9}, \overset{-}{10}, \overset{-}{11}, \overset{-}{12}, \overset{-}{13}, \overset{+}{14}, \overset{+}{15}, \overset{+}{16}, \overset{+}{17}, \overset{-}{18}, \overset{+}{19}, \overset{+}{20} \right\}$$

$$\text{Positives} = \left\{ \overset{+}{5}, \overset{+}{6}, \overset{+}{7}, \overset{+}{8}, \overset{+}{14}, \overset{+}{15}, \overset{+}{16}, \overset{+}{17}, \overset{+}{19}, \overset{+}{20} \right\}$$

$$\text{Negatives} = \left\{ \overset{-}{1}, \overset{-}{2}, \overset{-}{3}, \overset{-}{4}, \overset{-}{9}, \overset{-}{10}, \overset{-}{11}, \overset{-}{12}, \overset{-}{13}, \overset{-}{18} \right\}$$

Iteration: 2

Eigenvalue:

Second smallest Eigenvalue $\lambda_2 = 0.0750$

Eigenvector:

v_2 is used to recursively partition the graph by separating the components into negative and positive values.

$$v_2 = \left\{ \overset{-}{5}, \overset{-}{6}, \overset{-}{7}, \overset{-}{8}, \overset{+}{14}, \overset{-}{15}, \overset{-}{16}, \overset{+}{17}, \overset{+}{19}, \overset{-}{20} \right\}$$

$$\text{Positives} = \left\{ \overset{+}{14}, \overset{+}{17}, \overset{+}{19} \right\}$$

$$\text{Negatives} = \left\{ \overset{-}{5}, \overset{-}{6}, \overset{-}{7}, \overset{-}{8}, \overset{-}{15}, \overset{-}{16}, \overset{-}{20} \right\}$$ Finally we get three subtrees $T_1 = \{ 1, 2, 3, 4, 9, 10, 11, 12, 13, 18 \}$, $T_2 = \{ 5, 6, 7, 8, 15, 16, 20 \}$ and $T_3 = \{ 14, 17, 19 \}$ and initial cluster centers $C_1 = (6.48, 3.03, 0.61)$, $C_2 = (12.64, 6.50, 0.37)$ and $C_3 = (20.50, 13.50, 0.67)$

The new efficient MST based IFCM objective function LINEX measure the termination value is achieved, with very less iteration and with much better performance in getting membership (Table 7) than standard IFCM. Table 8 gives the number of iteration to achieve the results of cluster on the data points by standard IFCM and LINEX IFCM. It is clear from the final cluster, membership (Table 7), scatter diagram (Figure 1), that our proposed MST based LINEX IFCM is much faster than the standard FCM and the method is converged fast to terminate condition with less run time. To test the effectiveness of LINEX_IFCM, then MST initialization method based IFCM is used as center. This is done to find out the fuzzy membership and appropriate number of clusters. Thus, we have concluded the final optimal clusters formed as 3. This algorithm has also reduced the number of iterations. Best result is achieved by this measure fuzzy partition coefficient V_{pc} maximum and validation function V_p minimum (Table 9). The LINEX IFCM clustering algorithm has the following membership value intimacy (Table 7),

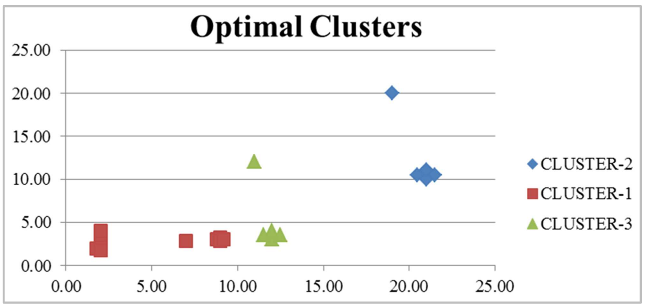

Figure 2. *Spectral initialization method based Intuitionistic FCM, final cluster three.*

Table 7. *Final membership of three clusters of Intuitionistic FCM method and object allocation.*

Co-ordinate (x,y)			**intensity**				**appropriate cluster**
S. No	**x**	**y**	**I(v)**	**Mem-1**	**Mem-2**	**Mem-3**	
1	1.80	2.00	0.50	0.9624	0.0369	0.0007	1
2	2.00	2.20	0.91	0.9607	0.0386	0.0007	1
3	2.00	1.80	0.12	0.9511	0.0479	0.0010	1
4	2.00	3.50	0.40	0.9505	0.0485	0.0009	1
5	8.80	3.00	0.50	0.8587	0.1392	0.0020	1
6	9.00	3.20	0.38	0.7944	0.2026	0.0031	1
7	9.00	2.80	0.60	0.8513	0.1466	0.0021	1
8	9.20	3.00	0.12	0.7331	0.2615	0.0054	1
9	7.00	2.80	0.80	0.9877	0.0121	0.0002	1
10	12.00	3.00	0.90	0.3227	0.6639	0.0135	2
11	12.00	4.00	0.80	0.1668	0.8228	0.0104	2
12	11.50	3.50	0.45	0.3028	0.6860	0.0111	2
13	12.50	3.50	0.55	0.2046	0.7798	0.0156	2
14	21.00	10.00	0.65	0.0006	0.0097	0.9897	3
15	21.00	11.00	0.25	0.0001	0.0020	0.9979	3
16	20.50	10.50	0.35	0.0001	0.0025	0.9974	3
17	21.50	10.50	0.75	0.0005	0.0088	0.9906	3
18	2.00	4.00	0.70	0.9464	0.0526	0.0010	1
19	19.00	20.00	0.60	0.0048	0.0641	0.9311	3
20	11.00	12.00	0.40	0.0687	0.7512	0.1801	2

Table 8. *Comparison of iteration count.*

	No. of iterations	**No. of clusters**
FCM	10	3
KFCM	5	3
MST initialization method based LINEX Intuitionistic FCM	2	3

Table 9. *Cluster validity function.*

	V_{pc}	V_p
FCM	0.8020	0.2750
KFCM	0.8058	0.2660
MST based LINEX Intuitionistic FCM	0.8103	0.2568

6. Conclusion

This paper studies the intuitionistic fuzzy C-means clustering algorithm. Several important parameters during the IFCM clustering process, such as the initial form of the partition matrix, the number of classification and the threshold of terminating the iteration, which significantly affect the clustering results, are analyzed and discussed. The proposed a novel integration of Fiedler value for MST based LINEX_IFCM to search for the optimal parameters to improve the performance. The algorithm overcomes problems involved with membership values of objects to each cluster by generalizing degrees of membership of objects to each cluster. For the initial partition matrix, if the membership degrees and non-membership degrees of the classified objects to the categories are obviously different, the iteration times will be reduced accordingly. When the number of classification increases, the clustering results change, and some objects are separated from the original category. Compared with FCM clustering algorithm, Fiedler value MST initialization method based LINEX_IFCM algorithm can classify objects more accurately. This study can help to understand and master the factors that affect the intuitionistic fuzzy clustering results, and help to promote the further research and application of LINEX_IFCM clustering method.

Acknowledgements

We would like to thank the reviewers for their constructive comments. We thank to Dr. R. David Chandrakumar, Professor, Department of Mathematics, Vickram College of Engineering for his encouragement and support given. I also thank Dr. A. Sathya, Assistant Professor, Department of Mathematics, Government College of Daman, Daman & Diu for her timely help throughout this work.

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