

Characterization of Associative PU-algebras by the Notion of Derivations

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Abstract: In this manuscript we insert the concept of derivations in associative PU-algebras and discuss some of its important results such that we prove that for a mapping being a (Left, Right) or (Right, Left)-derivation of an associative PU-algebra then such a mapping is one-one. If a mapping is regular then it is identity. If any element of an associative PU-algebra satisfying the criteria of identity function then such a map is identity. We also prove some useful properties for a mapping being (Left, Right)-regular derivation of an associative PU-algebra and (Right, Left)-regular derivation of an associative PU-algebra. Moreover we prove that if a mapping is regular (Left, Right)-derivation of an associative PU-algebra then its Kernel is a subalgebra. We have no doubt that the research along this line can be kept up, and indeed, some results in this manuscript have already made up a foundation for further exploration concerning the further progression of PU-algebras. These definitions and main results can be similarly extended to some other algebraic systems such as BCH-algebras, Hilbert algebras, BF-algebras, J-algebras, WS-algebras, CI-algebras, SU-algebras, BCL-algebras, BP-algebras and BO-algebras, Z- algebras and so forth. The main purpose of our future work is to investigate the fuzzy derivations ideals in PU-algebras, which may have a lot of applications in different branches of theoretical physics and computer science.

Keywords: PU-Algebras, (Left, Right)-derivations of PU-algebras, (Right, Left)-derivations of PU-algebras, Regular Derivations of PU-algebras.

1. Introduction

In 1966, Y. Imai and K. Isèki introduced two classes of abstract algebras: BCK-algebras and BCI-algebras [1-3]. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. Neggers et al. [4] introduced a notions, called Q-algebras, which is a generalization of BCH / BCI / BCK-algebras and generalized some theorems discussed in BCI-algebras. Megalai and Tamilarasi [5] laid down the foundation of a notion, called TM-algebra. Moreover, Mostafa et al. [6] introduced a new algebraic structure called PU-algebra, which is a dual for TM-algebra and investigated several basic properties. Moreover they derived new view of several ideals on PU-algebra and studied some properties of them. Derivation is a very interesting and important area of research in the theory of algebraic structures in mathematics. Several authors [7-11] have studied derivations in rings and near rings. Jun and Xin [12] applied the notions of derivations in ring and near-ring theory to BCI-algebras, and they also introduced a new concept called a regular Derivation in BCI-algebra. They investigated some of its

properties, defined a d -derivations ideal and gave conditions for an ideal to be d-derivations. Later, Abujabal and Al-Shehri [13], defined a left derivations in BCI-algebras and investigated a regular left derivations. Zhan and Liu [14] studied f-derivations in BCI-algebras and proved some results. Muhiuddin and Al-roqi [15, 16] introduced the notions of (α, β) -derivations in a BCI-algebras and investigated related properties. They provided a condition for a (α, β) - derivations to be regular. They also introduced the concepts of a d (α, β) - invariant (α, β) - derivations and α -ideal, and then they investigated their relations. Furthermore, they obtained some results on regular (α, β) - derivations. Moreover, they studied the notions of t-derivations on BCI-algebras [17] and obtain some of its related properties. Further, they characterized the notions of p-semisimple BCI-algebras X by using the notions of t-derivations. Abujabal and Shehri in their pioneer paper [18], defined the derivations as, for a self-map, d , for any algebra X, d is a left-right derivation (briefly (l, r) -derivation) of X if it satisfies the identity $d(a * b) = (d(a) * b) \wedge (a * d(b))$. For all $a, b \in X$. If d satisfies the identity $d(a * b) = (a * d(b)) \wedge (d(a) * b)$ for all $a, b \in X$,

then d is a right-left derivation (briefly (r, l) -derivation) of X . If d is both (l, r) -derivation and (r, l) -derivation, then d is a derivation of X . The aim of the paper is to complete the studies on PU-algebra; in particular, we aim to apply the notion of derivation on associative PU-algebra and obtain some related properties. We start with definitions and propositions on PU-algebra taken from [6].

Then, we redefine the notion of derivation in associative PU-algebra and prove that for ϕ being a (Left, Right) or (Right, Left) -derivation of an associative PU-algebra Z then ϕ is one-one map. If ϕ is a regular map then it is identity. If there exists an element $a \in Z$ such that $\phi(a)=a$ then the map ϕ is identity. We prove that If ϕ is (Left, Right) -regular derivation of Z then $\phi(a) = a \wedge \phi(a)$ also if ϕ is (Right, Left)-regular derivation of Z then $\phi(a)=\phi(a) \wedge a$, $\forall a \in Z$. We prove that if ϕ is a self-map of an associative PU-Algebra Z then $(a * (a * \phi(a))) * a = (\phi(a) * (\phi(a) * a)) * a$. We also prove that if ϕ is a regular (Right, Left)-derivation of an associative PU-algebra Z then $\text{Ker}(\phi) = \{a \in Z : \phi(a)=0\}$ is a subalgebra of Z .

