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Electromagnetic Problems Modeling Using Algebraic Topological Method

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Abstract: We can solve electromagnetic problems using two main mathematical tools: vector calculus and differential equations. These tools command the computational electromagnetic domain. However, these tools are not always needed for the realistic modeling of electromagnetic problems. In reality, we are interested in the measurement of scalar quantities in electromagnetics, not vector quantities. Conventional electromagnetic simulation approaches are proving to be more mathematical than physical. Furthermore, the use of differential equations leads us along a different route for modeling fundamental physics. Since computers need discrete formulations, we can't directly transform continuous differential equations into numerical algorithms. The algebraic topological method is a direct discrete and computationally ambitious technique that uses only physically measurable scalar quantities. This paper simulates a parallel plate capacitor using global variables and calculating and comparing the potentials with the analytical method. The measured results show a good agreement between the analytical and the algebraic topological methods.

Keywords: Topological, Scalar, Vector, Variables, Simplex, Primal, Dual, Capacitor

1. Introduction

Computational electromagnetic problems mainly deal with finite-difference methods, finite-element methods, finite-volume methods, or method of moments. These methods have some unique advantages and disadvantages for modeling certain problems in electromagnetics and computing approximate values of physical quantities. These methods mainly depend on differential equations and vector calculus [1, 2]. In this article, we address our standard methodology and can't help thinking about why we quite often go with vector-calculus and differential equations? [3, 4] Is there any choice for examining and demonstrating electromagnetic issues other than vector-calculus and differential equations? Indeed, the appropriate response is YES. [5] In the last 30-40 years, a better than ever approach to consider Electromagnetic was presented in [6]. So other than this, there is another tool called the algebraic topological method (ATM). Algebraic topology can be defined as a study of spatial objects like 0 simplex (nodes), 1 simplex (lines), 2 simplex (surfaces), 3 simplex (volumes) etc. In the Algebraic topological method, to avoid vector field and differential equations we will use only physically measurable scalar quantities. [7] Using the mathematical tools of algebraic topology, we directly get discrete formulations. So with these, we cannot change the problem from discrete to continuous we directly solve the problems with the discrete formulation. The physically measurable quantities like potential, current, electric & magnetic flux, and charge content are defined as co-chains on topological objects such as points, lines, surfaces, and volumes [8, 9]. For a better understanding of algebraic topology in electromagnetics, let's take an example; there are 4 metro stations, namely A, B, C, and D, from the figure 1 and l1, l2, l3, and l4 are the tracks on each the metro trains runs on both sides and this is called an un-oriented network, but some characteristics do not vary under topologically equivalent transformations. So for clarity purposes, let us see the right side of the figure 1, where all track distances are changed; however, it represents the same network. We have not added any new connections like new metro stations and existing connections between stations. Also, a particular parameter must belong to a particular network.

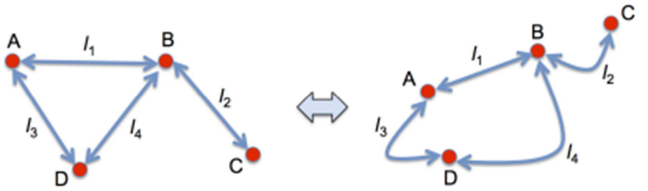

Figure 1. *Two topologically equivalent situations.*

We can also visualize the oriented structure from the office tree where the ending and starting points are well defined, for example- from CEO to research and development to project manager to team lead to employees. These examples are used to introduce and explain some topologies like homeomorphic transformation etc.

2. Source & Configuration Variables

Source variables, configuration variables and energy variables are the major types of variables that characterize physical phenomena.

2.1. Configuration Variables

Configuration variables are those that describe the configuration of the field, its potentials and all those variables are linked by algebraic and differential operations. Configuration variables are linked to the source one by the consecutive equations.

2.2. Source Variables

Source variables they are variables that characterize the field's origin or forces acting on the device. These variables are linked with one another by operations of sum, difference, limit, derivative and integral.

2.3. Energy Variables

Energy variables that are acquired by the product of a source variables by a configuration variable.

