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Connectedness Generalizations Using the Concept of Adherence Dominators

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Abstract: An *adherence dominator* on a topological space X is a function π from the collection of filterbases on X to the collection of closed subsets of X satisfying $\mathcal{A}\Omega \subset \pi\Omega$ where $\mathcal{A}\Omega$ is the adherence of Ω and $\pi\Omega = \bigcap_{\Omega} \pi F = \bigcap_{\sum F, F\in\Omega} \pi V$, where $\sum F$ represents the collection of open sets containing F. The π-adherence may be adherence, θ-adherence, u-adherence s-adherence, f-adherence, δ-adherence, etc., of a filterbase. Pervin deined a partition (or a separation) of a set in a topological space as a pair of subsets (P, Q) satisfying $P \cap clQ = clP \cap Q = \emptyset$, where clP represents the closure of P and a set K is said to be connected if $K = \emptyset$ or $K \neq P \cup Q$ where (P, Q) is a partition. In this paper, a *πpartition (or a πseparation)* is a pair of subsets (P, Q) satisfying $P \cap \pi Q = \pi P \cap Q = \emptyset$ where π is an adherence dominator and a subset K of a space X is πconnected relative to X If $K = \emptyset$ or there is no πpartition (P, Q) such that $K = P \cup Q$. This paper investigates these new forms of connectedness. Theorems due to A. D. Wallace and J. D. Kline are generalized. Geralizations of C-compact spaces and functionally compact spaces are also presented.

Keywords: Flters, Adherence Dominators, Connectedness, πseparation, πclosed

1. Introduction

An *adherence dominator on a topological space* is a function π from the set of filterbases to the set of closed subsets satisfying $\mathcal{A}\Omega \subset \pi\Omega$, where $\mathcal{A}\Omega$ is the adherence of Ω and and $\pi\Omega = \bigcap_{\Omega} \pi F = \bigcap_{\sum F, F\in\Omega} \pi V$, where $\sum F$ represents the collection of open sets containing the set F. If π is an adherence dominator, a nonempty subset B in a space X is *πclosed* if $\pi B = B$ and a space to be a πspace if $\{x\}$ is πclosed for every $x \in X$. A partition is defined in a topological space as a pair of subsets (P, Q) satisfying $P \cap clQ = clP \cap Q = \emptyset$ where clP represents the closure of P and K is said to be connected if $K = \emptyset$ or $K \neq P \cup Q$ where (P, Q) is a partition [19]. For a space X and $P \subseteq X$, if $P \neq \emptyset, clP \subseteq \pi P$.

The π-adherence may be adherence, θ-adherence, u-adherence s-adherence, f-adherence δ-adherence, etc., of a filterbase [22, 6, 7, 10, 20, 9, 11, 8, 12, 21]. These closure operators are not individually stated here. Please refer to the appropriate references for the definitions of each of these closure operators.

This article is divided into six sections. In thise section for introduction, concepts which motivated the study of connectedness via an adherence dominator are presented; next section on πconnectedness, πconnected sets and their properties are given. In the third section, generalizations of continuity are presented. The fourth section gives some generalizations of compactness through adherence dominator and in the fifth section components relative to a space are presented through adherence dominator. Sixth section remarks on conclusion of tthis study.

Following result is stated without proof as it can be easily verified.

Theorem 1 A T_1 space X is a πspace if and only if $\pi \sum\{x\} = \{x\}$ for every $x \in X$, where $\Sigma\{x\}$ represents the collection of open sets containing x and $\pi \sum\{x\} = \bigcap_{\sum\{x\}} \pi V$.

2. πConnectedness

The concept of a πconnected subset is defined in the model of the connected subset, using πclosure, instead of closure of a set.

Definition 1 A subset K is called *πconnected relative to X* if $K=\emptyset$ or $K\neq P\cup Q, P\cap\pi Q=\pi P\cap Q=\emptyset$, where π is an adherence dominator on X, that is to say *$K=\emptyset$ or K has no πpartition (or πseparation).*

It is clear that connected subsets are πconnected relative to the space. The following two examples illustrate the following: They are given here to show that (1) πconnected relative to a space need not be connected and (2) that these two classes may coincide in spaces which are not regular, using $\pi\Omega=ad_\theta\Omega$, where $ad_\theta\Omega$ represents the θ-adherence of Ω.

Example 1 [2]. Let I be the unit interval $[0,1], Y=I\times\{0\}, X=I\times I$ with the topology generated by the following base for the open sets: (1) the relative open sets from the plane in $X-Y$ (2) and for $x\in Y$, sets of the form $(V\cap(X-Y))\cup\{x\}$ where V is open in the plane with $x\in V$. It is clear that Y is discrete in the relative topology from X and hence is not connected. Suppose $\pi=cl_\theta$, the θ-closure operator, and that (P,Q) a πseparation relative to X, and $Y=P\cup Q$. Choose $(r,0)\in P$ and without loss of generality, assume that there is an $s\in I$ with $r<s$ and $(s,0)\in Q$. Let

$$c=sup\{r\in I:r<s \text{ and } (r,0)\in P\}.$$

We see easily that $(c,0)\in\pi P$. Hence $(c,0)\in P$. Since it is readily seen that $(c,0)\in\pi Q$, a contradiction is reached and Y is πconnectcd relative to $X(\pi=cl_\theta)$.

Example 2 [2]. Let E be the set of even positive integrs, O the set of odd positive integers, and let $X=\{0\}\cup E\cup O\cup\{j+n^{-1}: j,n\in(E\cup O)-\{1\}\}$ be endowed with the topology from the following open set base: (1) the relative open sets from the reals in $X-\{0,1\}$, (2) all subsets of the form $\{0\}\cup\{j+n^{-1}:n\in E, j\geq j_0\}$. where $j_0>1$ and (3) all subset of the form $\{1\}\cup\{j+n^{-1}:n\in O-\{1\}, j\geq j_0\}$. where $j_0>1$. It is not difficult to see that the collection of πconnected subsets and the collection of connected subsets are both the collection of singletons ($\pi=cl_\theta$).

