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Solving Triangular Fuzzy Transportation Problem Using Modified Fractional Knapsack Problem

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Abstract: A transportation problem (TP) is a specific part of a linear programming problem that arises in a collection of contexts and has received much attention in the literature. Minimizing transportation costs or time (one objective) is one of the primary goals of transportation problem-solving approaches. Supply, demand, and unit transportation costs may be uncertain in real-life applications due to many factors, such as multiple objectives. The goal of this paper is to look at the fuzzy transportation problem (FTP), which is crucial in TP with multiple objectives. In the literature, numerous techniques for dealing with FTPs are proposed. The cost, supply, and demand values of the FTPs are taken as symmetric triangular fuzzy numbers and then converted into crisp values using ranking techniques to solve the FTP. The initial solution is then obtained by Vogel's approximation method (VAM), and the optimal solution is obtained by the modified distribution method (MODI). The proposed method is based on the Modified Fractional Knapsack Problem and introduces a new approach to solving the triangular fuzzy transportation problem. This paper analyses an alternative method using the fractional knapsack problem, which was modified using a minimum ratio test. To express the efficiency of the proposed method, it is compared with existing methods in the literature.

Keywords: Fractional Knapsack Problem, Fuzzy Transportation Problem, Harmonic Mean, Initial Feasible Solution, Minimum Ratio Test, Triangular Fuzzy Numbers

1. Introduction

Transportation problems (TP) are one of the earliest and most notable implementations of linear programming problems (LPP). TP is an aspect of linear programming that deals with daily effectiveness in our lives. Like public life, a nominated quantity of homogeneous items is available in several provenances, and a certain amount is required to satisfy the demand in each of the consumption venues. Transportation models are concerned with the best way for a product from many plants or factories (supply assets) to be delivered to many warehouses or clients (demand destinations). The aim is to find an optimal solution for shipping the commodity while satisfying the demands of each destination. Hitchcock F. L. [1] initially presented the TP in 1941. T. C. Koopmans [2] offered his research to solve (TP) in 1947. The two research listed above are the most significant advances in various strategies for solving the transportation model. The TP can also be solved by describing it as an LP model using the simplex technique developed by Dantzig G. B. in 1951 [3], but it has a plurality of variables and constraints, and solving them using the simplex method needs a significant amount of effort and time. Much research has expanded replacement approaches to finding an IBFS. An IBFS for the TP can obtain using one of three traditional techniques: the northwest corner method (NWCM), the minimal cost method (MCM), or Vogel's approximation approach (VAM). According to the studies, Vogel's Approximation Method

(VAM) is the best of the three ways. In one of two approaches, we can improve the outcomes of the first solution to acquire the optimal solution to the transportation problem, either the Modified Distribution Method (MODI) or the Stepping Stone Method. Essentially, these approaches differ only in terms of the best solutions at the start, and a good solution primitively will produce a lower objective value.

The key traits of each transportation problem are the unit costs (profits) and the values of supply and demand (production and storage capacity). However, due to a lack of trustworthy information on the parameters, which are not always precisely known and steady due to these and other variables, such as fuel price inflation, weather fluctuations, and natural disasters, these parameters might not always be available. These could have an impact on an organization's management capability flexibility. Fuzzy numbers [4] are a common way to communicate imprecision. Bellman and Zadeh [5] established fuzzy decision-making in 1970. Chanas and Kucht [6] developed a method for handling fuzzy transport problems in 1996.

Gani and Razak [7] (2006) provided the perfect answer to the transportation problem using fuzzy cost coefficients. In 2011 Kaur A. and Kumar A. [8] proposed a novel solution to the fuzzy transportation problem. A novel method for addressing transportation problems with uncertain prices is goal programming. Furthermore, research by Bisht & Srivastava (2019) [9] and Srivastava et al. (2020) [10] has demonstrated that the fuzzy transportation problem (FTP) operates at its peak level. Dinagar and Palanivel (2009) [11] investigated both of these aspects by ranking fuzzy numbers using a maximizing set and a minimizing set. In an unpredictable environment, Pandian and Natarajan (2010) [12] looked into the transportation problem. Ahmed, Khan, Uddin, Md., and Ahmed (2016) [13] computed an improved fuzzy optimum Hungarian assignment problem with fuzzy costs using robust ranking algorithms.

The new approach uses the knapsack problem that was developed to solve the triangular Fuzzy Transportation Problem (FTP). Knapsack problems can be found in real-world decision-making processes in a wide range of fields, including determining the least wasteful way to cut raw materials selecting investments and portfolios, selecting assets for asset-backed securitization, and other knapsack cryptosystems.

Knapsack algorithms [14] were created to design and scoring of examinations that allowed test-takers to choose which questions to answer early on. Early research on the knapsack problem date back to 1897 and the subject has been studied for nearly a century. In the early writings of mathematician Tobias Dantzig, the phrase "knapsack problem" (referring to the everyday challenge of packing the most valuable or useful items without overloading the luggage) first appeared. It describes the challenge of carrying useable items without overloading the luggage. George Dantzig suggested a method to tackle the unbounded knapsack problem known as a greedy approximation [15]. In his interpretation, the items are arranged according to diminishing value per unit of approach. The essential concept underlying all families of knapsack problems is the selection of various items, with profit and weight values, to be packed into one or more knapsacks with capacity. The fractional knapsack problem and the 0/1 knapsack problem are two variations of the knapsack problem.

In this study, the proposed method was based on the fractional knapsack problem using the greedy method since it is an efficient method to solve it, where the items are sorted according to their ratio of value to weight. In a fractional knapsack, it can break objects to maximize the knapsack's total value. But instead of using the classical fractional knapsack problem, the method was modified using the minimum ration test, commonly used in the simplex method, and so on. In addition, numerical examples are given to illustrate the methodology.

