
Adaptation of the Principles of Classical Dynamics to Microeconomic

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To cite this article:

Levin Boris Arkadievich, Levin Arkadij Isaakovich. Adaptation of the Principles of Classical Dynamics to Microeconomic. *Economics*. Vol. 10, No. 4, 2021, pp. 152-163. doi: 10.11648/j.economics.20211004.15

Received: August 17, 2021; **Accepted:** September 11, 2021; **Published:** December 2, 2021

Abstract: The principles of classical dynamics are applied to economic processes. The author successfully overcame the main difficulty of the basic requirement for dynamics: a description of any process should be made in real (four-dimensional) «space-time». The economic meaning for all four coordinate axes is stated that allows to introduce the concept of force interaction. Besides, J. Hicks understood more than anyone that in the analysis of microeconomic models four dimensional space (including time axis) should be considered. Unfortunately, having denoted two axes as commodities and one as a time axis he found two axes to be interdependent that allowed him to turn to two-axes coordinate system. Such a conversion is only possible if initial space is heterogeneous. In reality, however, if price is a vector quantity it should act as a vector in any spatial dimension. As for the time axis is concerned it is definitely defined by scalar property. The nature of phenomena and substance we experience in our sensations is like this. Doubtfully, that market relations which had begun in human society from the time immemorial, are not specified by these properties. The consequence of homogeneity' of the space is that we are able to analyze free motion of the point-denoting commodity in the presence of force interaction in three-dimensional space. Free motion of the point-denoting commodity means the lack of any kind of forces that limit its trajectory in any direction. The motion of the point is defined as explicitly stated periodical fluctuations of each elementary axis parameter depending on time. That is why in our reasoning we proceeded from the fact that economic space is homogeneous. Later, using the principles of special theory of relativity we shall be able to set relationship between different coordinate systems of homogeneous space in economics modeling. The only purpose of this article is to find out the third previously unknown axis in economics. The technique used to find its meaning is following. Consider the nature of interaction linked with general utility of commodity' or with total costs using time axis of commodity' quantity in three-coordinate system. Then taking into account the nature of force interaction the projection of force interaction or rather projection of a resultant of a system of force interactions should be found. The later projections on to previously unknown axis three dimensional space will clarify its economic meaning.

Keywords: The Microeconomic, Space in the Coordinate System, Demand-Supply-Time Parameter, Force Interaction in Space, Acceleration Vector of Price in Space

1. Introduction

Modern economics based on a concept of “scarce resources” in principle is divided into experimental [1-3] and mathematical [4, 5]. This approach seems to represent sufficiently organized system of concepts, views, and relations, which are proved by experimental data.

Nevertheless, tire existing concepts of economics are very limited because economic science based on tire concept of “scarce resources” does not take into account allocation of some

recourses by so called “coercive force” (that sometimes happens in very significant scope). Coercive force in many cases is a decisive factor, especially in a society with underdeveloped democratic principles or during transition periods. Such circumstances are often insurmountable obstacles for an economic modeling or predicting economic parameters.

That is why it is a very pressing issue is to widen the current concept of “scarce resources” by the introduction of force

allocation of resources or so called in economics [6] “coercive force”. But it is impossible to imagine “coercive force” allocation without principles of force interaction. And this in turn requires principles of classical dynamics [7]. The author is not aware of any other non-force principles for the description of similar force processes.

So it is necessary to transform all models of modern economics by using directly the force principles of classical dynamics. These principles (although rather intuitively) were confirmed (and are being constantly confirmed) by the majority of modern economists in particular by P. Samuelson [1], S. Fischer [2], J. Sacks [6], M. Mankiw [7] and others.

At the same time utilization of general dynamics principles is impossible without “space-time” (spatiotemporal) coordinate system. On the first stage of the research there is a necessity to clear out two prerequisite conditions for dynamic principles. The first one is four-dimensional space as a coordinate system, the second condition is to determine the factors of economic forces interaction between economics parameters. Of course, the initial stage of proposed research is related to microeconomics processes, which are the most elementary at present.