Configuration variables	
Gauge-function	X
electric-voltage (impulse)	U,(U)
e. m. f. (impulse)	E, (E)
electric-field vector	E
magnetic-flux	Φ
electric-potential (impulse)	V, (V)
magnetic-vector potential	A
magnetic-induction	B
Source variables	
Electric-charge content	Q^C
Electric-charge flow	Q^f
Electric-current density	J
Electric-flux	Ψ
Electric-induction	D
Magnetic-field strength	H
Magnetic-voltage (impulse)	$U_{m,}$ (Um)
m. m. f. (impulse)	$F_{m,}$ ($F_{m)}$
magnetic-scalar potential	V_m
dielectric-polarization	P
Magnetization	M
Energy variables	
work, heat	W, Q
electric-energy density	w_e
magnetic-energy density	w_m
Poynting-vector	S
Electro-magnetic momentum	G
Momentum-density	G
electro-magnetic action	A

In this algebraic topology, we completely avoid using field variables (vector quantities). So we use only global variables; these variables are known as integral variables derived from the result of volume, surface, and line integration of field vectors. And they are six global variables, namely Electric charge flow, Magnetomotance, Electromotance, Magnetic-flux, Electric charge content, Electric flux, Qc, Qf, Ψ, represent U, V, and Φ, respectively. All these variables are scalar quantities. Source variables and configuration variables subdivide these global variables. The product of source variable and configuration variables will get energy-variables are derived. These source and configuration-variables are obtained from Lorentz equation.

$$F = q\ (E + v \times B) \tag{1}$$

E and B refer to the electric and magnetic fields of forces, respectively.

$$E = Fe/q,\ v \times B = Fm\ /p$$

Global variables are derived from line, surface, and volume association.

Line association:

$$V = \int E \cdot dl \tag{2}$$

$$U = \int H \cdot dl \tag{3}$$

Surface association:

$$\Psi = \int D \cdot ds \tag{4}$$

$$\Phi = \int B \cdot ds \tag{5}$$

$$I = \int J \cdot ds \tag{6}$$

Volume association:

$$Q_c = \int \rho\ dv \tag{7}$$

Magnetomotance, Electromotance equation (2), (3) are obtained by line integral of Electric field and Magnetic field and Electric flux, magnetic flux and electric charge flow equation (4), (5), (6) are obtained by surface integral. With the help of this knowledge, we can divide these global variables as source variables and configuration variables for field variables. So, electromotance and magnetic flux are configuration variables, and electric flux, magnetic-flux, and electric-charge flow are source variables.

3. Global Variables VS Field Variables

Global variables are not the volume, surface, or line density of another variable. Most global variables are also known as integral variables, derived from volume, surface, or line of field variables. Here, we use V, U, Φ, Ψ, Qc, and Q_f of six global variables. One might ask why field quantities were introduced into Maxwell equations in the first place. The explanation is clear, but many people are unaware of it. Field variables were created mostly for mathematical reasons rather than physical ones. Mathematically has already developed elaborate which can be used to analyze differential formulation employing field variables - E, B, D, H, and J. field variables can only deal by the differential formulation. These Global variables are always uninterrupted across the boundary surfaces and depending on the geometrical objects known as domain functions. Geometrical objects play the best role in algebraic topological formulations that will get much detailed in further secessions.

4. Algebraic Topological Parameters

4.1. Simplex

The dimension of a spatial object k is its most basic feature. For example, the spatial dimensions of points, lines, surfaces, and volumes are k = 0, 1, 2, and 3, respectively. A simplex k, abbreviated as s_k, is a set of k+1 points in R^n with n k. As a result, s^0 denotes a vertex (point), s^1 denotes a line, s^2 denotes a surface, and s^3 denotes a volume. In a model, the i^{th} point, j^{th} line, and l^{th} surface are denoted as $s^0_{i,}$ s^1_j, and s^2_l and so forth. The ordinal numbers of a simplex are denoted by the subscripts i, j and l. Simplexes can be thought of as the building blocks of all geometric models. In the introduction, we defined algebraic topology as the study of spatial objects such as points, lines, surface, volume, etc. Simplexes are used to define basic topological objects generally call as 0-, 1-, 2-, 3-simplex in [10] Here 0, 1, 2, 3 are called as dimensions. Simplex are represented by s_i^k 0, 1, 2, 3 simplex represents points, lines, surfaces and volume respectively.