2. Preliminaries

2.1. Definition Interval

When an interval is defined on real number $\mathfrak{R}$, this interval is said to be a subset of $\mathfrak{R}$. For instance, if the interval is denoted as A = $[a_1, a_3]$, $a_1, a_3 \in \mathfrak{R}$, $a_1 < a_3$, we may regard this as one kind of set. Expressing the interval as a membership function is shown in the following (Figure 1): [16].

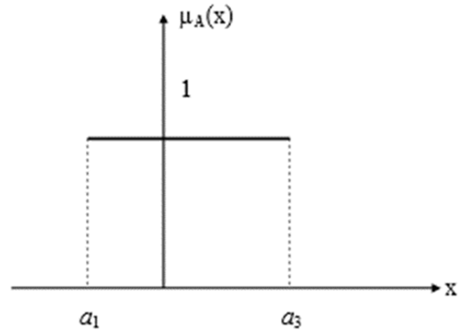

Figure 1. *Interval A = $[a_1, a_3]$, [16].*

2.2. Definition Fuzzy Number

A Fuzzy number is expressed as a fuzzy set defining a fuzzy interval in the real number $\mathfrak{R}$. Since the boundary of this interval is ambiguous, the interval is also a fuzzy set. Generally, a fuzzy interval is represented by two endpoints a_1,a_3, and a peak point a_2 as $[a_1, a_2, a_3]$ (Figure 2). The α-cut operation can be also applied to the fuzzy number. If we denote the α-cut interval for fuzzy number A as Aα, the obtained interval A_α is defined as, [16].

$$A_\alpha = [a_1{}^\alpha \ a_3{}^\alpha]$$

We can also know that it is an ordinary crisp interval (Figure 3).

It is a fuzzy set of the following conditions:

1. convex fuzzy set
2. normalized fuzzy set
3. its membership function is piecewise continuous.

4. It is defined as a real number.

Fuzzy numbers should be normalized and convex. Here the condition of normalization implies that the maximum membership value is 1.

$$\exists x \in \Re,\ \mu A(x) = 1$$

The convex condition is that the line by α-cut is continuous and the α-cut interval satisfies the following relation.

$$A\alpha = [a_1^{\alpha}\ a_3^{\alpha}]$$

$$(\alpha' < \alpha) \Rightarrow (a_1^{\alpha'} \leq a_1^{\alpha},\ a_3^{\alpha'} \geq a_3^{\alpha})$$

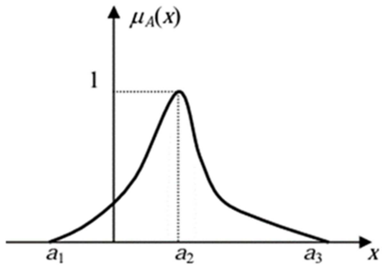

Figure 2. *Fuzzy Number A = [a₁, a₂, a₃] [16].*

The convex condition may also be written as,
(α′< α) => ($A_\alpha \subset A_{\alpha'}$)

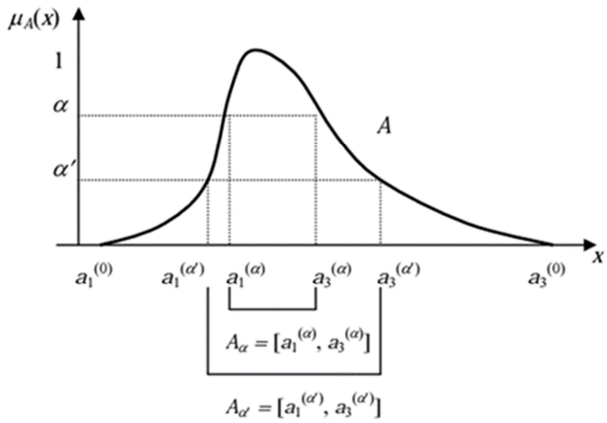

Figure 3. *α – cut of Fuzzy Number (α′< α) => ($A_\alpha \subset A_{\alpha'}$) [16].*

2.3. Definition Triangular Fuzzy Number

Among the various shapes of fuzzy numbers, triangular fuzzy number (TFN) is the most popular one. [16]

It is a fuzzy number represented by three points as follows:

$$A = (a_1,\ a_2,\ a_3)$$

This representation is interpreted as membership functions,

$$\mu_{(A)}(x)\begin{cases} 0\ x < a_1 \\ \frac{x-a_1}{a_2-a_1}, a_1 < x < a_2 \\ \frac{a_3-x}{a_3-a_2}\ a_2 < x < a_3 \\ 0\ x > a_3 \end{cases}$$

Now if you get a crisp interval by α-cut operation, interval Aa shall be obtained as follows ∀α ∈ [0, 1]. From,

$$\frac{a_1^{(\alpha)}-a_1}{a_2-a_1} = \alpha, \frac{a_3-a_3^{(\alpha)}}{a_3-a_2} = \alpha$$

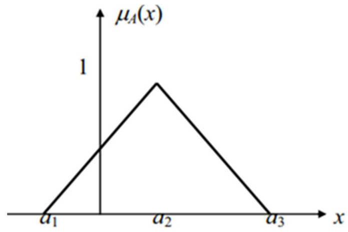

Figure 4. *Triangular fuzzy number A= [a₁, a₂, a₃] [16].*

We get,

$$a_1^{(\alpha)} = (a_2 - a_1)\alpha + a_1$$

$$a_3^{(\alpha)} = -(a_3 - a_2)\alpha + a_3$$

Thus,

$$A_\alpha = [a_1^{\alpha}\, a_3^{\alpha}] = [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3]$$

2.4. Definition Harmonic Mean

If the sequence is in harmonic mean, the harmonic mean (HM) is computed by using the formula.

$$HM = \frac{n}{\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\ldots+\frac{1}{a_n}}$$