2. The Space of Economics and Its Coordinate System - Time Axis

Of all four axes of the economic space time axis is a crucial one because all economic processes run with time. Nobody has ever denied the correlation between the time each market works and calendar time (connected with the Earth spin). Indeed, each person, every family or household lives and works according to calendar time and this must be reflected in functioning of the markets. But there is a question to be answered: whether calendar time determines the fundamental parameter (price) of the market on the microlevel of demand and supply interaction. The answer should be negative since otherwise dependence of calendar time upon price should be implied. Market price clearly reveals properties of speed parameter and at the same time it does not include calendar time parameter. War time experience and famine periods in the USSR history (and the history of other countries) show that when there is a deficit of goods or services or substantial price reductions, calendar time is completely excluded from the market interaction of demand and supply, because consumers are ready to spend days and nights in lines for necessary goods.

Now in the foreground comes out the flow of goods supplied that in our marginal case equals the flow of goods bought by customers. Due to Marshall's works [8] modern economics stated indefinite dependence of price parameter (equilibrium price) on calendar time that allowed Marshall to introduce three different kinds of time periods distinguished by possibilities for entrepreneurs to change an output in response to price changes. Therefore, Marshall's market periods demonstrate the change in time axis scale as a result of process properties and its conditions. In other words, each of

Marshall's periods has different scale of time axis. This is nothing else but the fundamental principle of special theory of relativity [8], which says that every process or event must have its own scale of a time parameter.

Is it possible to begin the study of the economic processes (even the elementary ones) by using the principles of special theory of relativity? The answer is negative because the technique of the research in this area is well known and consists of time synchronization of absolutely all processes on the first stage. It means that the scale of time for all analyzed processes should be the same. In economics it is accomplished by selecting as a time axis the axis which characterizes the flow of goods demanded (q_d) and supplied (q_n). This intuitive usage of the axis of quantity of goods supplied as time axis originates from A. Comot, W. Jevons [9], A. Marshall [8], who started to analyze the interactions using not the axis of time but the axis of quantity of goods on the market q . Thus, time axis of quantity of goods supplied (q_n) or demanded (q_d) allows us to synchronize time axis. Quite naturally, the second step of the study of economic processes is to take into account the changes in the scale of time axis as it is required by special theory of relativity quantum and percussive waves. Riders Companies A. S., [9]. The time axis and parameter of speed.

If time synchronization at economic analysis favors the selection of quantity axis as time axis, according to general dynamics principles we must realize the existence of a parameter of speed (speed parameter). It is functionally dependent on the

$$P \text{ mdl. } d = \sum X_i / \sum q_i, \quad (1)$$

where parameter q and describes motion i. e. movement of goods from supplier to consumer. The only such parameter in this case is a price of a commodity. The price of a commodity is defined as amount of money per unit. It complies with the essence of speed parameter ($\$/\text{kg}$, $\text{€}/\text{bushel}$). The average price of marketed commodity can be calculated as follows: $\sum X_i$ - the total amount of utility derived by consumer from commodity bought $\sum q_i$.

Similarly, an average price can be calculated,

$$P \text{ mdl. } s = \sum_{i=1}^n Y_i / \sum_{i=1}^n q_i \quad (2)$$

where $\sum Y_f$ - the total amount that should be paid to an entrepreneur $\sum q_i$.

If we now move to the consideration of infinitely divisible commodities like flour, sugar, or milk, we can buy/sell infinitely small amount of commodity⁷. Thus, let us move to the model description.

Demand

$$Pd = dX / dq, \quad (3)$$

where dX - the infinitely small change in amount of money received by the producer from infinitely small piece of commodity dq .

Supply

$$Ps = dY / dq, \quad (4)$$

where dY - the infinitely small change in amount of money received by the producer from a sale of infinitely small piece of commodity dy .

3. The Law of Demand in “Pd – q” Coordinate System

The law of demand (Figure discovered experimentally for different kinds of goods can be represented by: linear dependence [1, 2], non-linear dependence (concave down curve) [6], step or discrete function when quantity of goods could not be expressed as a continuous function of price [6].

In our case we choose linear dependence. It does not reduce the level of generality at all. We could take nonlinear or discrete function. It will make calculations more complex but by no means will change tire generality of the model.

The linear dependence of demand in “P-q” coordinates (Figure 1) may be expressed as follows:

$$P_d = P_0 - k_0 \cdot q \tag{5}$$

Where

P_d is the current market price; P_0 - the initial maximal price provided infinitely small amount of commodity ($q \rightarrow 0$), k_0 - tire slope of the straight lure to the q axis, if α_0 is a sharp angle (Figure 2).

$$k_0 = \text{tga} \alpha_0 \tag{6}$$

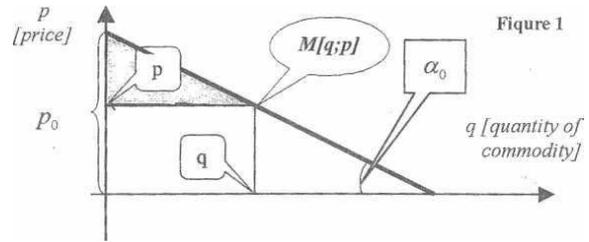


Figure 1. Coordinate axis of microeconomic space, which represents demand.

