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Finding the Photon Mass from Maxwell's Equations and Compton Scattering Theory

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Abstract: The question of whether particles of light have mass has been asked in natural philosophy for centuries, starting with theories such as the corpuscular theory of Newton and contemporaries, based in turn on older ideas back to classical times. In the early twentieth century, Planck and Einstein introduced the concept of the photon, the quantum of light energy. It was proposed by Einstein and others, notably in about 1906, that the photon has mass. The behavior of a photon is strange. The aim of this work is to attempt to theoretically investigate the rest mass of a photon from Maxwell's equation and Compton scattering theory. In this paper the equation of the electric field intensity in the presence of polarization in vacuum is derived. Maxwell's equations can describe state of electromagnetic waves in any medium. Their physical content is familiar; these equations are derived from the laws of electricity and magnetism. According to electromagnetic theory, the rest mass of photon in free space is zero and also photon has non-zero rest mass, as well as wavelength-dependent. To apply Maxwell's equations and Compton scattering theory this is modified to find the photon rest mass is of the order $\sim 10^{-49}$ g which is comparable to that obtained by Coulomb experiment which is of the order $\sim 10^{-44}$ g. Our theoretical work on the speculative mass of the photon must be experimentally verified and may open up plausible new applications in different fields.

Keywords: Maxwell's Equations, Polarization, Photon Mass

1. Introduction

The idea that light consists of rapidly moving particle can be traced from the writing of ancient authors to Descartes and Newton. The wave theory of light was put forward by Huygens and was later decisively proved to be correct through discovery of interference and diffraction by Young and Fresnel. Maxwell's theory of light as electromagnetic waves was one of the greatest achievements of the 19th century [1].

The history of the photon in 20th century started in 1901 with the formula by Planck for radiation of a black body and introduction of what was called later the quantum of action h. In 1902 Lenard discovered that energy of electrons in photo-effect does not depend on the intensity of light but depend on the wavelength of the later [2].

In 1905 Einstein pointed out that discovery of Lenard meant that energy of light is distributed in space not uniformly, but in a form of localized light quanta called photons.

The proof that Einstein's light quanta behave as a particles, carrying not only energy, but also momentum was given in 1923 in the experiments by Compton on scattering of x-ray. From Compton effect it clear that photon scattering supports the idea that a photon has a quantity of energy and momentum. Also it supports the Planck-Einstein hypothesis that there exists a particle which carries electromagnetic energy.

The Scotsman James Clerk Maxwell (1831–1879) described light as a propagating wave of electric and magnetic fields. More generally, he predicted the existence of electromagnetic radiation: coupled electric and magnetic fields traveling as waves at a speed equal to the known speed of light.

The enormous successes of quantum electrodynamics (QED) have led to an almost total acceptance of this concept of the mass- less photon.

It is almost certainly impossible to do any experiment that would firmly establish that the photon rest mass is exactly zero. According to the uncertainty principle, the ultimate upper limit on the photon rest mass m_γ can be estimated to be of the order a magnitude of 10^{-66} g [3].

However, a photon has a real non-zero mass, which depends on the photon's wavelength in free space and is inversely proportional to the wavelength when the photon's speed does not depend on the wavelength, i.e. the constant speed [4, 5]. Even experimentally, it has been proven that the electromagnetic wave (photon) has an imaginary rest mass in the medium (dispersed) similar to the mass of the electron and neutrino [6].

1.1. Maxwell's Equations

The Maxwell's equations which describe the behavior of electromagnetic wave in the presence of electric and magnetic fields are given by the set of equations [7]:

$$\nabla . D = \rho$$

$$\nabla . B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (1)$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

Where:

D = the electric flux density B = the magnetic flux density E = the electric field

H = the magnetic field J = the current density

Satisfying the following relations:

$$B = \mu_0 H$$

$$J = \sigma E \quad (2)$$

$$D = \varepsilon_0 E + P$$

Here P is the macroscopic polarization of the medium

$\varepsilon_0$is the permittivity of free space $\mu_0$is the permeability of free space.