The addherence operator π subsumes the adherence of a filterbase based on different closure operators. As we have seen above, a πconnected relative to the space need not be connected. However, note that if for a space X, the collection of closed sets coincide with the collection of πclosed sets, then πconnected sets will be connected. Therefore, investigating on additional condition(s) which will make sets which are πconnected relative to the space to be connected, one notice that neither θ-closure operator nor u-closure operator are Kuratowki closure operators [13]. In the cited paper, it was shown that if the topology on the space agree with the semiregularization topology T_s, that is, the topology generated by the regularly open subsets of the space, where the collection of closed sets is the collection of regularly closed sets, then $cl_\theta(cl_\theta A)=cl_\theta A$ and $cl_u(cl_u A)=cl_u A$. (A set is regularly open (regularly closed), if it is the inerior of a closed (closure of an open) set. Complement of a regularly open set is regularly closed. Also, for any space X, if T_k denotes the topology on X generated by the θ-closures of regularly closed subsets and T_u denotes the topology generated by u-closed subsets, then $T_k=T_u$ [13]. So, when π-closure is a Kuratowski operator for a space X, if T_π dnotes the the topology generated by the π-closed subsets of X, then for (X,T_π), the class of πconnected relative to X subsets will coincide with the class of connected subsets of X. Note that the space in Example 2 is semiregular and as is observed, in that the collection of connected sets coincide with the collection of πconnecccted sets, where $\pi=cl_\theta$.

The following concept and results are used in the sequal and hence are included in this section

Definition 2 A set $A\subseteq X$ is said to be *πrigid*, if each filter base Ω on X satisfying the property that $F\cap\pi V\neq\emptyset$, for all $F\in\Omega$ and $V\in\Sigma A$ also satisfies that $\pi\Omega\cap A\neq\emptyset$ [4].

The above definition of a πrigid set subsumes the concepts of θ-rigid, $u$rigid, $s$rigid sets when the adherence dominator π is the θ-closure or u-closure or s-closure operator respectively [4].

It is to be noted that in a π space, $\pi\{x\}=\{x\}$ for each $x\in X$ and for any set $A\subseteq X, \bigcup_A\pi\{x\}\subseteq\pi A$. It can be shown that $\pi A=\bigcup_A\pi\{x\}$ for any πrigid $A\subset X$.

Theorem 2 If A is πrigid, then $\pi A=\bigcup_A\pi\{x\}$.

Proof Let A be πrigid and let $x\in\pi A$. Then the constant net $\{x\}$ is frequently in πV for all $V\in\Sigma A$. Hence there is a $y\in A$ such that $\{x\}$ is frequently in πV for all $V\in\Sigma\{y\}$. So, $x\in\pi\{y\}$. Therefore, $\pi A\subseteq\bigcup_A\pi\{x\}$ and hence $\pi A=\bigcup_A\pi\{x\}$.

The following cahracterization of πrigid subsets is used later in this article.

Theorem 3 A subset K of a space X is *πrigid* if and only if for every cover Ω of K with open subsets of X there is a finite $\Omega^*\subset\Omega, K\subset int(\bigcup_{\Omega^*}\pi V)$.

Proof Suppose that A is πrigid and Ω is an open cover of A with subsets of X. Suppose that for any finite subfamily Ω^*, of Ω, $A\not\subseteq int(\bigcup_{\Omega^*}\pi F)$. Then $A\cap(X-int(\pi F))=A\cap cl(X-\pi F)\neq\emptyset$, for each $F\in\Omega$. The family $\mathcal{F}=\{A\cap cl(X-\pi F), F\in\Omega\}$ is a filterbase on X and $(X-\pi F)\cap\pi V\neq\emptyset$ for $V\in\Sigma(A)$. Hence $\pi\mathcal{F}\cap A\neq\emptyset$. So, $(\cap(X-F))\cap A=(X-\cup_\Omega F)\cap A\neq\emptyset$, a contradiction.

Conversely, suppose that the open cover condition holds and Ω is a filterbase on X with $\pi\Omega\cap A=\emptyset$. Then there is an $F\in\Omega$ such that $\pi F\cap A=\emptyset$ which imples that $\pi V\cap F=\emptyset$ for some $V\in\Sigma(A), F\in\Omega$. Hence A is πrigid.

Definition 3. Let (X,τ) be a topological space and let $\tau\subset\tau^*$. Then τ^* will be called a *simple extension* of τ if there exists an $A\notin\tau$ such that $\tau^*=\{U\cup(V\cap A)$, with $U,V\in\tau\}$ and $\tau^*=\tau(A)$, a simple extension of τ through A [18].

Theorem 4 The following statements are equivalent for a T_1space X:

1. The space X is a πspace;
2. Each πrigid subset of X is πclosed;
3. Each compact subset of X is πclosed;

4. Each continuous function on X to a πspace maps compact sets onto πclosed sets;
5. Each continuous bijection onto X maps compact subsets onto πclosed sets.

Proof (1) $\Rightarrow$ (2). Let A be πrigid. Then $\pi A = \bigcup_A \pi(\{x\}) = \bigcup_A \{x\} = A$, since X is a πspace. So, A is πclosed.

(2)$\Rightarrow$ (3). A compact subset is πrigid.

(3) $\Rightarrow$ (4) $\Rightarrow$ (5). Obvious.

(5) $\Rightarrow$ (1). Let $x \in X$ and let X^* be X with the simple extension of X through the set $\{x\}$[18]. The identity function $g : X^* \to X$ is a continuous bijection onto X and $\{x\}$ is compact in X^*. So $\pi g(x) = \pi\{x\} = \{x\}$.

Corollary 1 The following statements are equivalent for a space X:

1. The space X is a Hausdorff (Urysohn) [regular] space;
2. Each θ-rigid ($u$rigid)[$s$rigid] subset of X is θclosed ($u$closed) [$s$closed];
3. Each compact subset of X is θclosed ($u$closed) [$s$closed];
4. Each continuous function on X to a Hausdorff (Urysohn) [regular] space maps compact sets onto θclosed ($u$closed) [$s$closed] sets;
5. Each continuous bijection onto X maps compact subsets onto θclosed ($u$closed) [$s$closed] sets.