2.5. Fractional Knapsack Problem

The common problem being solved is the Fractional Knapsack Problem, which restricts the number x_i of copies of each kind of item to zero or one. Given a set of items numbered from 1 up to n, each with a weight w_i and a value v_i, along with a maximum weight capacity W,

$$\text{Maximize } \sum_{i=1}^{n} v_i x_i$$

subject to

$$\sum_{i=1}^{n} w_i x_i \leq W \text{ and } x_i \in Z, x_i \geq 0$$

2.6. Definition Mathematical Formulation of the FTP

The mathematical formulation of the FTP is as follows.

$$Min\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}\, X_{ij}$$

$$\text{Subject to } \sum_{j=1}^{n} X_{ij} = S_i\ for\ i = 1,2,3 \ldots .n$$

$$\sum_{i=1}^{m} X_{ij} = D_j\ for\ j = 1,2,3 \ldots .m$$

$$where\ X_{ij} \geq 0\ for\ all\ i,j$$

A transportation problem is considered a balanced problem if the total supply is equal to the total demand as shown.

$$\sum_{i=1}^{m} S_i = \sum_{j-1}^{n} D_j$$

3. Research Methodology

The transportation costs and values of the supplies and demands are represented by non-negative triangular fuzzy numbers and covert FTP into crisp values using harmonic mean.

Step 1: Formulate the fuzzy transportation cost matrix.

Step 2: Compute the harmonic mean table using the harmonic mean definition. If the problem is unbalanced, make it a balanced problem by introducing a dummy source or a dummy demand row or column accordingly.

Step 3: Find the maximum of either one of the demand or supply cells. Then allocate the particular max $(\tilde{s_i}, \tilde{d_j})$ to the cell corresponding to the minimum unit cell in a peculiar row or column.

Step 4: If the demand in the column or supply in the row is satisfied, remove the column or row and move to the next maximum max $(\tilde{s_i}, \tilde{d_j})$ value.

Step 5: If the termination condition is satisfied $(\tilde{s_i} = \tilde{d_j} = 0)$, afterward go to Step 6. Otherwise, go to Step 3.

Step 6: Stop and determine a solution.

4. Numerical Illustration

This session provides a numerical illustration of the Proposed Method compared to Yager's ranking method, Naresh Kumar and S. Kumara Ghuru's ranking method, and the Uthra ranking method.

Step 1: Formulate the fuzzy transportation cost matrix.

Table 1. *Detailed data representations of the problem [17].*

	$\widetilde{D_1}$	$\widetilde{D_2}$	$\widetilde{D_3}$	$\widetilde{D_4}$	Supply
$\widetilde{S_1}$	(2,3,3)	(2,3,3)	(2,3,3)	(1,4,4)	(0,3,3)
$\widetilde{S_2}$	(4,9,9)	(4,8,8)	(2,5,5)	(1,4,4)	(2,13,13)
$\widetilde{S_3}$	(2,7,7)	(0,5,5)	(0,5,5)	(4,8,8)	(2,8,8)
Demand	(1,4,4)	(0,9,9)	(1,4,4)	(2,7,7)	

Using the harmonic mean, convert the crispy value and compute the harmonic mean table.

Table 2. *Harmonic mean table.*

	$\widetilde{D_1}$	$\widetilde{D_2}$	$\widetilde{D_3}$	$\widetilde{D_4}$	Supply
$\widetilde{S_1}$	2.57	2.57	2.57	2	4.50
$\widetilde{S_2}$	6.35	6	3.33	2	4.59
$\widetilde{S_3}$	3.82	7.5	7.5	6	4
Demand	2	13.51	2	3.82	

Since the problem is unbalanced, make the matrix a balanced problem by introducing a dummy source.

Table 3. *Harmonic mean table with a dummy row.*

	$\widetilde{D_1}$	$\widetilde{D_2}$	$\widetilde{D_3}$	$\widetilde{D_4}$	Supply
$\widetilde{S_1}$	2.57	2.57	2.57	2	4.50
$\widetilde{S_2}$	6.35	6	3.33	2	4.59
$\widetilde{S_3}$	3.82	7.5	7.5	6	4
Dummy row	0	0	0	0	8.24
Demand	2	13.51	2	3.82	21.32

Step 2: Select the 1st row of the cost matrix with the row of demand and Supply at a particular source. here Supply cell is the sack size. Then in Step 3: Divide ith row by row of demand and using the minimum ratio test allocate unit cost to the sell by following the fractional knapsack method and write down x_i values.

Table 4. *Allocation of the third step.*

	$\widetilde{D_1}$	$\widetilde{D_2}$	$\widetilde{D_3}$	$\widetilde{D_4}$	Supply
$\widetilde{S_1}$	2.57	2.57	2.57	2	4.50
Demand	2	13.51	2	3.82	
$\widetilde{S_1}/\widetilde{D_\iota}$	1.285	0.190	1.285	0.523	
x	[0	4.50/13.5	0	0]	

Sack size = 4.50

Unit Cost= 4.50*2.57=11.565

Step 4: Select another row of the cost matrix with the row of demand and Supply at a particular source. In this step and so on allocation of the unit cost must subtract by the particular demand cell. Then follow Step 5: Repeat Step 3 until all the demand cells are equal to zero. And Step 6: Stop and Calculate the solution.