The Maxwell's equations can be utilized to derive the equation of E. Now applying the curl operator to both sides of the third equation in (1) one can obtain:

$$\nabla \times (\nabla \times E) = -\nabla \times \frac{\partial B}{\partial t} = -\frac{\partial}{\partial t}(\nabla \times B) \quad (3)$$

Using the identity:

$$\nabla \times (\nabla \times E) = \nabla(\nabla . E) - \nabla^2 E \quad (4)$$

Thus (3) becomes:

$$\nabla(\nabla . E) - \nabla^2 E = -\frac{\partial}{\partial t}(\nabla \times B) \quad (5)$$

Using (2) $B = \mu_0 H$

So eq. (5) becomes

$$\nabla(\nabla . E) - \nabla^2 E = -\frac{\partial}{\partial t}(\nabla \times \mu_0 H) \quad (6)$$

From equation (5) since $\nabla \times H = J + \frac{\partial D}{\partial t}$ from (1) gets:

$$\nabla(\nabla . E) - \nabla^2 E = -\frac{\partial}{\partial t}\left(\mu_0 J + \mu_0 \frac{\partial D}{\partial t}\right) \quad (7)$$

Utilizing (2) again

$$D = \varepsilon_0 E + P$$

Therefore (7) becomes:

$$\nabla(\nabla . E) - \nabla^2 E = -\mu_0 \frac{\partial J}{\partial t} - \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} - \mu_0 \frac{\partial^2 P}{\partial t^2} \quad (8)$$

Also (2) gives:

$$J = \sigma E \text{ So}$$

$$\nabla(\nabla . E) - \nabla^2 E + \mu_0 \sigma \frac{\partial E}{\partial t} + \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} = -\mu_0 \frac{\partial^2 P}{\partial t^2} \quad (9)$$

The polarization $P_{(r,t)}$ thus acts as a source term in the equation for the radiation field. Since

$$D = \varepsilon_0 E \text{ And } \nabla . D = \rho \; \rho = 0$$

Therefore

$$\varepsilon \nabla . E = \rho = 0 \; \nabla . E = 0$$

So the equation (9) becomes:

$$\nabla^2 E + \mu_0 \sigma \frac{\partial E}{\partial t} + \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} = -\mu_0 \frac{\partial^2 P}{\partial t^2} \quad (10)$$

This equation represents Maxwell's equation for the electric field [8].

1.2. Theory of the Photon with Finite Mass

Consider a photon having angular frequency ω incident on an electron of mass M at rest (see figure 1). If the electron quires a momentum p', then the principle of relativistic energy conservation reads [9, 10]:

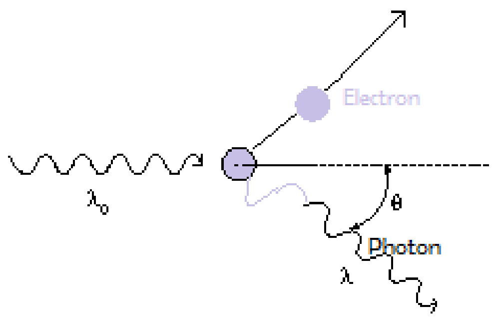

Figure 1. *Schematic plot of Compton scattering process.*

$$\hbar\omega + Mc^2 = \hbar\omega' + (M^2c^4 + c^2p'^2)^{1/2} \quad (11)$$

Where the initial and final energies of the photon are respectively $\hbar\omega$ and $\hbar\omega'$. The initial energy of the electron is its rest energy in special relativity:

$$E = M c^2 \tag{12}$$

And the final energy of the electrons is:

$$E = (M^2 c^4 + c^2 p'^2)^{1/2} \tag{13}$$

Where P' is its final momentum. From the conservation of total momentum of photon and electron the electron momentum is given by:

$$P' = \hbar(k - k') \tag{14}$$

Where k is the initial wave-vector and k' the final wave-vector of the photon. From elementary vector analysis:

$$P'^2 = \hbar^2 (k - k').(k - k') = \hbar^2 (k^2 + k'^2 - 2\, kk' \cos\theta) \tag{15}$$

Where θ is the angle between the vectors k and k'

If the photon has mass m then its energy is given by

$$E = \hbar\,\omega = \gamma\, m\, c^2 \tag{16}$$

While its momentum is given according to de Broglie hypothesis

$$P = \hbar k = \gamma m\, v \tag{17}$$

Which are simple combination of special relativity and quantum theory. Here E is the total relativistic energy of one photon, and P is the relativistic momentum of one photon. By considering the magnitude of the velocity of photon *is* v, instead of c which is maximum speed attainable by the photon, c being the usual fundamental constant of the standards laboratories, fixed by treaty. The Lorentz factor is given by