The proof of the following result is easy and follows in the same line of proof as for a connected set A.

Theorem 5 The following statements are equivalent for $A \subset X$:

(1) A is πconnected relative to X;
(2) For each two points $x, y \in A$, there is a πconnected relative to X set $B \subset A$ with x, y in B;
(3) If $(P, Q))$ is a πseparation relative to X and $A \subset P \cup Q$ then either $A \subset P$ or $A \subset Q$.

Theorem 6 The following statements hold for spaces X, Y :

(1) Connected subsets of a space X are πconnected relative to X;
(2) A pair V, W of disjoint open subsets of X is a πseparation relative to X;
(3) If A is πconnected relative to X and $A \subset B \subset \pi A$, then B is πconnected relative to X;
(4) If Ω is a family of subsets πconnected relative to X no pair of which is a πseperation relative to X then $\bigcup_\Omega F$ is πconnected;
(5) If Ω is a family of subsets πconnected relative to X and there is an $F_0 \neq \emptyset, F_0 \in \Omega, (F_0, F)$ fails to be a a πseperation relative to X for each $F \in \Omega$, then $\bigcup_\Omega F$ is πconnected. relative to X;
(6) If F_n is a sequence of πconnected relative to X subsets and (F_n, F_{n+1}) fails to be a πseparation relative to X for all n then $\bigcup_{n \in N} F_n$ is πconnected relative to X;
(7) If A, B are πconnected relative to X and Y respectively, then $A \times B$ is πconnected relative to $X \times Y$;
(8) If A is πconnected relative to X and and $X_0 \subset X$ satisfies $A \cap intX_0 \neq \emptyset, A \cap int(X - X_0) \neq \emptyset$ then $A \cap bdX_0 \neq \emptyset$;
(9) If Ω is a family of πconnected relative to X subsets and $\bigcap_\Omega F \neq \emptyset$ then $\bigcup_\Omega F$ is πconnected.

Proof Only proofs of (3) and (7) are given here.

Proof of (3). Let (P, Q) be a πseparation relative to X and suppose $B \subset P \cup Q$. Therefore $A \subset P$ or $A \subset Q$. So $B \subset \pi Q$ or $B \subset \pi P$.

Proof of (7): Let $(x, y), (u, v) \in A \times B$. Then $(x, v) \in (A \times \{v\}) \cup (\{x\} \times B)$ where both $(A \times \{v\})$ and$(\{x\} \times B)$ are πconnected relative to $X \times Y$ since πconnectivity is a topological invariant, as is shown in Theorem 11. It follows that

$$(x, v) \in (A \times \{v\}) \cap \{x\} \times B)$$

and

$$(x, y), (u, v) \in (A \times \{v\}) \cap (\{x\} \times B) \subset A \times B.$$

Hence, in view of Theorem 5 (2), the proof is complete.

Theorem 7 If $\{X(n)\}_\Lambda$ is a family of spaces and $\{A(n)\}_\Lambda$ is a family of sets such that $A(n)$ is πconnnected relative to $X(n)$ for each $n \in \Lambda$ then $\prod_\Lambda A(n)$ is πconnected relative to $\prod_\Lambda X(n)$

Proof Let (P, Q) be a πseparation relative to $\prod_\Lambda X(n)$ such that

$$\prod_\Lambda A(n) \subset P \cup Q.$$

Let $x \in \prod_\Lambda A(n)$, and for each $n \in \Lambda$, Let $P_n : \prod_\Lambda X(n) \to X(n) \cap P_n(P)$ be the projection intersected with the projection of P. There is a basic open set $V = \bigcap_{\Lambda(x)} p_n^{-1} W(n)$ with $x \in V$, $\Lambda(x)$ a finite subset of Λ, and $Q \cap \pi V = \emptyset$. Choose $y \in \prod_\Lambda A(n)$ and let

$$J(y) = \{z \in \prod_\Lambda A(n) : z(n) = y(n), n \in \Lambda - \Lambda\{x\}\}.$$

$J(y)$ is homeomorphic to $\prod_{\Lambda(x)} A(n)$ and Theorem 6(7) may be used with induction to show that $\prod_{\Lambda(x)} A(n)$ is πconnected relative to $\prod_\Lambda X(n)$. This implies $J(y) \subset P$ since we see z defined by $z(n) = x(n)$ when $n \in \Lambda\{x\}, z(n) = y(n)$ when $n \in \Lambda - \Lambda(x)$, it follows that $z \in J(y) \cap V \subset P$.

The following theorem is a generalization of a result of J. R. Kline remarking on a paper of Knaster and Kuratowski [16], [17].

Definition 4. A set X is *totally πdisconnected* if the singletons are the only non-empty π conncted subsets of X.

Theorem 8 If X is πconnected relative to X and for $y \in X$, $X - \{y\}$ is totally πdisconnected then $X - \{x\}$ is πconnected relative to X for every $x \neq y, x \in X$.

Proof For every $A \subset X - \{y\}$, with more than one point, there is a πseparation (P, Q) of A. Let $x \in X - \{y\}$. The set $X - \{y\}$ has at least two elements x, z. If there is a πseparation (P, Q) of $\{x, z\}$, for each $z \notin \{x, y\}$, we get a π separation for the π conncted set X, a contrtadiction. Hence $X - \{x\}$ is πconnected relative to X for every $x \neq y$.

3. Genralizations of Continuity Through Adherence Dominators

Below, some generalizations of continuous functions are provided in terms of adherence dominator. One of the well-known charcterizations of continuity of a function $f : X \to Y$ is that $f(clA) \subseteq clf(A)$ for each subset A of X. It is this characterization which is used as a model to define the notion of a πcontinuous function.