2. Preliminaries

This section consists of some preliminary definitions and basic facts about PU-algebra which are useful in the proofs of our results. Throughout this research work we denote the PU-algebra always by Z .

Definition 2.1: [6] PU-algebra $(Z, *, 0)$ is a class of the type $(2, 0)$ algebras which satisfies the (P_1) and (P_2) conditions for all $p, q, r \in Z$, where

$$(P_1) 0 * a = a \quad (P_2) (a * c) * (b * c) = b * a$$

While the binary relation ' \leq ' on Z is defined as $a \leq b \Leftrightarrow b * a = 0$.

Proposition 2.2: [6] In PU-algebra $(Z, *, 0)$ the following results are true for all $a, b, c \in Z$.

$$(P_3) a * a = 0$$

$$(P_4) (a * c) * c = a$$

$$(P_5) a * (b * c) = b * (a * c)$$

$$(P_6) a * (b * a) = b * 0$$

$$(P_7) \text{ The following three results are similar in } (Z, *, 0)$$

$$(1): b = c \quad (2): b * a = c * a \quad (3): a * b = a * c$$

(P_8) Both (left and right) cancellation properties hold in $(Z, *, 0)$.

Definition 2.3: [6] PU-algebra $(Z, *, 0)$ is said to be associative if it satisfies the condition $a * (b * c) = (a * b) * c$ for all $a, b, c \in Z$.

3. Main Results

Definition 3.1:- Let $(Z, *, 0)$ is an associative PU-algebra and $\phi: Z \rightarrow Z$ is a self-map then ϕ called (Left, Right) -derivation on Z if $\phi(a * b) = (\phi(a) * b) \wedge (a * \phi(b))$.

Definition 3.2:- Let $(Z, *, 0)$ is an associative PU-algebra and $\phi: Z \rightarrow Z$ is a self-map then ϕ is called (Right, Left)-derivation on Z if $\phi(a * b) = (a * \phi(b)) \wedge (\phi(a) * b)$.

Definition 3.3: If ϕ is both (Left, Right)-derivation and (Right, Left)-derivation on Z then ϕ is called derivation on Z .

Definition 3.4: A self-map $\phi: Z \rightarrow Z$ on associative PU-

algebra Z is called regular if $\phi(0) = 0$.

Example 3.5: Let the set $Z = \{0, a, b, c\}$ defined by the following table.

Table 1. Tabular arrangement of the values of the set satisfying the axioms of associative PU-algebra.

*	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Is an associative PU-algebra and a map, $\phi: Z \rightarrow Z$ defined by

$\phi(0)=c, \phi(a)=b, \phi(b)=a$ and $\phi(c)=0$ is both (Left, Right)-derivation and (Right, Left)-derivation on Z and thus a derivation of Z .

Proposition 3.6: Let ϕ be a (Left, Right)-derivation of an associative PU-algebra Z then

$$(P_9): \phi(0)=\phi(a) * a, \forall a \in Z.$$

$$(P_{10}): \phi \text{ is one-one map.}$$

$$(P_{11}): \text{If } \phi \text{ is a regular map then it is identity.}$$

$(P_{12}):$ If there exists an element $a \in Z$ such that $\phi(a)=a$ then the map ϕ is identity.

$(P_{13}):$ If $\phi(b) * a = 0$ or $a * \phi(b) = 0$ then $\phi(b)=a, \forall a, b \in Z$ i.e. ϕ is constant.

Proof (P_9) :

$$\phi(0)=\phi(a * a), \because \text{by } (P_3)$$

$$=(\phi(a) * a) \wedge (a * \phi(a))$$

$$=(a * \phi(a)) * [(a * \phi(a)) * (\phi(a) * a)]$$

$$=[(a * \phi(a)) * (a * \phi(a))] * (\phi(a) * a), \because Z \text{ is associative}$$

$$=0 * (\phi(a) * a), \because \text{by } (P_3)$$

$$=(\phi(a) * a), \because \text{by } (P_1)$$

Proof (P_{10}) : Let $a, b \in Z$ such that

$$\phi(a)=\phi(b) \quad (1)$$

From (P_9) we have

$$\phi(0)=\phi(a) * a \quad \forall a \in Z \quad (2)$$

Also from (P_9) we have

$$\phi(0)=\phi(b) * b \quad \forall b \in Z \quad (3)$$

From (2) and (3) we get

$$\phi(a) * a = \phi(b) * b \quad (4)$$

Using the result of equation (1) in equation (4) we get

$$\phi(a) * a = \phi(a) * b \quad (5)$$

By (P_8) left cancellation law holds in Z therefore from (5) we get $a = b$. Hence ϕ is one to one.