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \tag{18}$$

At different angular frequencies ω and ω', the photon has different velocities v and v', but the mass m of the photon is the fixed mass of an elementary particle. Therefore:

$$\hbar\,\omega = \gamma\, m\, c^2, \hbar k = \gamma\, m_0\, v \tag{19}$$

$$\hbar\omega' = \gamma' m\, c^2, \hbar k = \gamma'\, m_0\, v \tag{20}$$

And the relation between ω and k is:

$$k = \frac{v}{c^2}\omega, k' = \frac{v'}{c^2}\omega' \tag{21}$$

The two Lorentz factors γ and γ' are related to the angular frequencies as

$$\frac{\gamma'}{\gamma} = \frac{\omega'}{\omega} \tag{22}$$

The equation of conservation of total energy for massive photon is given by

$$\gamma\, m_0\, c^2 + M_0\, c^2 = \gamma' m_0 c^2 + (c^2\, P'^2 + M_0^2\, c^4)^{1/2} \tag{23}$$

Where the left side represents the frame in which the electron is at rest and the equation of conservation of momentum is equation (15), in which Eqs (18) to (22) are used to give

$$c^2 P'^2 = \frac{\hbar^2}{c^2}\left(\omega^2 v^2 + \omega'^2 v'^2 - 2\omega\omega' vv' \cos\theta\right) \tag{24}$$

$$c^2 p'^2 = \frac{\hbar^2}{c^2}\left(k^2 + k'^2 - 2\, kk' \cos\theta\right) \tag{25}$$

Eliminating the electron momentum from (23) and (24)

$$(c^2\, m_0\, (\gamma - \gamma') + M_0\, c^2)^2 - M_0^2 c^2$$

$$= \frac{\hbar^2}{c^2}\left(\omega^2 v^2 + \omega'^2 v'^2 - 2\omega\omega' vv' \cos\theta\right) \tag{26}$$

$$[(\gamma - \gamma')\, m_0\, c^2 + M_0 c^2]^2 = c^2\, P'^2 + M_0^2\, c^4$$

$$= \frac{\hbar^2}{c^2}(k^2 + k'^2 - 2kk' \cos\theta) + M_0^2\, c^4$$

$$\frac{\hbar^2}{c^2}\left(\omega^2\, v^2 + \omega'^2\, v'^2 - 2\omega\omega' vv' \cos\theta\right) + M_0^2\, c^2 \tag{27}$$

But from equation (22)

$$\gamma' = \frac{\omega'}{\omega}\gamma$$

So

$$\gamma - \gamma' = \left(1 - \frac{\omega'}{\omega}\right)\gamma = \Omega\,\gamma \tag{28}$$

From Eqs (22) which also gives:

$$\left(1 - \frac{v'^2}{c^2}\right) = \left(\frac{\omega}{\omega'}\right)^2 \left(1 - \frac{v^2}{c^2}\right) \tag{29}$$

The fundamental energy equation of the de Broglie Einstein theory gives:

$$\hbar\,\omega = m\, c^2 \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \tag{30}$$

From Eqs (26), (29) and (30) the following is obtained

$$\omega^2 v^2 + \omega'^2 v'^2 - 2\omega\omega' vv' \cos\theta = A \tag{31}$$

Where:

$$A = \Omega^2 \omega^2 c^2 \left(1 + \frac{2Mc^2}{\hbar\,\omega\,\Omega}\right) \tag{32}$$

Eqs (21) can be solved simultaneously with:

$$1 - \frac{v'^2}{c^2} = \left(\frac{\omega}{\omega'}\right)^2 \left(1 - \frac{v^2}{c^2}\right) \tag{33}$$

To eliminate v'. leaving as follows an equation for v and therefore m in experimental observation.