Definition 5. A function $f : X \to Y$ is *πcontinuous* if $f(\pi\Omega) \subset \pi f(\Omega)$ for every filterbase Ω. A function $f : X \to Y$ is *weakly-πcontinuous* if $f(\mathcal{A}\Omega) \subset \pi f(\Omega)$ for every filter Ω. A function $f : X \to Y$ is *$\theta\pi$continuous* if $f(\theta\Omega) \subset \pi f(\Omega)$ for every filter Ω, where $\theta\Omega$ is used here for the θ adherence of Ω and $\mathcal{A}\Omega$ denotes the adherence of Ω.

Theorem 9 The following are equivalent:

1. The function $f : X \to Y$ is πcontinuous;
2. The function $f : X \to Y$ satisfies $f(\pi A) \subset \pi f(A)$ for each nonempty $A \subset X$;
3. The function $f : X \to Y$ satisiies $\pi f^{-1}(B) \subset f^{-1}(\pi B)$ for each nonempty $B \subset Y$;
4. The function $f : X \to Y$ satisiies $\pi f^{-1}(\Omega) \subset f^{-1}(\pi\Omega)$ for each filterbase Ω on Y;
5. The function $f : X \to Y$ satisfies $\bigcap_{\sum A} \pi f^{-1}(V) \subset f^{-1}(\pi A)$ for each $A \subset X$.

Proof (1) $\Leftrightarrow$ (2). For (1)$\Rightarrow$(2), consider the filterbase $\{A\}$, for $A \subseteq X$. For (2)$\Rightarrow$(1), note that for each $F \in \Omega, f(\pi F) \subseteq \pi f(F)$ and hence (1) follows.

(2) $\Leftrightarrow$ (3). Assume (2). Let $B \subset Y$. Then $f(\pi f^{-1}(B)) \subset \pi f(f^{-1}(B)) \subset \pi B$. So, $\pi f^{-1}(B) \subset f^{-1}(\pi B)$.

Now, suppose we have (3). let $A \subset X$. Clearly, $A \subseteq f^{-1}(f(A))$. So, $\pi A \subset \pi f^{-1}(f(A)) \subset f^{-1}(\pi f(A))$. Hence, $f(\pi A) \subset \pi f(A)$.

Proofs of (3) $\Rightarrow$(4) $\Rightarrow$(5) $\Rightarrow$(3) are clear.

Corollary 2 If X, Y are spaces and π is an adherence dominator on X, a function $f : X \to Y$ is πcontinuous if and only if $\pi f^{-1}(W) \subset f^{-1}(\pi W)$ for each open W.

Proof The function $f : X \to Y$ satisiies $\pi f^{-1}(B) \subset f^{-1}(\pi B)$ for each $B \subset Y$.

For weakly πcontinuous ($\theta\pi$ continuous) functions, we have the following:

Theorem 10 The following are equivalent for spaces X, Y and function $f : X \to Y$.

1. $f : X \to Y$ is weakly πcontinuous ($\theta\pi$ continuous);
2. $clf^{-1}(B) \subset f^{-1}(\pi B)(cl_\theta f^{-1}(B) \subset f^{-1}(\pi B)$ for each non-empty $B \subset Y$;
3. $f(cl(A) \subset \pi f(A)(f(cl_\theta(A) \subset \pi f(A))$ for every $A \subset X$;
4. The function $f : X \to Y$ satisiies $\mathcal{A}f^{-1}(\Omega) \subset f^{-1}(\pi\Omega)(ad_\theta f^{-1}(\Omega) \subset f^{-1}(\pi\Omega))$ for each filterbase Ω on Y;
5. The function $f : X \to Y$ satisfies $\bigcap_{\sum A} clf^{-1}(V) \subset f^{-1}(\pi A)$
   $(\bigcap_{\sum A} cl_\theta f^{-1}(V) \subset f^{-1}(\pi A))$ for each $A \subset X$.

Proof Proof is similar to the proof of Theorem 9.

Theorem 11 If $f : X \to Y$ is a πcontinuous function and K is πconnected relative to X, then $f(K)$ is πconneted relative to Y.

Proof Let (P, Q) be a πpartition such that $f(K) \subset P \cup Q, P \cap \pi Q = \emptyset, Q \cap \pi P = \emptyset$. Then $K \subset f^{-1}(P) \cup f^{-1}(Q)$. However, f being πcontinuous, $\pi f^{-1}(P) \cap f^{-1}(Q) \subset f^{-1}(\pi P) \cap f^{-1}(Q) = f^{-1}(Q \cap \pi P) = \emptyset$. Similarly, $\pi f^{-1}(Q) \cap f^{-1}(P) = \emptyset$. That is, $f^{-1}(P)$ and $f^{-1}(Q)$ are π separated. Thus, K being πconnected, $K \subset f^{-1}(P)$ or $K \subset f^{-1}(Q)$ according to Theorem 5(3). So, $f(K)$ is πconnected.

4. Some Generalizations of Compactness

Before introducing the generalizations compactness through adherence dominators, the following results of filter convergence, which will be used in the sequel, are provided. The concepts of functionaly compact spaces and C-compact spaces are some of the generalizations of compactness considered here. See [1], [23].

Theorem 12 If $\mathcal{F}$ is a filter on a πspace X and $\mathcal{O} = \{V \text{open in X} : F \subset V \text{for some } F \in \mathcal{F}\}$, then $\mathcal{O}$ is an open filter and $\pi\mathcal{F} = \pi\mathcal{O}$.

Proof $\mathcal{O}$ is an open filter and $\mathcal{O} \subseteq \mathcal{F}$. So, $\pi\mathcal{F} \subset \pi\mathcal{O}$. For the reverse inclusion, if $x \notin \pi\mathcal{F}$, then $x \notin \pi F$ for some $F \in \mathcal{F}$. Therefore, there exist $F \in \mathcal{F}$and $V \in \sum\{x\}$ such that $\pi V \cap F = \emptyset$. Therefore, $F \subset X - \pi V$. Hence, $X - \pi V \in \mathcal{O}$, and $\pi V \cap (X - \pi V) = \emptyset$. So $x \notin \pi\mathcal{O}$.