Substituting (1, 2, 3) into part 2

$$dX / dq = p_0 - k_0 \cdot q \text{ or } dX = p_0 \cdot dq - k_0 \cdot q \cdot dq$$

Integrating

$$\int dX = \int p_0 \cdot dq - \int k_0 \cdot q \cdot dq \text{ or } \tag{7}$$

$$X = p_0 \cdot q - k_0 \cdot q^2 / 2 + C \tag{8}$$

where

X - the total amount of utility derived by the buyer from consuming of commodity;

C - the constant of integration.

If we not that by $q > 0, X > 0$ we obtain: $0 = p_0 \cdot 0 - k_0 \cdot 0^2 / 2 + C \Rightarrow C = 0$

Substituting (9) into (8) and finally.

$$X = p_0 \cdot q - k_0 \cdot q^2 / 2 \tag{9}$$

The curve (3) is a parabola (Figure 2) that crosses q axis at 0 and $2p_0 / k_0$ with maximum at $q_{\max} = p_0 / k_0; X_{\max} = p_0^2 / 2k_0$.

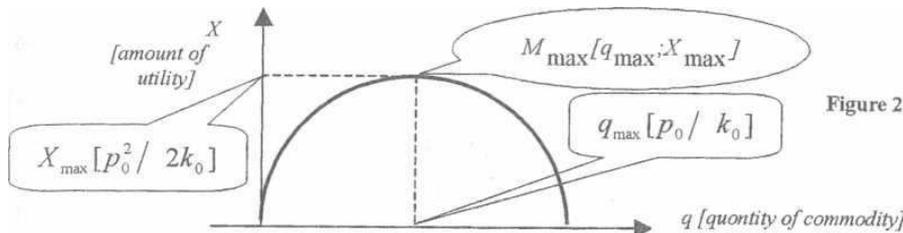
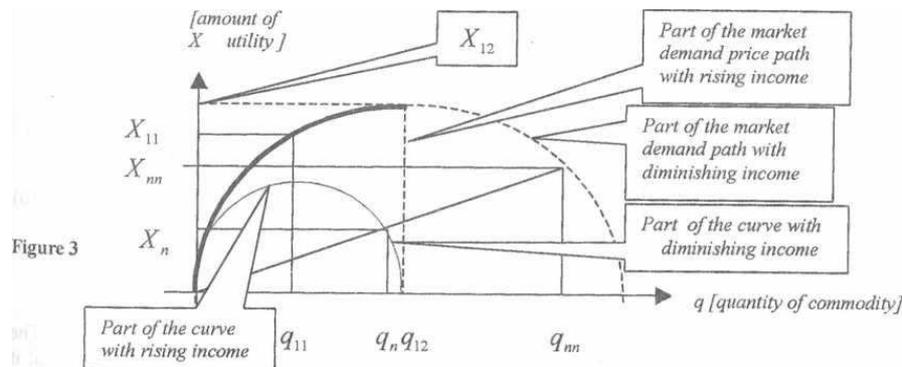


Figure 2. Parabola of the producers income.



$$\text{Parabola 1 - } X_{11} = 3p_0^2 / 8k_0; [X_{11} = p_0 \frac{p_0}{2k_0} - \frac{k_0}{2} (\frac{p_0}{2k_0})^2]$$

$$\text{Parabola 2 - } X_{12} = p_0^2 / 4k_0; [X_{12} = p_0 \frac{p_0}{2k_0} - \frac{k_0}{1} (\frac{p_0}{2k_0})^2]$$

Figure 3. Ordinates of its intersection points with axis p, q.