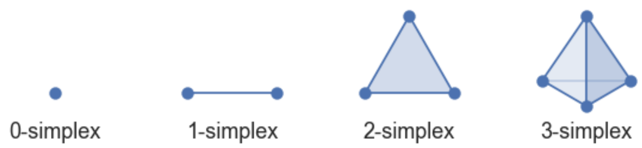

Figure 2. *3-dimenssional objects that contains 0-, 1-, 2-simplex.*

4.2. Chains & Co-chains

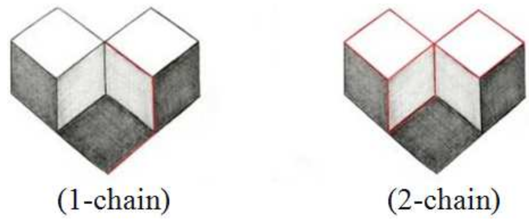

Figure 3. *Representation of chains and co-chains.*

The variables and the geometrical elements (k-simplexes) have a close relationship, as we discovered. A collection of simplexes is called a chain. [11] The representation of k-chains are

$$C_k^{\,i} = \Sigma a_i s_i^{k}$$

Where a_i is the weighted-orientation coefficient.

The representation of chains and co-chains are c_k and c_k, respectively. However, in Algebraic topology, we cannot connect different dimensions (simplex) of a topological object. A k-chain has strictly only a collection of k-simplexes. [12, 13] The respective co-chains are potentials, electromotances, fluxes, and charge-contents defined on these chains.

4.3. Boundary & Co-boundary Operators

A boundary operator is a fundamental operator that gives the boundary of certain objects. The boundary values will be obtained through this equation in [14, 15].

$$\partial\,(s_{ij}^{\,k+1}) = \sum_{i=1}^{nk} a_{ij}^{\,k} s_i^{\,k}$$

Here a_k is called incidence coefficients. If we take the boundary of 1-simplex we will get 0-simplex.

The co-boundary operator will be the exact opposite of the boundary operator and it is represented by delta(δ), it takes k co-chains to k+1 co-chains.

$$\delta\text{: } c_k \rightarrow c_{k+1}$$

The relation between co boundary operator and boundary operator is the adjoint of boundary operator.

$$\delta = (a^{KT})^{*} = a^{k} \; (a^{k}\text{=incidence coefficient})$$

$$[c_{\,k}, c^{k}] = [(a_k^{\,T})^{*} c_{k-1}, c^{k}] = [c_{k-1}, a_k^{\,T}\, c^{k}]$$

$$\int_{ck} \delta c^{k-1} = \int_{\partial ck} c^{\,k-1}$$

The co-boundary operator uses the node potentials to calculate the electromotance (potential difference between nodes). We receive the flux travelling through the surface when it works on the potential difference on a chain of lines.

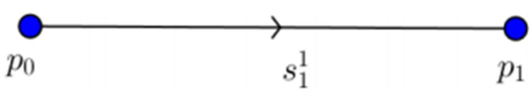

Figure 4. *Defining positive and negative orientations of a point and a line.*

Let us take 1-simplex that having a boundary of two 0-simplex. The above figure shows the $s_1^1 = [p_0, p_1]$. Here s11 called as 1-simplex is moving away from the po so the coefficient of this taken as negative as the same time the s11 is moving towards p1 hence it taken as positive. In special orientation we generally talk about two dimensional space called as R^2 and three dimensional space called as R3. These orientation may vary according to the dimensions and according to inner and outer orientation. If the inner orientation is k-simplex it becomes outer orientation of (n-k) simplex, n gives the dimension of space. The below table gives much information on special orientation.