Table 5. *Allocation of the fourth step.*

	$\widetilde{D_1}$	$\widetilde{D_2}$	$\widetilde{D_3}$	$\widetilde{D_4}$	Supply
$\widetilde{S_2}$	6.35	6	3.33	2	4.59
Demand	2	9.01	2	3.82	
$\widetilde{S_2}/\widetilde{D_\iota}$	3.175	0.666	1.665	0.524	
x	[0	0.77/9.01	0	3.82]	

Sack Size= 4.59

Unit Cost=0.77*6+3.82*2=12.26

Table 6. *Allocation of the fourth step.*

	$\widetilde{D_1}$	$\widetilde{D_2}$	$\widetilde{D_3}$	$\widetilde{D_4}$	Supply
$\widetilde{S_3}$	3.82	7.5	7.5	6	4
Demand	2	8.24	2	0	
$\widetilde{S_3}/\widetilde{D_\iota}$	1.910	0.91	3.75	0	
x	[0	4/8.24	0	0]	

Sack Size= 4

Unit Cost=7.5*4=30

Table 7. *Allocation of the fourth step.*

	$\widetilde{D_1}$	$\widetilde{D_2}$	$\widetilde{D_3}$	$\widetilde{D_4}$	Supply
Dummy row	0	0	0	0	8.24
Demand	2	4.24	2	0	
$\widetilde{S_4}/\widetilde{D_\iota}$	0	0	0	0	
x	[1	1	1	0]	

Sack Size= 8.24

Unit Cost=0*2+0*4.24+0*2=0

Step 6: determine a solution

Initial Feasible Solution=11.565+12.26+30=53.825

The following table compares Yager's ranking approach, Naresh Kumar & S. Kumara Ghuru's ranking method, and the Uthra ranking method to demonstrate the effectiveness of the proposed method tested in this study.

Table 8. *Performance measure over the proposed method with Yager's ranking method, Naresh Kumar & S. Kumara Ghuru's ranking method, and the Uthra ranking method.*

Methods	Initial Basic Feasible Solution
Yager's Ranking method	72.5625
S. Naresh Kumar & S. Kumara Ghuru Ranking method	84.33
Uthra Ranking method	60.664
Proposed method	53.825
Optimal Solution	41

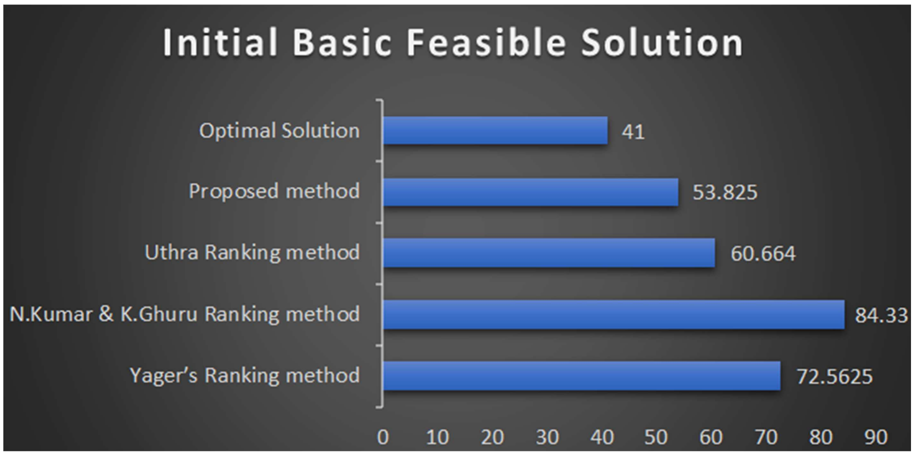

Figure 5. *Comparative Study of the Result obtained by existent approaches.*

Table 8 and Figure 5 shows that compared to Yager's ranking approach, Naresh Kumar and S. Kumara Ghuru's ranking method, and the Uthra ranking method, the proposed method offers the best IFS.

In this session, the proposed method is contrasted with existing methods.

5. Results and Discussion

5.1. Comparative Study of the Proposed Method over the Ramesh Kumar Method, VAM, NWCM, and LCM

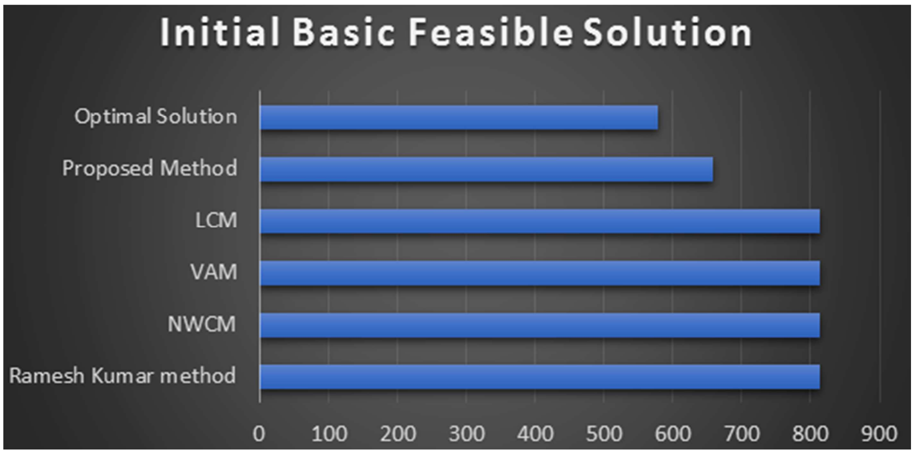

Figure 6. *Comparative Study of the result obtained by Ramesh Kumar Method, NWCM, VAM, LCM, and the proposed method.*

Table 9. *Detailed data representations of the problem. [18].*

	$\widetilde{D_1}$	$\widetilde{D_2}$	$\widetilde{D_3}$	$\widetilde{D_4}$	Supply
$\widetilde{S_1}$	(5,10,15)	(5,10,20)	(5,15,20)	(5,10,15)	(10,15,20)
$\widetilde{S_2}$	(5,10,20)	(5,15,20)	(5,10,15)	(10,15,20)	(5,10,15)
$\widetilde{S_3}$	(5,10,20)	(10,15,20)	(10,15,20)	(5,10,15)	(20,30,40)
$\widetilde{S_4}$	(10,15,25)	(5,10,15)	(10,20,30)	(10,15,25)	(15,20,25)
Demand	[25,30,35]	[10,15,20]	[5,15,20]	[10,15,25]	

Table 10 shows the performance ratio of the proposed method compared to the Ramesh Kumar method, NWCM, VAM, and LCM.