Corollary 3 If $\mathcal{F}$ is an ultrafilter on a πspace X and $\mathcal{O} = \{V \text{open in X} : F \subset V \text{for some } F \in \mathcal{F}\}$, then $\mathcal{O}$ is an open filter and $\pi\mathcal{F} = \pi\mathcal{O}$. [3]

Proof Note that $\mathcal{O} \subset \mathcal{F}$.

Definition 6. If $\mathcal{O}$ is an open filter, then $\pi\mathcal{O} = \bigcup_{\mathcal{O}} \pi\{x\}$.

Corollary 4. If $\mathcal{F}$ is a fillter on a πspace X, then $\pi\mathcal{F} = \bigcup_{\mathcal{F}} \pi\{x\}$.

Proof If $\mathcal{O}$ is an open filter, then $\pi\mathcal{O} = \bigcup_{\mathcal{O}} \pi\{x\}$. Theorem 12 then completes the proof.

Definition 7. If π is an adherence dominator on a πspace X, then X is πclosed if $\pi\Omega \neq \emptyset$ for each filterbase Ω on X.

The following result is immediate.

Theorem 13 If X is πclosed and Ω is a filterbase on X, then $\pi\Omega$ is quasi πclosed relative to X, where a set X is said to be quasi πclosed if $\pi\Omega \neq \emptyset$ for each filterbase Ω on X, but X is not necessarily a πspace.

Corollary 5. If X is an H-closed (Urysohn-closed)[regular-closed] space and Ω is a filterbase on X then $adh_\theta\Omega(adh_u\Omega)[adh_s\Omega]$ is quasi H-closed (quasi Urysohn-closed) [quasi regular-closed] relative to X. [2]

Definition 8. If π is an adherence dominator on a space X, then X is *rim πclosed* if each $x \in X$ has a base of open sets with πclosed boundaries.

Theorem 14 If X is a space, A is a nonempty compact subset of X and Ω is a filter on X such that $A \cap F \neq \emptyset$ for every $F \in \Omega$, there exists $x \in A$ such that $F \cap V \neq \emptyset, F \in \Omega,\ V \in \sum\{x\}$.

Proof Suppose that for each $x \in A, F_x \cap V_x = \emptyset, F_x \in \Omega, V_x \in \sum\{x\}$. Then, there exists finite $B \subset A$ such that

$A \subset \bigcup_B V_x = V, F = \bigcap_B F_x \in \Omega, V \in \sum A, F \cap V = \emptyset$. Therefore, there exists $x \in A$ such that $F \cap V \neq \emptyset, F \in \Omega,\ V \in \sum\{x\}$.

Theorem 15 If X is a πspace, A is a nonempty compact subset of X and $\mathcal{U}$ is an ultrafilter on X such that $A \cap F \neq \emptyset$ for every $F \in \mathcal{U}$ there exists $x \in A$ such that $\mathcal{U} \to x$.

Proof Follows easily from Theorem 14, for an ultrafilter $\mathcal{U}$ on X.

Theorem 16 The following are equivalent for a πspace X and adherence dominator π on X.

1. X is compact;
2. Each nonempty closed subset of X is πclosed, and $\pi\mathcal{U}$ is compact for each ultrafilter $\mathcal{U}$ on X;
3. The boundary bdV is πclosed for each open, non-empty proper subset of X and $\pi\mathcal{U}$ is compact for each ultrafilter $\mathcal{U}$ on X;
4. X is rim πclosed, and $\pi\mathcal{U}$ is compact for each ultrafilter $\mathcal{U}$ on X.

Proof (1)⇒(2)⇒(3)⇒(4). Immediate.

(4)⇒(1). Let $\mathcal{U}$ be an ultrafilter on X and let $V \in \sum\pi\{x\}, x \in \pi\mathcal{U}$. Then $F - V = \emptyset$ for some $F \in \mathcal{U}$. If not, $V, bdV \in \mathcal{U}$, a contradiction. So $\mathcal{U} \to \pi\{x\}$. Since $\pi\{x\}$ is a closed, and therefore compact subset of $\pi\mathcal{U}$, there exists $z \in \pi\{x\}, \mathcal{U} \to z$, in view of Theorems 14 and 15.

Recall that a Hausdorf space X is *C-compact (functionally compact)[1]* if each closed set is an H-set (if each open filterbase Ω on X satisfying $\mathcal{A}\Omega = I\Omega$ is an open set base for $I\Omega$) [23], [1]. Below we define and charecterize these concepts using adherence dominators.

Definition 9. If X is a space, a non-empty set $A \subset X$ is a *πset* if each filterbase Ω on A satisfies $\pi\Omega \cap A \neq \emptyset$.

Definition 10. A space X is *πC-compact* if every non-empty closed set $A \subset X$ is a πset.

Definition 11. A filterbase $\mathcal{F}$ is said to *πconverge to x (to a set A)*, if for each $V \in \Sigma x\ \ (\Sigma A)$, there is an $F \in \mathcal{F}$ such that $F \subseteq \pi V$.

Theorem 17 If X is a πclosed space and $\emptyset \neq A \subset X$, then πA is a πset.

Proof Let Δ be a filterbase on πA. Since for any filterbase Ω on X, $\pi\Omega \cap \pi A \neq \emptyset$, and Δ is a filterbase on X, $adh\Delta \cap \pi A \neq \emptyset$.

Proposition 1. Let X be a space and $A \subset X$ be πclosed. If (P, Q) is a πseparation of A relative to X, then P and Q are πclosed in X.

Proof Let $A = P \cup Q$, with $P \cap \pi Q = \pi P \cap Q = \emptyset$. Since A is πclosed, $\pi A = \pi P \cup \pi Q = A = P \cup Q$. Since $P \cap \pi Q = \pi P \cap Q = \emptyset$, this implies that $\pi P \subseteq P$ and $\pi Q \subseteq Q$.