Table 1. *Outer-orientation and inner-orientations in 2&3-dimensional embedding space.*

	Object dimension, K (simplex)	Inner orientation	Outer orientation
2-dimensional space, R^2	K=0	Point	Surface
	K=1	Line	Line
	K=2	Surface	Point
	K=3	Volume	Not-defined
3-dimensional space, R3	K=0	Point	Volume
	K=1	Line	Surface
	K=2	Surface	Line
	K=3	Volume	Point

In 2-D space, the outer-orientation of a1-simplex, for example, is also a line. In three-dimensional space, however, a line's outer-orientation is a surface.

4.4. Primal and Dual Complex

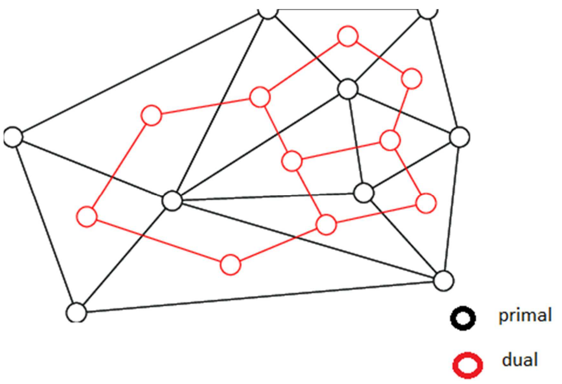

Figure 5. *Representation 0-, 1-, 2-simplex primal and dual complex.*

The above diagram describes the primal and dual complex so that all the inner oriented can be taken as primal and all the outer oriented can be taken as dual so that in the 2D geometry, The dual 0-simplex is equaled to primal 2-simplex and dual 1-simplex is equals to primal 1-simplex and dual 2-simplex is equals to primal 0-simplex. [16, 17].

5. Maxwell's Equations

1) ∇. D =ρv → Gauss's-law
2) ∇. B =0 → Gauss's law for magnetism
3) ∇XE = -∂t B → Faraday's-law
4) ∇XH = ∂tD + J → Ampère's-law

The above equations are Maxwell's equations. [18, 19] These equations operate on curl, gradient, Divergence and differential equations. However, while modeling in ATM we will use mainly 6 global variables in space and time. All these global variables are scalar quantities. Global variables are further divided into source variables and configuration variables. The cause of electromagnetic fields is related to the source variables. Configuration-variables correspond to the effect produced due to the source. The combination of these source-variables and configuration-variables is called energy-variables. There is another model called primal and dual, which relates to source-variable and configuration-variable to model the ATM formulations [20] for electromagnetics.

Table 2. *Algebraic notation for global variables.*

Global variable	Notation
Charge-content	$Qc(\tilde{s}^3, t)$
Charge-flow	$Qf\ (\tilde{s}^2, \tau)$
Electric-flux	$\Psi(\tilde{s}^2, t)$
Magnetomotance	$U(\tilde{s}^1, \tau)$
Electromotance	$V\ (s^1, \tilde{\tau})$
Magnetic-flux	$\Phi(s^2, \tilde{t})$
Electric-potential	$\Phi(s^{0,} \tilde{t})$

6. Algebraic Topological Formulations

Laplace's equation
Algebraic equation:

$$\Phi\ (s_l^0, \tilde{t}_n) = 0$$

Algebraic mathematical equation:

$$\Sigma\ \tilde{a}^3 .\ \Sigma\ a_{ml}^1\ \Phi\ (s_l^0, \tilde{t}_n) = 0$$

Where a_{ml}^1 is the primal line*point incidence matrix. We have $\tilde{a}_{ml}^0 = 0$ if the point $\tilde{s}_l^0$ does not connect to the boundary of the line. If the $\tilde{s}_l^0$ does belong to the boundary of the $\tilde{s}_m^3$, then $a_{ml}^1 = 1$. $\tilde{a}^3$ is called dual 3-simplex*2-simplex incidence matrix.