Table 10. *Performance measure over the proposed method with Ramesh Kumar Method, NWCM, VAM, and LCM.*

Methods	Initial Basic Feasible Solution
Ramesh Kumar method	812.49
NWCM	812.49
VAM	812.49
LCM	812.49
Proposed Method	658.3112
Optimal Solution	577.77

According to the results in Table 10 and Figure 6, the proposed method bested Ramesh Kumar's method, NWCM, VAM, and LCM in terms of results.

5.2. Comparative Study of the Proposed Method over the Deepa Method

Table 11. *Detailed data representations of the problem. [19].*

	$\widetilde{D}_1$	$\widetilde{D}_2$	$\widetilde{D}_3$	$\widetilde{D}_4$	Supply
$\widetilde{S}_1$	(3,6,9)	(3,6,12)	(3,9,12)	(3,6,9)	(6,9,12)
$\widetilde{S}_2$	(3,6,12)	(3,9,12)	(3,6,9)	(6,9,12)	(3,6,9)
$\widetilde{S}_3$	(3,6,12)	(6,9,12)	(6,9,12)	(3,6,9)	(12,18,24)
$\widetilde{S}_4$	(6,9,15)	(3,6,9)	(6,12,18)	(6,9,15)	(9,12,15)
demand	(15,18,21)	(6,9,12)	(3,9,12)	(6,9,15)	

To investigate the efficiency of the Proposed method with the Deepa method, the results are summarized in Table 12 and Figure 7.

Table 12. *Performance measure over the proposed method with the Deepa method.*

Methods	Initial Basic Feasible Solution
Deepa Method	371.8125
Proposed Method	236.93
Optimal Solution	181

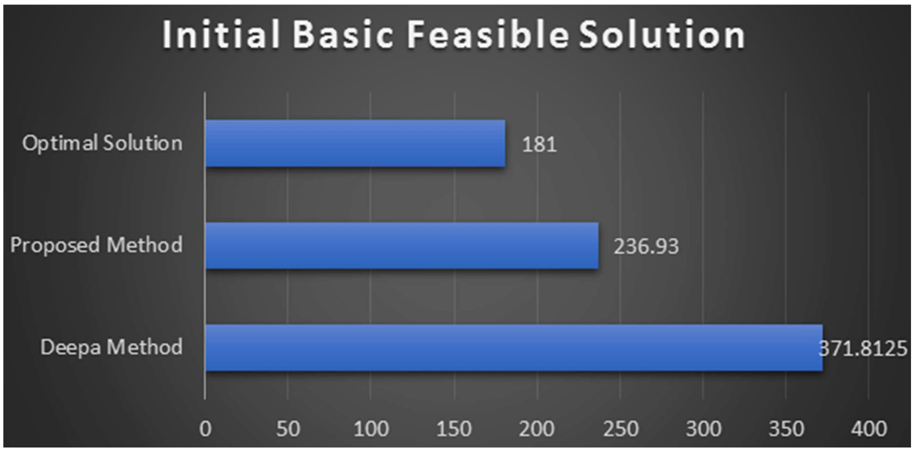

Figure 7. *Comparative Study of the result obtained by the Proposed method and the Deepa method.*

According to the results in Table 12 and Figure 7 above, the proposed method outperformed the Deepa Method.

5.3. Comparative Study of the Proposed Method over the Fuzzy Approach, Interactive Approach, Trust Region Approach, and the Product Approach

Table 13. *Detailed data representations of the problem. [20].*

	$\widetilde{D}_1$	$\widetilde{D}_2$	$\widetilde{D}_3$	$\widetilde{D}_4$	$\widetilde{D}_5$	Supply
$\widetilde{S}_1$	(9,2,2)	(12,9,4)	(9,8,6)	(6,1,3)	(9,4,6)	5
$\widetilde{S}_2$	(7,1,4)	(3,9,8)	(7,9,4)	(7,5,9)	(5,2,2)	4
$\widetilde{S}_3$	(6,8,5)	(5,1,3)	(9,8,5)	(11,4,3)	(3,5,6)	2
$\widetilde{S}_4$	(6,2,6)	(8,8,9)	(11,6,6)	(2,9,3)	(2,8,1)	9
Demand	4	4	6	2	4	20

To investigate the efficacy of the proposed method over The Fuzzy Approach, Interactive Approach, Trust Region Approach, and Product Approach results are abridged in Table 14.

Table 14. *Performance measure over the proposed method with The Fuzzy Approach, Interactive Approach, Trust Region Approach, and Product Approach.*

Method	$f_1(x)$	$f_2(x)$	$f_3(x)$	**Mean**
The Fuzzy Approach	112	106	80	99.33
Interactive Approach	127	104	76	102.33
Trust Region Approach	144	104	73	107
Parallel Method	157	72	86	105
Product Approach	157	72	86	105
Proposed Method	156	80	82	106
Optimal Solution	102	72	64	79.33

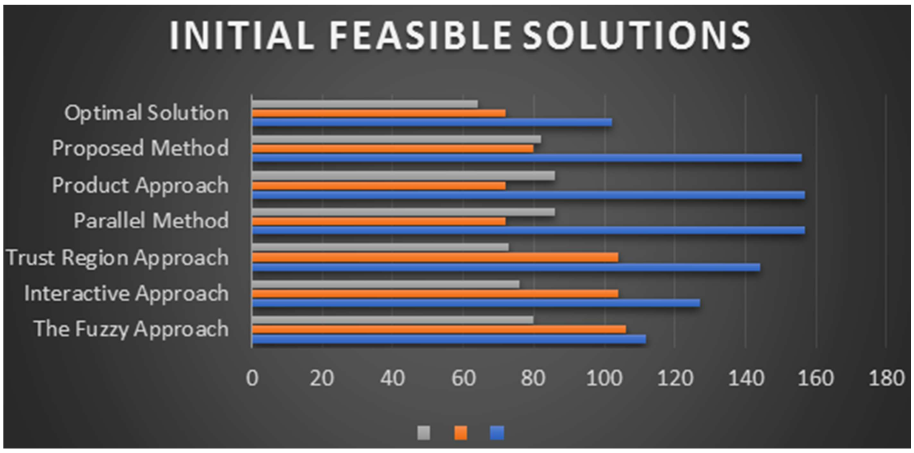

Figure 8. *Comparative Study of the Result obtained by The Fuzzy Approach, Interactive Approach, Trust Region Approach, Product Approach, and the proposed method.*