Proof (P_{11}) : Let ϕ is regular then we have

$$\phi(0)=0 \quad (6)$$

From (P₉) we have

$$\phi(0)=\phi(a) * a \quad \forall a \in Z \quad (7)$$

From (6) and (7) we get

$$\phi(a) * a=0 \quad \forall a \in Z \quad (8)$$

Now by using (P₃) in the right hand side of equation (8) then (8) becomes

$$\phi(a) * a = a * a \quad \forall a \in Z, \because a * a=0 \quad (9)$$

By (P₈) right cancellation law holds in Z therefore (9) becomes $\phi(a)=a \quad \forall a \in Z$.

Hence ϕ is the identity map.

Proof (P₁₂): Let

$$\phi(a)=a \quad (10)$$

Now by proposition (P₇) equation (10) is equivalent to

$$\phi(a) * a = a * a \Rightarrow \phi(a) * a=0, \because \text{by (P}_3\text{)} \quad (11)$$

From (P₉) we have

$$\phi(0)=\phi(a) * a \quad \forall a \in Z \quad (12)$$

So now using the result of equation (12) in the left hand side of equation (11) we get

$\phi(0)=0 \Rightarrow \phi$ is regular which by (P₁₁) ϕ is the identity map.

Proof (P₁₁): Let

$$\phi(b) * a=0 \quad (13)$$

by proposition (P₃) equation (13) becomes

$$\phi(b) * a = a * a \quad (14)$$

by (P₈) right cancellation law holds in Z therefore (14) becomes $\phi(b)=a$.

Similarly for

$$a * \phi(b)=0 \quad (15)$$

by (P₃) equation (15) becomes

$$a * \phi(b)=a * a \quad (16)$$

by (P₈) left cancellation law holds in Z therefore (16) becomes $\phi(b)=a$.

Proposition 3.7:- Let ϕ be a (Right, Left)-derivation of an associative PU-algebra Z then

(P₁₄): $\phi(0)=a * \phi(a), \forall a \in Z$

(P₁₅): ϕ is one-one map.

(P₁₆): If ϕ is a regular map then it is identity.

(P₁₇): If there exists an element $a \in Z$ such that $\phi(a)=a$ then the map

ϕ is identity.

(P₁₈): If $\phi(b) * a=0$ or $a * \phi(b)=0$ then $\phi(b)=a, \forall a, b \in Z$ i.e. ϕ is constant.

Proof (P₁₄):

$$\phi(0)=\phi(a * a), \because \text{by (P}_3\text{)}$$

$$=(a * \phi(a)) \wedge (\phi(a) * a)$$

$$=(\phi(a) * a) * [(\phi(a) * a) * (a * \phi(a))]$$

$$=[(\phi(a) * a) * (\phi(a) * a)] * (a * \phi(a)), \because Z \text{ is associative}$$

$$=0 * (a * \phi(a)), \because \text{by (P}_3\text{)}$$

$$=a * \phi(a), \because \text{by (P}_1\text{)}$$

Proof (P₁₅): Let $a, b \in Z$ such that

$$\phi(a)=\phi(b) \quad (17)$$

From (P₁₄) we have

$$\phi(0)=a * \phi(a) \quad \forall a \in Z \quad (18)$$

Also from (P₁₄) we have

$$\phi(0)=b * \phi(b) \quad \forall b \in Z \quad (19)$$

From (17) and (18) we get

$$a * \phi(a)=b * \phi(b) \quad (20)$$

Using (17) in the right hand side of equation (20) we get

$$a * \phi(a)=b * \phi(a) \quad (21)$$

By (P₈) right cancellation law holds in Z therefore from (21) we get $a = b$ that is ϕ is one to one.

Proof (P₁₆): Let ϕ is regular then we have

$$\phi(0)=0 \quad (22)$$

From (P₁₄) we have

$$\phi(0)=a * \phi(a) \quad \forall a \in Z \quad (23)$$

From (22) and (23) we get

$$a * \phi(a)=0 \quad \forall a \in Z \quad (24)$$

Now by using (P₃) in the right hand side of equation (24) then (24) becomes

$$a * \phi(a)=a * a \quad \forall a \in Z \because a * a=0 \quad (25)$$

By (P₈) left cancellation law holds in Z therefore (iv) becomes $\phi(a)=a, \forall a \in Z$.