The following Theorem gives characterizations of πC-compact spaces in terms of ultrafilters.

Theorem 18 The following are equivalent for a πspace X :

1. X is πC-compact;
2. For each closed subset $B \subset X$ each ultrafilter $\mathcal{U}, B \in \mathcal{U}, \mathcal{U}$ πconverges to some point of B;
3. $\bigcap_{\mathcal{B}} \pi B$, where $\mathcal{B}$ is the collection of closed sets in $\mathcal{U}$, is a singleton for each ultrafilter $\mathcal{U}$.

Proof (1) ⇒(2). Follows from the fact that in a πC-compact space, every non-empty closed set is a πset. Hence the ultrafilter $\mathcal{U}$ has non-empty πadherence in B and therefore, $\mathcal{U}$ π converges to some point of B.

(2) ⇒ (3). Let $\mathcal{U}$ be an ultrafilter on X and let $\mathcal{B}$ be the collection of closed sets in $\mathcal{U}$. Suppose that $\{x, y\} \subseteq \bigcap_{\mathcal{B}} \pi B$, with $x \neq y$. Then, in view of (2) and since πB being closed, $\mathcal{U} \to x$ and $\mathcal{U} \to y$, a contradiction. Hence (3).

(3) ⇒(1). Let A be a closed set and let Ω be a filterbase on A. let $\mathcal{U}$ be an ultrafiler containing Ω. In view of (3), there is a singlton set $\{x\} = \bigcap_{\mathcal{B}} \pi B$, where $\mathcal{B}$ is the collection of closed sets in $\mathcal{U}$. Since A is closed and Ω is a filterbase on A, $x \in A$. Hence A is a π set.

Corollary 6. The following are equivalent for a Hausdorff space X :

1. X is C-compact;
2. For each closed subset $B \subset X$ and each ultrafilter $\mathcal{U}$ with $B \in \mathcal{U}, \mathcal{U}$ θconverges to some point of B;
3. $\bigcap_{\mathcal{B}} cl_\theta B$, where $\mathcal{B}$ is the collection of closed sets in $\mathcal{U}$, is a singleton for each ultrafilter $\mathcal{U}$.

Theorem 19 In a πC-compact space, for any two disjoint πclosed sets P and Q, there are open sets V and W such that $P \subset V, Q \subset W, \pi V \cap \pi W = \emptyset$.

Proof Let P and Q be two disjoint π closed sets of the πC-compact space X. Since each π closed set is closed, P and Q are π sets. Suppose that for each $V \in \Sigma P$ and $W \in \Sigma Q$, with $\pi V \cap \pi W \neq \emptyset$. Then $\Omega = \{\pi V \cap \pi W, V \in \Sigma P$ and $W \in \Sigma Q\}$ is a filterbase in πP as well as in πQ and has non-empty adherence in πP and in πQ, a contradiction since $\pi P \cap \pi Q = \emptyset$.

Corollary 7. In a C-compact space, any two disjoint θ-closed sets are separated by disjoint open subsets.

Theorem 20 Let X be a πC-compact space and let Ω be a filterbase of πconnected relative to X subsets. Then $\pi\Omega$ is πconnected relative to X if $\pi\Omega$ is πclosed in X.

Proof Let (P, Q) be a π separation relative to X of $\pi\Omega$. Then P and Q are disjoint and πclosed in X from the Proposition 1. Since X is πC-compact, there are sets $V \in \sum P, W \in \sum Q, V \cap \pi W = \pi V \cap W = \emptyset$. Note that $\pi\Omega$ is πclosed and hence is closed in the C-compact space X. Also $\pi\Omega \subseteq V \cup W$. So there is an $F \in \Omega, F \subset V \cup W$. Since F is πconnected relative to X and (V, W) is a π separation relative to $X, F \subset V$ or $F \subset W$. Hence $\pi F \subset \pi V$ or $\pi F \subset \pi W$. So $\pi\Omega \subset \pi V$ or $\pi\Omega \subset \pi W$. Therefore, $\pi\Omega \cap P = \emptyset$ or $\pi\Omega \cap Q = \emptyset$, a contradiction.

Corollary 8. Let X be a C-compact space and let Ω be a filterbase of θconnected relative to X subsets. Then $ad_\theta\Omega$ is θconnected relative to X if $ad_\theta\Omega$ is θclosed in X. [2]

Arguments similar those in the proof of the last theorem lead to the following.

Theorem 21 Let X be a πC-compact space and let Ω be a filterbase of connected subsets. Then $\pi\Omega$ is connected if open and closed subsets are πclosed in X.

Definition 12. A πspace X is *πfunctionally compact* if each open filterbase Ω of the space satisfying the contion $\pi\Omega = I\Omega$ is an open set base for $I\Omega$, where $I\Omega$ represents the intersection of members of Ω.

Theorem 22 The following are equivalent for a πspace X:

1. The space X is πfunctionally compact;
2. Each open filterbase Ω on X satisfying $\pi\Omega = I\Omega$ converges to Ω;
3. Each filterbase Ω on X satisfying $\pi\Omega = I\Omega$ converges to $I\Omega$.

Proof (1)$\Rightarrow$ (2). Follows from the definition of πfunctionaly compact spaces. (2) $\Rightarrow$ (3). Note that $\pi\Omega = \mathcal{A}_\Omega(\Sigma\pi F)$.

(3) $\Rightarrow$(1). Follows easily.

Corollary 9. A Hausdorff space X is functionally compact if and only if each filterbase Ω on X with $ad_\theta\Omega = I\Omega$ converges to $I\Omega$. (Theorem 3.12 [13])

When the π adherence represents $u$adherence, we have the following result.

Corollary 10 [13] The following are equivalent for a Urysohn space X:

1. X is $u$functionally compact;
2. Each open filterbase Ω on X with $ad_u\Omega = I\Omega$ converges to $I\Omega$;
3. Each filterbase Ω on X satisfying $ad_u\Omega = I\Omega$ converges to $I\Omega$.