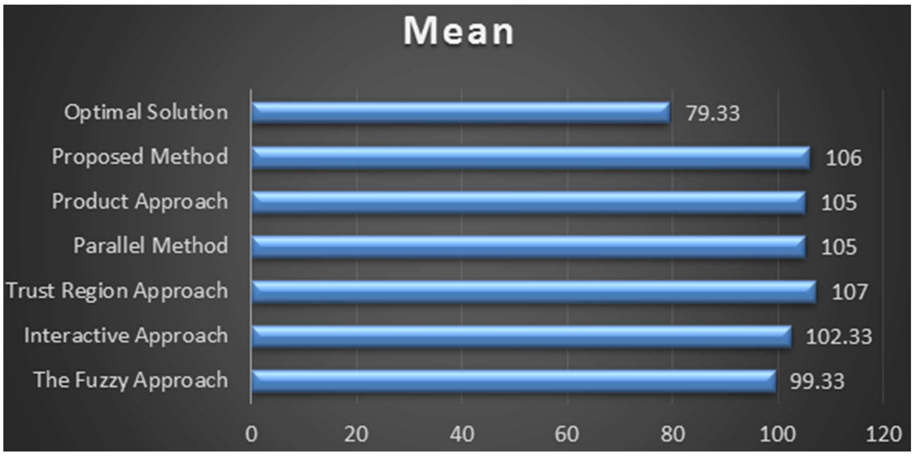

Figure 9. *Comparative Study of Mean obtained by The Fuzzy Approach, Interactive Approach, Trust Region Approach, Product Approach, and the proposed method.*

Table 14, Figures 8 and 9 portray that the proposed method for finding IFS for the triangular fuzzy transportation problem is more capable than formalized inspection methods.

5.4. Comparative Study of the Proposed Method over with Fuzzy Version of Vogel's Approximation Algorithm (FVAM)

Table 15. *Detailed data representations of the problem. [23].*

	$\widetilde{D_1}$	$\widetilde{D_2}$	**Supply**
$\widetilde{S_1}$	(1,3,5)	(3,5,13)	(300,399,504)
$\widetilde{S_2}$	(2,3,10)	(3,4,11)	(250,301,346)
$\widetilde{S_3}$	(5,6,13)	(1,3,5)	(300,399,504)
Demand	(400,448,508)	(300,351,396)	

The Performance proportion of the proposed method over the Fuzzy Version of Vogel's Approximation Algorithm (FVAM) is exhibited in Table 16:

Table 16. *Performance measure over the proposed method with the Fuzzy Version of Vogel's Approximation Algorithm (FVAM).*

Methods	Initial Basic Feasible Solution
FVAM	2392
Proposed method	1771.3135
Optimal Solution	920

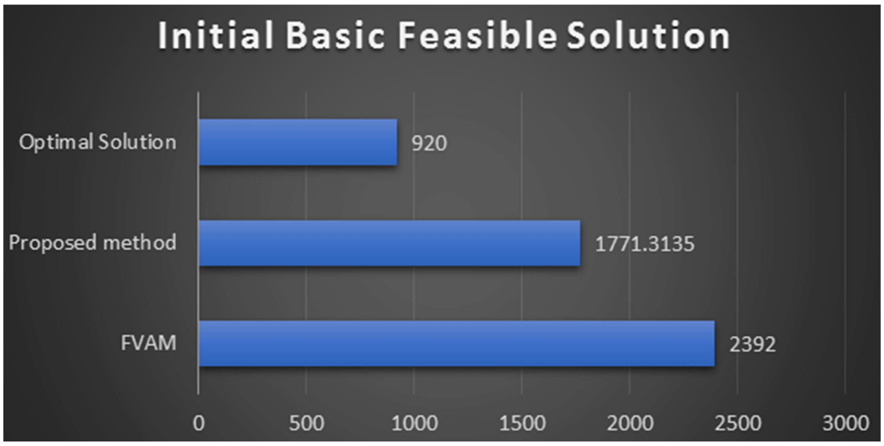

Figure 10. *Comparative Study of the Result obtained by existent approach.*

Table 16 and Figure 10 show that the proposed method outperforms the Fuzzy Version of Vogel's Approximation Algorithm (FVAM) method.

5.5. Comparative Study of the Proposed Method over with Fuzzy Zero Point Method

Table 17. *Detailed data representations of the problem [22].*

	$\widetilde{D_1}$	$\widetilde{D_2}$	$\widetilde{D_3}$	Supply
$\widetilde{S_1}$	(1,2,3)	(2,5,8)	(2,4,6)	(2,5,8)
$\widetilde{S_2}$	(2,6,10)	(1,3,5)	(0,1,2)	(3,6,9)
$\widetilde{S_3}$	(4,8,12)	(3,9,15)	(1,2,3)	(3,9,15)
Demand	(4,8,12)	(8,10,12)	(3,5,7)	

The Performance proportion of the proposed method over the Fuzzy Zero Point method is shown in Table 18:

Table 18. *Performance measure over the proposed method with the Pandian method.*

Method	$f_1(x)$	$f_2(x)$	$f_3(x)$	Mean	Deviation
Fuzzy Zero Point method	-30	58	184	70.67	87.82
Proposed Method	17.28	43.76	70.24	43.76	21.62
MODI Method	8	71	183	87.33	72.37