From Eqs (33)

$$v'^2 = c^2 \left(1 - \left(\frac{\omega}{\omega'}\right)^2 \left(1 - \frac{v^2}{c^2}\right)\right) \tag{34}$$

And using this equation in Eqs (33) gives the result:

$$\omega^2 v^2 - \omega\omega' v v' \cos\theta = B \quad (35)$$

Where:

$$B = \frac{1}{2}\left(A - c^2\left(1 - \left(\frac{\omega}{\omega'}\right)^2\right)\omega'^2\right) \quad (36)$$

Therefore:

$$v' = \frac{\omega^2 v^2 - B}{\omega\omega' v \cos\theta} \quad (37)$$

From Eqs (3.12)

$$\frac{v^2}{c^2} = 1 - \left(\frac{\omega'}{\omega}\right)^2\left(1 - \frac{v'^2}{c^2}\right) \quad (38)$$

i.e.:

$$\frac{v^2}{c^2} = 1 - \left(\frac{\omega'}{\omega}\right)^2 + \frac{1}{c^2}\left(\frac{\omega'}{\omega}\right)^2\left(\frac{\omega^2 v^2 - B}{\omega\omega' v \cos\theta}\right)^2 \quad (39)$$

This is quadratic equation in v^2:

$$\frac{v^4}{c^2}(1 - \cos^2\theta) + \left(\left(1 - \left(\frac{\omega'}{\omega}\right)^2\right)\cos^2\theta - \frac{2B}{c^2\omega^2}\right)v^2 + \left(\frac{B}{c\omega^2}\right)^2 = 0 \quad (40)$$

The solution of this quadratic is:

$$v^2 = \frac{1}{2a}\left(-b \pm (b^2 - 4ac)^{1/2}\right) \quad (41)$$

Where:

$$a = \frac{1}{c^2}(1 - \cos^2\theta) \quad (42)$$

$$b = \left(1 - \left(\frac{\omega'}{\omega}\right)^2\right)\cos^2\theta - \frac{2B}{c^2\omega^2} \quad (43)$$

$$c = \left(\frac{B}{c\omega^2}\right)^2 \quad (44)$$

Finally the photon mass is found unequivocally from:

$$m = \frac{\hbar\,\omega}{c^2}\left(1 - \frac{v^2}{c^2}\right)^{1/2} \quad (45)$$

Since v^2 must be positive and real valued, it is straight forward to select the relevant root of the quadratic.

1.3. Photon Mass and Maxwell's Equations

Maxwell's equations are used to describe the behavior of electromagnetic waves, to describe the nature of electromagnetic waves in free space, it is better to bear in mind that its conductivity is very small and can be neglected. The equation of the electric field in free space in the presence of polarization is given by [11]:

$$-\nabla^2 E + \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} + \mu_0 \frac{\partial^2 P}{\partial t^2} = 0 \quad (46)$$

The electric polarization P is defined to be [12]

$$P = e\,x \quad (47)$$

Where e is the electron charge while x is the electron displacement. To solve equation (46), one can assume the solution:

$$E = E_0 e^{i(kx - \omega t)} \quad (48)$$

Where E_0 stands for the amplitude of the electric wave, while k and ω represents the wave number and angular frequency respectively.

$$k = \frac{2\pi}{\lambda}, \omega = 2\pi f$$

Where λ is the wavelength, while f is the frequency.

With the aid of (48) one finds

$$\frac{\partial E}{\partial x} = ikE$$

$$\frac{\partial^2 E}{\partial x^2} = -k^2 E \quad (49)$$

$$\frac{\partial E}{\partial t} = -i\omega\, E$$

$$\frac{\partial^2 E}{\partial t^2} = i^2\omega^2 E = -\omega^2 E \quad (50)$$

To relate polarization P to E it is important to find the equation of motion of electron of mass m and charge e in the presence of the electric field E. This equation takes the form:

$$m_e\,\ddot{x} = -eE\,,\ \ddot{x} = -\frac{e}{m_e}\,E \quad (51)$$

But

$$\mu_0 \frac{\partial^2 P}{\partial t^2} = \mu_0 e \frac{\partial^2 x}{\partial t^2} = \mu_0 e\ddot{x}$$

With the aid of (3-25)

$$\mu_0 \frac{\partial^2 P}{\partial t^0} = \mu_0 e\ddot{x} = -\mu_0 \frac{e^2}{m_e}\,E \quad (52)$$