The Wallace Theorem states that if A and B are compact sets in X and Y respectively and $A \times B \subset H, H$ open in $X \times Y$, there exists V open in X, W open in Y such that $A \subset V, B \subset W, V \times W \subset H$. (see Theorem 12 p. 142 [15])

The following is a restatement of the Wallace Theorem in terms of closures.

Theorem 23 Let X, Y be spaces and A, B be compact subsets of X, Y respectively. Let $K \subset X \times Y$ satisfy $(A \times B) \cap clK = \emptyset$. Then there are $V \in \sum A, W \in \sum B$ satisfying $(V \times W) \cap clK = \emptyset$.

The next theorems generalize Wallace's theorem to πrigid subsets.

Theorem 24 Let X, Y be spaces and let $B \subset Y$ be πrigid. Let $x \in X, K \subset X \times Y$ satisfy $(\{x\} \times B) \cap \pi K = \emptyset$. Then there are open sets $V \in \sum\{x\}, W \in \sum B$ such that $(V \times W) \cap \pi K = \emptyset$.

Proof For each $y \in B$, there are open sets $V(y) \in \sum\{x\}, W(y) \in \sum(y), (V(y) \times W(y)) \cap \pi K = \emptyset$. There is a finite $B^* \subset B$ with

$$B \subset int\left(\bigcup_{B^*}\pi W(y)\right) = W,$$

in view of Theorem 3. Let $V = \bigcap_{B^*} V(y)$. Then $V \in \sum\{x\}, W \in \sum B$. Therefore $(V \times W) \cap \pi K = \emptyset$.

Theorem 25 Let X, Y be spaces and A, B be πrigid subsets of X, Y respectively. Let $K \subset X \times Y$ satisfy $(A \times B) \cap \pi K = \emptyset$. Then there are $V \in \Lambda A, W \in \Lambda B$ satisfying $(V \times W) \cap \pi K = \emptyset$, where $\Lambda A = \sum \pi A$.

Proof From the prof of the last theorem, Theorem 24, for each $x \in X$ there exists $V(x) \in \sum(x), W(x) \in \Lambda(B)$ satisfying $(V(x) \times W(x)) \cap \pi K = \emptyset$. There is a finite $A^* \subset A$ with $A \subset int\pi\left(\bigcup_{A^*} V(x)\right) = V$. Let $W = \bigcap_{A^*} W(x)$, Then V, W have the required properties.

Definition 13. A relation (or a multifunction) λ from X to Y is a function $\lambda : X \to 2^Y - \{\emptyset\}$. A relation λ from a space X to a space Y is *upper semicontinuous (u.s.c.)* if for every $W \in \sum\lambda(x)$ there is a $V \in \sum(x)$ such that $\lambda(V) \subset W$. A multifunction λ from a space X to a space Y has a π-strongly closed graph if $\pi\lambda(\sum(x)) = \lambda(x)$ for each $x \in X$.

Theorem 26 If λ is an u.s.c. multifunction on X and π is an adherence dominator then $\pi\lambda\Sigma\{x\} = \pi\lambda\{x\}$.

Proof Clearly $\pi\lambda\{x\} \subset \pi\lambda(\Sigma\{x\})$ and for each $W \in \sum\lambda(x)$, some $V \in \sum\{x\}$ satisfies $\lambda(V) \subset W$, since λ is u.s.c. and thus $\pi\lambda\Sigma\{x\} \subset \pi\lambda\{x\}$.

Corollary 11. An u.s.c. multifunction λ has a π strongly-closed graph if and only if λ has πclosed point images.

It is established for θ-closures that *if x and y are points in a space X, then $y \in cl_\theta(x)$ if and only if $x \in cl_\theta(y)$ [5],* The following definition is modelled after this property of θ-closures.

Definition 14. We say that *x is equvalent to y* if πx=πy and use the notation $x \equiv y$. For the adherence dominators being considered, $x \in \pi y$ if and only if $y \in \pi x$. If $v \in X$, a πspace, then $\pi\bigcap_{\sum\{v\}} W = \{v\}$.

In a πspace the πclosure of each point is trivially compact and maximal in the set of πclosures of points ordered by inclusion. We use the last theorem to prove that in any space, the πclosures of points satisfy a maximally condition, when the πclosure of some point is compact.

Theorem 27 Let Y be a space and let $y_0 \in Y$ with πy_0 compact. Then there is a $y \in Y$ such that (1) $\pi y_0 \subset \pi y$ and (2) πy is maximal in the set of π closures of points when this set is ordered by inclusion.

Proof Let $X = \{y \in Y : \pi y_0 \subset \pi y\}$. For each $y \in X$ we have $y \in \pi y_0$. Moreover if $v \in \mathcal{A}X$ and $W \in \sum\{v\}$ then some $y \in W$ satisfies $\pi y_0 \subset \pi y \subset \pi W$. Hence $\pi y_0 \subset \pi v$. So, $v \in X$ and X is closed in Y. Moreover, for each $y \in X, y \in \pi y_0$ since $\pi y_0 \subseteq \pi y$. That is, $y_0 \in \bigcap_X \pi y$. Hence if $y \in X, y \in \pi y_0$ and πy_0 is compact. Therefore X is a compact subset of Y. Since the identity function from X to Y is u.s.c., the proof is complete, in view of Theorem 11 (1)$\Rightarrow$ (2) [4].

5. πComponents Relative to a Space

A πcomponent relative to a space X is a πconnected relative to X subset which is not properly contained in any πconnected relative to X subset. Since πA is πconnected relative to X whenever A is πconnected relative to X, it follows that a π component relative to X is πclosed. It is easy to see that each πconnected relative to a space X is contained in a πcomponent relative to X. It is not difficult to see that if H, K are two distinct πcomponents relative to a space X then either $H \cap \pi K = \emptyset$ or $K \cap \pi H = \emptyset$. If $\emptyset \neq A \subset X$ and if $x, y \in A$, we say that x and y are equivalent if whenever (P, Q) is a πseparation of A relative to $X, \{x, y\} \subset P$ or $\{x, y\} \subset Q$. If Ax represents the equivalence class of x with respect to this relation on A, call Ax a πquasicomponent of A relative to X. A πquasicomponent of A relative to A is a quasicomponent of A. If $A \subset X$ and $x \in A$ let $\pi\mathcal{S}(A, x) = \{P \subset X :$ for some $Q \subset$

$X, (P, Q)$ is a πseparation of$A$ relative to X, satisfying $A = P \cup Q$, and$x \in P\}$. The following proposition of Ax follows easily.