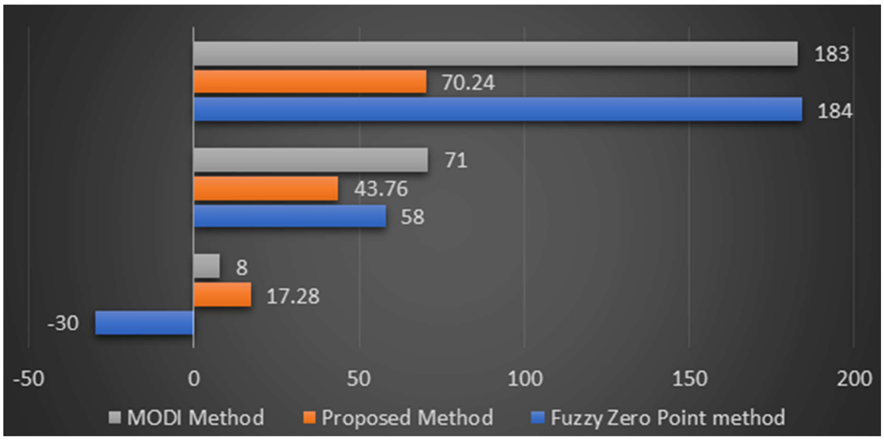

Figure 11. *Comparative Study of the Result obtained by existent approach.*

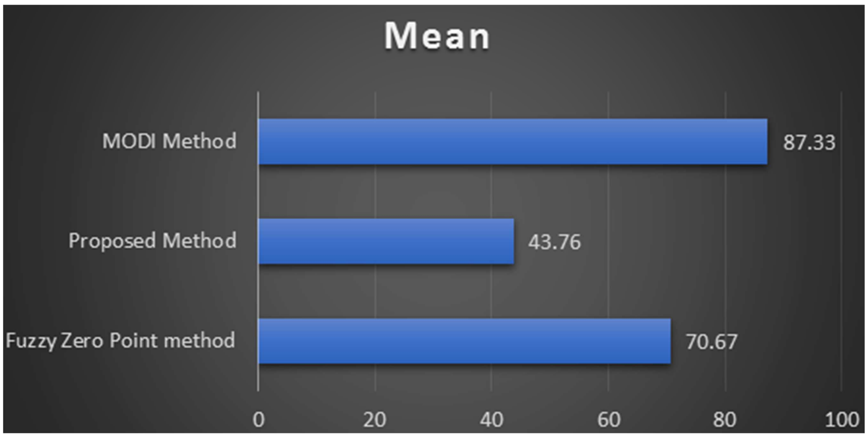

Figure 12. *Comparative Study of Mean obtained by existent approach.*

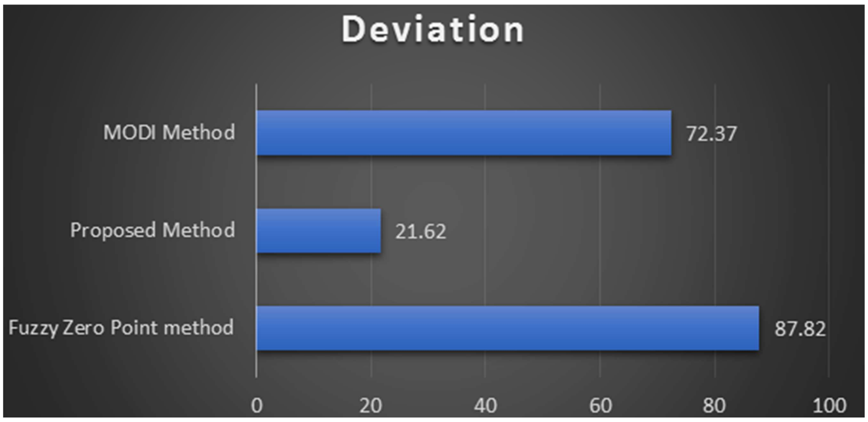

Figure 13. *Comparative Study of Deviation obtained by existent approach.*

Table 18, and Figures 11,12 and 13 show that the proposed method outperforms the other approach.

5.6. Comparative Study of the Proposed Method over the Balasubramanian Method

Table 19. *Detailed data representations of the problem [23].*

	$\widetilde{D_1}$	$\widetilde{D_2}$	$\widetilde{D_3}$	**Supply**
$\widetilde{S_1}$	[4,5,6]	[3,4,5]	[6,7,8]	[3,4,5]
$\widetilde{S_2}$	[1,2,3]	[5,6,7]	[4,5,6]	[5,6,7]
$\widetilde{S_3}$	[3,4,5]	[7,8,9]	[2,3,4]	[4,5,6]
Demand	[4,5,6]	[5,6,7]	[3,4,5]	

To investigate the effectiveness of the Proposed method compared to the Ranking method used in Balasubramanian, the results are summarized in Table 20.

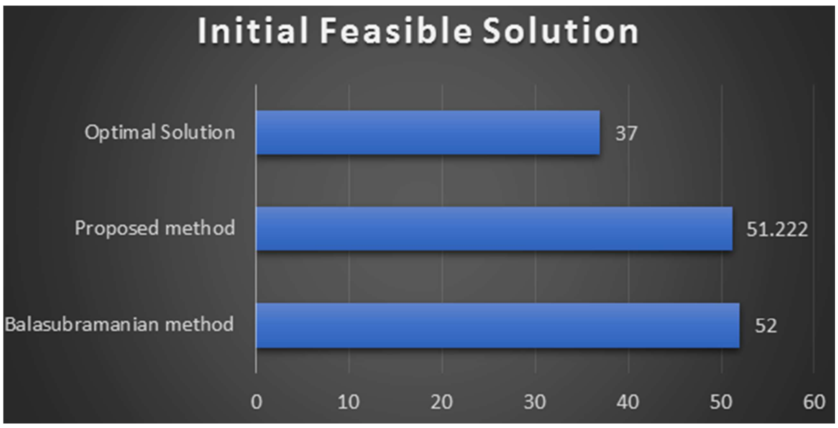

Figure 14. *Comparative Study of the Result obtained by existent approach.*

Table 20. *Performance measure over the proposed method with the Balasubramanian method.*

Method	Initial Feasible Solution
Balasubramanian method	52
Proposed method	51.222
Optimal Solution	37

Table 20 and Figure 14 represent that the proposed method for finding IFS for the triangular fuzzy transportation problem is more efficient and provides the best solution.