When utilizing the differential equations technique, we often begin with a discrete collection of scalar measurements. Then we'll move on to physics, where we'll solve continuous partial differential equations with field vectors. Finally, it's within the variety of a discrete formulation. This twisting way is avoided by taking the algebraic topological method (ATM), which begins with discrete scalar measurements and directly yields discrete algebraic topological equations. This is often obtained by boundary and co-boundary operators in algebraic topology. [21, 22] The physical quantities we define on objects like 0, 1, 2 & 3 simplex are called 0, 1, 2, 3-cochains, respectively.

7. Parallel Plate Capacitor Modeling

The energy of electrons is stored in the form of an electrical charge on the plates of capacitors. The larger the plates and/or the narrower their separation, the greater the charge held by the capacitor for any given voltage across its plates. So, we designed a parallel plate capacitor in the cad software that gives a 3-dimensional structure of the capacitor. In Algebraic topological method the domain is discretized into cells known as volumes, which can be of any form. To best model a multi-scale problem, we can use tetrahedral cells. So, to retrieve primal a^1 matrix and dual a^3 matrix for the Laplace equation we are going to generate a mesh grid for the designed parallel plate capacitors.

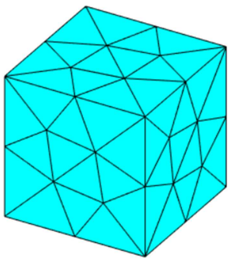

Figure 6. *Generating mesh for parallel plate capacitor.*

The above figure shows the formation of mesh grid for retrieving the primal and dual simplex. So, with nodes and elements we can able to retrieve 0-simplex (nodes) and 3-simplex respectively. In 2D space elements gives 2-simplex (surface) but here we are using 3D space hence we can able to retrieve 3-simplex (volume). So, from Laplace Algebraic topological formulation is the combination of primal a^1 and dual a^3 matrix. Primal matrix is the combination of 0-simplex and 1-simplex with the data of 0-simplex and 3-simplex performing matrix formulation we can able to retrieve 1-simplex (line) and 2-simplex (surface) So by forming the primal and dual matrix from the data that we retrieved and calculating the coefficients we will get the electric potential in the parallel plate capacitor as shown below diagram.

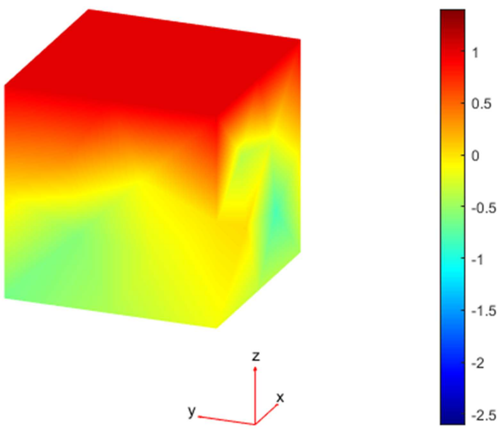

Figure 7. *Field in parallel plate capacitor through simulations.*

So, from the above diagram we are able to observe that the potential drastically decreases from the top plate to bottom plate only by using global variables and topological objects.

Table 3. *Comparison of algebraic topological method and analytical method.*

(V = $\frac{V0}{d}z$) Analytical method	Algebraic topological method
Z = -1 = V = -0.5	V = -0.56
Z = -0.8 = V=-0.4	V= -0.52
Z = -0.5 = V=-0.25	V=-0.36
Z = -0.3 = V = -0.15	V = -0.3
Z = 0 = V = 0	V = 0
Z = 0.2 = V = 0.1	V=0.24
Z = 0.4 = V = 0.2	V = 0.43
Z = 1 = V = 0.5	V = 0.7

8. Conclusion

As we saw in this study, algebraic topology is dealing with fairly basic but fundamental properties of space. while simulating the electromagnetic Unlike traditional methods such as finite-difference, finite-element, and finite-volume, the algebraic topological method provides more simple and discrete formulations using global variables. with this knowledge we had simulated parallel plate capacitor using laplace equation and compared the potentials with analytical method and Algebraic topological method.