Hence ϕ is the identity map.

Proof (P₁₇): Let

$$\phi(a)=a \quad (26)$$

Now by proposition (P₇) equation (26) is equivalent to

$$a * \phi(a)=a * a \Rightarrow a * \phi(a)=0, \because \text{by (P}_3\text{)} \quad (27)$$

From (P₁₄) we have

$$\phi(0)=a * \phi(a) \quad \forall a \in Z \quad (28)$$

so now using (28) in (27) we get $\phi(0)=0 \Rightarrow \phi$ is regular which by (P₁₆) ϕ is the identity map.

Proof (P₁₈):

$$\text{Let } \phi(b) * a = 0 \quad (29)$$

by proposition (P₃) the right hand side of equation (29) becomes

$$\phi(b) * a = a * a \quad (30)$$

by (P₈) right cancellation law holds in Z therefore (30) becomes $\phi(b) = a$

Similarly

$$a * \phi(b) = 0 \quad (31)$$

by (P₃) equation (31) becomes,

$$a * \phi(b) = a * a \quad (32)$$

by (P₈) left cancellation law holds in Z therefore (32) becomes $\phi(b) = a$.

Theorem 3.8: Let Z is an associative PU-algebra

(P_a): If ϕ is (Left, Right) -regular derivation of Z then $\phi(a) = a \wedge \phi(a) \forall a \in Z$.

(P_b): If ϕ is (Right, Left) -regular derivation of Z then $\phi(a) = \phi(a) \wedge a \forall a \in Z$.

Proof (P_a): Since ϕ is regular therefore we have

$$\phi(0) = 0 \quad (33)$$

Now consider for some $a \in Z$ we have

$$\phi(a) = \phi(0 * a), \because \text{by (P}_1)$$

$$= (\phi(0) * a) \wedge (0 * \phi(a)), \because \text{by definition 3.1}$$

$$= (0 * a) \wedge (0 * \phi(a)), \because \text{by using (33)}$$

$$= a \wedge \phi(a), \because \text{by (P}_1)$$

Proof (P_b): Since ϕ is regular therefore we have

$$\phi(0) = 0 \quad (34)$$

Now consider for some $a \in Z$, $\phi(a) = \phi(0 * a)$, \because by (P₁)

$$= (0 * \phi(a)) \wedge (\phi(0) * a) \because \text{by definition 3.1}$$

$$= (0 * \phi(a)) \wedge (0 * a) \because \text{by using (34)}$$

$$= \phi(a) \wedge a \because \text{by (P}_1)$$

Theorem 3.9: Let ϕ is a self-map of an associative PU-Algebra Z then $(a * (a * \phi(a))) * a = (\phi(a) * (\phi(a) * a)) * a$.

Proof: Since by (theorem 3.8 (P_b)) we have,

$$\phi(a) = \phi(a) \wedge a = a * (a * \phi(a)) \quad (35)$$

By (P₇) equation (35) is equivalent to

$$\phi(a) * a = (a * (a * \phi(a))) * a \quad (36)$$

on the other hand from (theorem 3.8 (P_a)) we have

$$\phi(a) = a \wedge \phi(a) = \phi(a) * (\phi(a) * a) \quad (37)$$

Similarly by (P₇) equation (37) is equivalent to

$$\phi(a) * a = (\phi(a) * (\phi(a) * a)) * a \quad (38)$$

from (36) and (38) we get

$$(a * (a * \phi(a))) * a = (\phi(a) * (\phi(a) * a)) * a.$$

Theorem 3.10: If ϕ is a derivation on an associative PU-algebra Z then $\forall a \in Z$

$$(P_c): \phi(a * \phi(a)) = 0$$

$$(P_d): \phi(\phi(a) * a) = 0$$

Proof (P_c): Let ϕ is a (Left, Right)-derivation on Z then

$$\phi(a * \phi(a)) = (\phi(a) * \phi(a)) \wedge (a * \phi(\phi(a))) = 0 \wedge (a * \phi(\phi(a))), \because \text{by using (P}_3)$$

$$\Rightarrow \phi(a * \phi(a)) = (a * \phi(\phi(a))) * [(a * \phi(\phi(a))) * 0] \quad (39)$$