Proposition 2. If X is a space and $\emptyset \neq A \subset X$, then for each $x \in A, Ax = \bigcap_{\mathcal{S}(A,x)} P$.

Proposition 3. If X is a space and $\emptyset \neq B \subset A \subset X$, then $Bx \subset Ax$ for each $x \in B$.

Proof Let $x \in B, y \in Bx, (P, Q)$ a πseraration of A relative to X. If $P \cap B = \emptyset$ or $(Q \cap B) = \emptyset)$, then $\{x, y\} \subset Q$ or $(\{x, y\} \subset P)$. Otherwise, $(P \cap B, Q \cap B)$ is a πseraration of B relative to X. So $\{x, y\} \subset Q \cap B$ (or $\{x, y\} \subset P \cap B$). Therefore, $y \in Ax$.

The following improves the well-known fact that quasicomponents are closed.

Proposition 4. If X is a space and $\emptyset \neq A \subset X$, then Ax is πclosed in A for each $x \in A$.

Proof Let $x \in A, y \in A - Ax$. Then there is a πpartition (P, Q) of A relative to $X, y \in P, Ax \subset Q$. Hence $y \notin \pi Ax$ in X. It foffows that $y \notin \pi Ax$ in A

Corollary 12. In a πclosed space X, πquasi components are πrigid.

Proof Follows from the fact that in a πspace, πclosed sets are πrigid.

Theorem 28 If X is a space and A is a non-empty subset of X, then Ax is a πcomponent of A relative to X for each $x \in A$ for which Ax is πconnected relative to X.

Proof Suppose that $x \in A$ and Ax is πconnected relative to X. Let $B \subseteq A$ be a πconnected relative to X and suppose that $Ax \subset B$. Let $y \in B$ and $A = P \cup Q$, where (P, Q) is a π separation relative to X for A. Then $B \subset P$ or $B \subset Q$. So, either $\{x, y\} \subset P$ or $\{x, y\} \subset Q$. Hence $y \in Ax$. Hence Ax is a π component of relative to X, since, thus, Ax πconnected relative to X.

Corollary 13. πconnected πquasicomponents are πcomponents.

We conclude this article with the following result which states that in a πC-compact space X, each πquasicomponent relative to X of a πclosed set A is a πcomponent relative to X of A.

Theorem 29 Let X be πC-compact and let $A \subset X$ be a non-empty πclosed set. Then each πquasicomponent Ax of A relative to X is a πcomponent of A relative to X.

Proof Let $A \subset X$ be non-empty and πclosed and suppose that $x \in A$. We shall show that Ax is πconnected relative to X and then in view of Theorem 28, Ax will be a πcomponent of A relative to X. Let $y \in Ax$ and let $\Omega = \{B \subset A$ such that B is πclosed in $X, x \in B, y \in Bx\}$. Then $\Omega \neq \emptyset$ since $A \in \Omega$. Order Ω by set inclusion and let Ω^* be a chain in Ω. Let $C = \bigcap_{\Omega^*} F$. Since each $F \in \Omega^*$ is πclosed and $x \in F$ for each $F \in \Omega^*, x \in C$. Let (P, Q) be a π separation relative to X of C where both P and Q are πclosed in X and $x \in P, y \in Q$. Since X is πC-compact and P and Q are two disjoint πclosed sets, in view of Theorem 19, there are disjoint open sets V and W containing P and Q respecively with $\pi V \cap \pi W = \emptyset$. Since $C \subset V \cup W$, there is an $F_0 \in \Omega^*$ such that $F_0 \subset (V \cup W)$. This gives a πseparation for F_0 as $F_0 = (F_0 \cap V) \cup (F_0 \cap W), x \in (F_0 \cap V), y \in (F_0 \cap W)$. This is a contradiction since $F_0 \in \Omega$. Therefore, C is a lowerbound for Ω and hence by Zorn's Lemma, has a minimal element, say C_0. To show that C_0 is πconnected relative to X, suppose that (P, Q) is a πseparation relative to X with $C_0 = P \cup Q$. Assume that $P \neq C_0$ and that $x, y \in P$. Since P is πclosed in X, there exist P_1 and P_2 such that (P_1, P_2) is a πseparation for P relative to X, with $x \in P_1$ and $y \in P_2$. This gives a π separation relative to X for C_0 with $C_0 = P_1 \cup (P_2 \cup Q)$, and $x \in P_1, y \in (P_2 \cup Q)$. This is a contradiction since $C_0 \in \Omega$. Thus C_0 is πconnected relative to X and $C_0 = Cx$ and $\{x, y\} \subset C_0 \subset Ax$. Therefore, inview of Theorem 5 (2), Ax is πconnected relative to X. Hence in view of Theorem 28, Ax is a πcomponent relative to X.

6. Conclusion

Connectedness of a set in a topological space depends on how close are the points of that set to each other and hence depends on the closure operator of that space. The adherence dominator concepts subsumes different closure operators. So, this article brings the study of different topological concepts such as continuity, convergence, compactness and their generalizations in a unified frame work. Recently, the authors have studied and compiled several topological properties using the adherence dominator operator [14].

Every attempt is made to give the appropriate citation whenever a result or concept is used from a source. Connectedness, compactness, maximality conditions using the Zorn's lemma, continuous function, multifunctions, regularly open sets, regularly closed sets, semiregularization etc are concepts and results which could be found and have been in literature on general topology and related topics and hence when these are stated, no particular citation is given. However we do not claim authorship for them.

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