9. Future Identification

When working in several dimensions, algebraic methods become increasingly relevant, and increasingly sophisticated elements of algebra are now being used. [23-27] Algebraic topology, a Field of mathematics that uses algebraic structures to study geometric object's transformations. It uses functions to represent continuous transformations. Some new uses of ATM tools in biomedicine, thermo-electrics, quantum burrowing, radar distant detecting worth mentioning.

References

[1] Sommerfeld, A., Electrodynamics - Lectures on Theoretical Physics, Vol. 3, Academic Press, Inc, 1952.

[2] W. L. Burke, Applied Differential Geometry. Cambridge: Cambridge University Press, 1985.

[3] Stratton, J. A., Electromagnetic Theory, McGraw-Hill Book Company, 1941.

[4] G. A. Deschamps, "Electromagnetics and differential forms," in IEEE Proceedings, vol. 69, pp. 676–696, 1981.

[5] Tonti, E., "Why starting with differential equations for computational physics," Journal of Computational Physics, Vol. 257, 1260–1290, 2014.

[6] Algebraic Topological Method: An Alternative Modelling Tool for Electromagnetics.

[7] Burke, W. L., Div, grad, curl are dead, version 2.0, Oct 1995.

[8] K. Sankaran and B. Sairam, “Modelling of nanoscale quantum tunnelling structures using algebraic topology method,” in AIP Conference Proceedings.

[9] K. Sankaran, “Old tools are not enough: Recent trends in computational electromagnetics for defense applications,” DRDO Defence Science Journal, 2018. In review.

[10] K. Sankaran, “Perspective: Are you using the right tools in computational electromagnetics?” Journal of Applied Physics, 2018.

[11] Aakash, A. Bhatt, and K. Sankaran, “How to model electromagnetic problems without using vector calculus and differential equations?” IETE Journal of Education, vol. 59, no. 2, 2018.

[12] E. Tonti, “Why starting with differential equations for computational physics,” J. Comput. Phys., Vol. 257, no. Part B, pp. 1260–1290, 2014.

[13] R. Feynman, Lectures in Physics vol. 2. New York: Addison-Wesley.

[14] T. Karunakaran, “Algebraic structure of network topology,” in Proceedings of the Indian National Science Academy, vol. 41, pp. 213–215, 1975.

[15] ALGEBRAIC TOPOLOGY AND COMPUTATIONAL ELECTROMAGNETISM E. Tonti University di Trieste, 34127 Trieste, Italia.

[16] T. Tarhasaari and L. Kettunen, “Topological approach to computational electromagnetism,” Progress Electromagn. Res., Vol. 32, pp. 189–206, 2001.

[17] H. M. Schey, Div, Grad, Curl and All That - An Informal Text on Vector Calculus. 1 ed., W. W. Norton & Company, 1973.

[18] A brief history of maxwells equation document-8685744.

[19] Deschamps, Georges. A.,“Electromagnetics and Differential Forms,” Proceedings of the IEEE, Vol. 69, No. 6, 676–696, Jun 1981.

[20] Fan, Ting-Jun, Describing and Recognizing 3D Objects Using Surface Properties, Berlin-New York, 1990.

[21] Hatcher, Allen, Algebraic topology, Cambridge University Press, Cambridge, 2002.

[22] Langefors, B., Algebraic Topology and Networks, Technical Report TN 43, Svenska Aeroplan Aktiebolaget, 1959.

[23] An algebraic topological method and future identification Erik Carlsson, Gunnar Carlsson, and Vin de Silva.

[24] Recent Trends in Computational Electromagnetics for Defence Applications._Computational_Electromagnetics_for_Defence Application.

[25] J. C. Maxwell, “A dynamical theory of the electromagnetic field,” Philos. Trans. R. Soc. London, Vol. 155, pp. 459–512, 1865.

[26] E. Tonti, The Mathematical Structure of Classical and Relativistic Physics - A General Classification Diagram. Basel: Birkhäuser Basel.

[27] A. Shaji and K. Sankaran, “Thermal integrity modelling using finite-element, finite-volume, and algebraic topological methods”, inAIP Conference Proceedings.