As Z is an associative PU-algebra therefore we can write equation (39) as

$$\phi(a * \phi(a)) = [a * \phi(\phi(a))] * (a * \phi(\phi(a))) * 0 = 0 * 0 = 0,$$

\because by using (P₃)

Proof (P_d): Let ϕ is a (Right, Left)-derivation on Z then

$$\phi(\phi(a) * a) = (\phi(a) * \phi(a)) \wedge (\phi(\phi(a)) * a) = 0 \wedge (\phi(\phi(a)) * a), \because \text{by using (P}_3)$$

$$\Rightarrow \phi(\phi(a) * a) = (\phi(\phi(a)) * a) * [(\phi(\phi(a)) * a) * 0] \quad (40)$$

As Z is an associative PU-algebra therefore (39) can be written as

$$\phi(\phi(a) * a) = [(\phi(\phi(a)) * a) * (\phi(\phi(a)) * a)] * 0 = 0 * 0 = 0, \because \text{by using (P}_3)$$

Theorem 3.11: Let ϕ is a regular (Left, Right)-derivation of an associative PU-algebra Z then the following results hold in Z .

$$(P_e): \phi(a) = a$$

$$(P_f): \phi(a) * b = a * \phi(b) \forall a, b \in Z$$

$$(P_g): \phi(b * a) = \phi(a) * b = \phi(a) * \phi(b) = a * \phi(b)$$

$$(P_h): \text{Ker}(\phi) = \{a \in Z: \phi(a) = 0\} \text{ is a subalgebra of } Z.$$

Proof (P_e): $\phi(a) = \phi(0 * a)$,

$$\because \text{by using (P}_1)$$

$$= (\phi(0) * a) \wedge (0 * \phi(a))$$

$$= (0 * a) \wedge (0 * \phi(a)), \because \phi \text{ is regular}$$

$$= a \wedge \phi(a), \because \text{by using (P}_1)$$

$$= \phi(a) * (\phi(a) * a)$$

$$= (\phi(a) * \phi(a)) * a, \because Z \text{ is associative}$$

$$= 0 * a = a \because \text{by using (P}_1)$$

Hence $\phi(a) = a$.

Proof (P_f): As ϕ is a regular (Left, Right)-derivation of an

associative PU-algebra \mathcal{Z} then by (P_e) we have

$$\phi(a)=a, \forall a \in \mathcal{Z} \quad (41)$$

And

$$\phi(b)=b, \forall b \in \mathcal{Z} \quad (42)$$

By using the results of equations (41) and (42) we get $\phi(a) * b = a * b = a * \phi(b)$.

Proof (P_g) : As ϕ is a regular (Left, Right)-derivation of an associative PU-algebra \mathcal{Z} then by (P_e) we have

$$\phi(a)=a, \forall a \in \mathcal{Z} \quad (43)$$

Therefore for any $a, b \in \mathcal{Z}$,

$$\text{We have } \phi(a * b)=a * b \because \text{by using } (P_e) \quad (44)$$

$$\Rightarrow \phi(a * b)=\phi(a) * \phi(b), \because \text{by equation (43)} \quad (45)$$

$$\Rightarrow \phi(a * b)=\phi(a) * b, \because \text{by equation (43)} \quad (46)$$

$$\text{And } \phi(a * b)=a * \phi(b) \because \text{by equation (43)} \quad (47)$$

The equations (45), (46) and (47) imply that

$$\phi(b * a)=\phi(a) * b = \phi(a) * \phi(b)=a * \phi(b)$$

$$\text{Proof } (P_h): \text{ Let } a, b \in \text{Ker}(\phi) \text{ then } \phi(a)=0 \quad (48)$$

And

$$\phi(b)=0 \quad (49)$$

As ϕ is a regular derivation therefore from (P_g) we have,

$$\phi(a * b)=\phi(a) * \phi(b) \quad (50)$$

Using (48) and (49) in the right hand side of equation (50) we get $\phi(a * b)=0$

$$\Rightarrow a * b \in \text{Ker}(\phi) \Rightarrow \text{Ker}(\phi) \text{ is a subalgebra of } \mathcal{Z}.$$

4. Conclusion

We see that derivations with special properties play a central role in the investigation of the structure of an algebraic system.

The forthcoming study of derivations in PU-algebras may be the following topics are worth to be taken into account.

To describe left derivations in PU-algebras and investigate a regular left derivations by using this concept.

To introduce the concept of f-derivations, t-derivations, t-bi-derivations and (α, β) -derivations in PU-algebras.

To refer this concept to some other algebraic structures.

To consider the results of this concept to some possible applications in information systems and computer sciences.

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