5.7. Comparative Study of the Proposed Method over the Jayaraman Method

Table 21. *Detailed data representations of the problem. [24].*

	$\widetilde{D_1}$	$\widetilde{D_2}$	$\widetilde{D_3}$	Supply
$\widetilde{S_1}$	(1, 4, 9)	(16,25, 36)	(9, 36,49)	(4,25, 36)
$\widetilde{S_2}$	(16, 25, 64)	(36, 64,81)	(4, 49,64)	(16, 36,49)
$\widetilde{S_3}$	(4,25,81)	(25, 36, 64)	(49,64,81)	(25, 49,81)
Demand	(16, 25,36)	(4,49, 81)	(25,36,49)	

The Performance proportion of the proposed method over the Jayaraman method is exhibited in Table 22:

Table 22. *Performance measure over the proposed method with the Jayaraman method.*

Method	Initial Feasible Solution
Jayaraman method	3584.19
Proposed method	1358.52
Optimal Solution	952

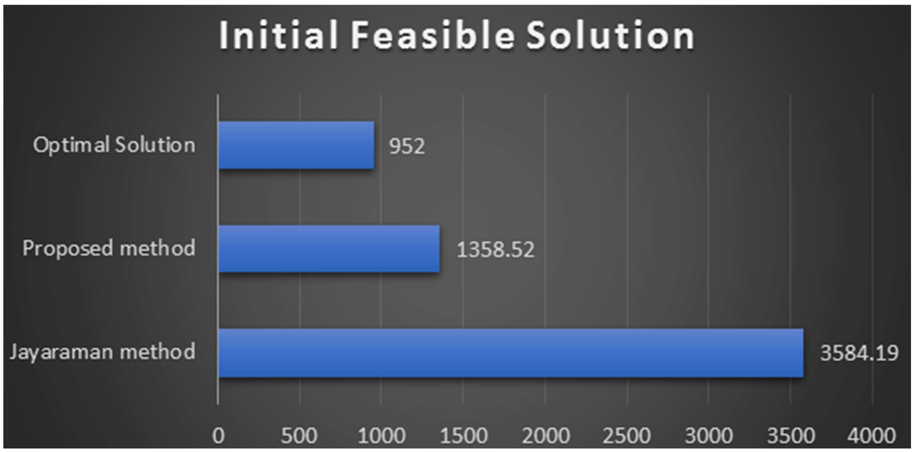

Figure 15. *Comparative Study of the Result obtained by existent approach.*

Table 22 and Figure 15 show that the proposed method outperforms the Jayaraman method.

5.8. Comparative Study of the Proposed Method over the Santhosh Kumar Method

Table 23. *Detailed data representations of the problem. [25].*

	$\widetilde{D_1}$	$\widetilde{D_1}$	$\widetilde{D_1}$	$\widetilde{D_1}$	Supply
$\widetilde{S_1}$	(10,15,20)	(41,51,61)	(32,42,52)	(23,33,43)	(13,23,33)
$\widetilde{S_1}$	(70,80,90)	(32,42,52)	(16,26,36)	(71,81,91)	(34,44,54)
$\widetilde{S_1}$	(80,90,100)	(30,40,50)	(56,66,76)	(50,60,70)	(23,33,43)
Demand	(13,23,33)	(21,31,41)	(6,16,26)	(20,30,40)	

The Performance proportion of the proposed method over the Santhosh Kumar Method is exhibited in Table 24:

Table 24. *Performance measure over the proposed method with the Santhosh Kumar Method.*

Method	Initial Feasible Solution
Santhosh Kumar Method	7005
Proposed method	3860.30
Optimal Solution	3322.32

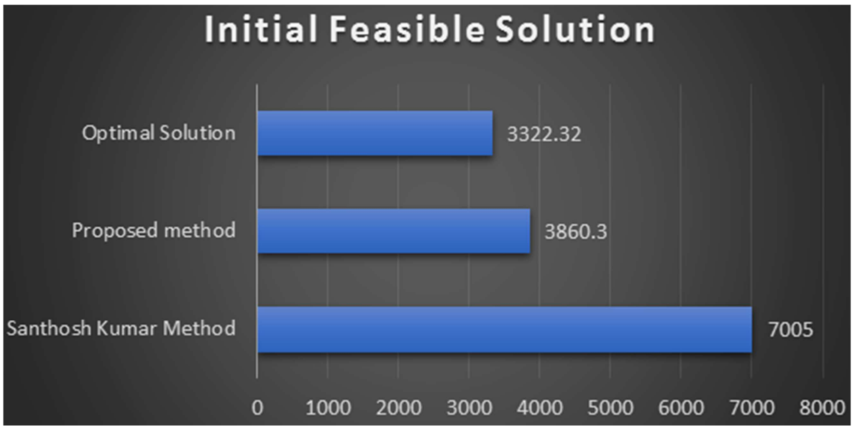

Figure 16. *Comparative Study of the Result obtained by existent approach.*

As shown in the above results indicated in Table 24 and Figure 16, the proposed method has produced the best results compared to the Santhosh Kumar Method.

6. Conclusion

In this paper, a simple yet effective new method was introduced to solve the triangular fuzzy transportation problem by using a modified knapsack problem. The proposed method can be used for all kinds of fuzzy transportation problems, whether triangular or other fuzzy numbers with crisp or fuzzy data and is especially beneficial for larger dimensions. Finally, demonstrated the proposed approach with a numerical example. When compared to other approaches, its results are very close to the optimal solution. The new approach is a systematic process that is simple to use for any kind of transportation problem, regardless of whether it maximizes or minimizes the objective function, and is easy to understand and put into practice.

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