But

$$c^2 = \frac{1}{\mu_0 \varepsilon_0}$$

Substituting (51) and (49) in (46) yields

$$c^2 k^2\,E - \omega^2 + \mu_0 \frac{e^2 c^2}{m_e}\,E = 0 \quad (53)$$

1.4. Photon Rest Mass

Eliminating E from both sides of (53) yields

$$c^2 k^2 - \omega^2 + \mu_0 \frac{e^2 c^2}{m_e} = 0 \quad (54)$$

Multiplying both sides by $\hbar^2$ yields:

$$c^2\hbar^2\,k^2 - \hbar^2\omega^2 + \hbar^2\,\mu_0 \frac{e^2 c^2}{m_e} = 0 \quad (55)$$

According to the laws of quantum mechanics

$$E = \hbar\,\omega, P = \hbar\, k \quad (56)$$

Then inserting (56) in (55) yields

$$c^2P^2 - E^2 + \hbar^2\,\mu_0\frac{e^2c^2}{m_e} = 0$$

$$E^2 = c^2P^2 + \hbar^2\,\mu_0\frac{e^2c^2}{m_e} \quad (57)$$

In other hand the energy E is related to the momentum P and rest mass m_0 in special relativity according to the relation

$$E^2 = c^2P^2 + m_0^2c^4 \quad (58)$$

Where m_0 here stands for the photon rest mass. Comparing (56) and (57) the photon rest mass is given by:

$$m_0 = \frac{\hbar e}{c}\sqrt{\frac{\mu}{m_e}} \quad (59)$$

Mathematically:

$$m_0 = \frac{6.63*10^{-34}*1.6*10^{-19}}{2*3.14*3*10^8}\sqrt{\frac{4*3.14*10^{-7}}{9.1*10^{-31}}} = 0.6587\times 10^{-49}\,g \quad (60)$$

1.5. Photon Mass from Compton Theory

The photon mass according to equation (45) can be found by determining the velocity of photon v in any medium. According to Maxwell's equations v is given in terms of electric permittivity ε and magnetic permeability μ to be

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0(1+\chi_m)\,\varepsilon_0(1+\chi_e)}}$$

$$= \frac{1}{\sqrt{\mu_0\varepsilon_0}(1+\chi_m\,\chi_e+\chi_m+\chi_e)} \quad (61)$$

Her χ_e and χ_m stands for electric and magnetic susceptibility. Since χ_e,and χ_m are `small, one can negligent the term $\chi_e\,\chi_m$ in eqn (61) to get

$$v = \frac{1}{\mu_0\varepsilon_0}\frac{1}{\sqrt{1+\chi_e\,\chi_m}} = c\,(1+\chi_e\,\chi_m)^{-1} \quad (62)$$

From (45) the photon mass is given by

$$m = \frac{\hbar\omega}{c^2}\,[(1+\chi_m+\chi_e)^{-2}]^{1/2} \quad (63)$$

2. Discussion

The derivation of the photon mass from Compton effect is of great importance because it provides an unequivocal test of modern physics by evaluating m at various scattering angles in a Compton effect experiment [13]. Maxwell's equations in free space is solved in the presence of polarization. The quantum expression for energy and momentum are used to relate energy to the momentum. This relation is compared to the expression of energy in special relativity. this comparison is utilized to find the mass of the photon [14].

In view of equation (60) the photon rest mass is of the order $\sim 10^{-49}$ g which is comparable to that obtained by Coulomb experiment which is of the order $\sim 10^{-44}$g, it also comparable to result for Mahendra Goray, Ramesh Naidu Annavarapu, they obtained the value of photon rest mass is 10^{-54} Kg [15].

In contrast to this result, Compton theory shows that the photon rest mass is strongly dependent on its frequency f as well as the electric and magnetic properties of the medium. Strictly speaking m_0 depends on electric and magnetic susceptibility [16].

3. Conclusion

Maxwell's equations and Compton scattering theory have been used to find the photon mass. Maxwell's equations can describe state of electromagnetic waves in any medium. From Compton effect used different velocities of photon to find the photon rest mass. The work based on Maxwell equations indicates that the photon rest mass is very small. But Compton theory shows that this mass dependent on the wave frequency as well as electromagnetic properties of the medium. Our work shows some new directions for further research in areas of theoretical physics where the rest mass of a photon becomes fictional. It may also open the door to more research in the field of theoretical and computational physics.

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