In order to find out the economic meaning we shall analyze the income of producer from commodity sold at a price p (Figure 1). The producer's income [1, 2, 6]:

$$X_1 = p \cdot q \tag{10}$$

Substituting (5) into (11):

$$X = (P_0 - k_0 \cdot q) \cdot q - p_0 \cdot q - k_Q \cdot q^2 \tag{11}$$

Comparing X with X_1 we find their dimensional representations to be equal. Moreover the curve (11) is an income curve. Ordinates of its intersection points with axes p , q are twice as small as of parabola (10).

$$\Delta X = X_{11} - X_{12} = p^2 / 8k_0 \tag{12}$$

If we now turn to Figure 1 the same q_x corresponds to the area of shaded triangle,

$$(\Delta X = p^2 / 8k_0, S = 0,5 \cdot q_i \cdot \Delta p)$$

Indeed, in our case $q = q_l$

where $\Delta p = p_0 - p$

p -M's ordinate; $p = p_0 - k_0 \cdot q_l = p_0 - k_0 \cdot (p_0 / 2k_0) = p_0 / 2$ but $\Delta p = p_0 - p_0 / 2 = p_0 / 2$

$$S = 0,5 \cdot (p_0 / 2k_0) \cdot (p_0 / 2) = p_0^2 / 8k_0 \tag{13}$$

Thus, ordinates of 1-st parabola (9) numerically equal to the area of formula (13) under the demand curve. The economic meaning of latest is known to be general utility of commodity. So, the economic meaning of the second axis X of microeconomic space is general utility of commodity.

4. The Law of Supply in "P-q" Coordinates

Dependence of supply of commodity in "P-q" coordinates is known in economics as the law of supply [1, 2, 6]. We will use linear dependence as we did for demand function. Linear dependence of supply is represented by

$$p_s = p_1 + k_1 \cdot q \tag{14}$$

where p_s - the price of supplies at quantity q , p_1 - the initial price that represents initial production costs necessary to start production, k_1 - the slope of supply line ($k_1 = \text{tga } \alpha$).

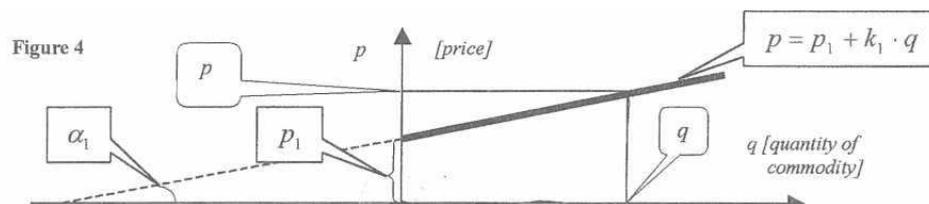


Figure 4. Coordinate axis that represents market supply.

$$dY / dq = p_1 + k_1 \cdot q \text{ or } dy = p_1 \cdot dq + k_1 \cdot q \cdot dq \tag{15}$$

Integrating $\int dy = \int p_1 \cdot dq + \int k_1 \cdot q \cdot dq$ or

$$Y = p_1 \cdot q + k_1 \cdot q^2 / 2 + C \tag{16}$$

where Y - the total costs; C - the constant of integration; (for other variables check 4).

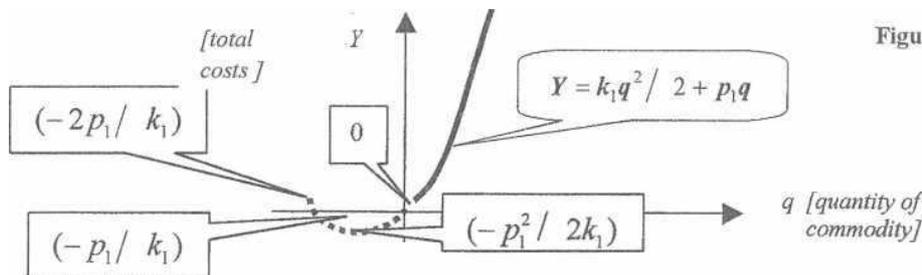


Figure 5. Total cost goods-quantity of goods.

If we note that by $q \Rightarrow 0$, we always need to cover initial costs of production, we substitute these costs into (16),

$$(-Y_1) = p_1 \cdot 0 + k_1 \cdot 0^2 / 2 + C \Rightarrow C = (-Y_1) \tag{17}$$

Substituting (17) into (16)

$$Y = P_1 \cdot q + k_1 \cdot (q^2 / 2) - Y_1 \tag{18}$$

If $Y=0$ function (18) is a parabola (Figure 5), which ordinates correspond to the area below supply line (Figure 4). The economic meaning of this area is total costs of production, so the economic meaning of the third axis Y is that it denotes total costs of entrepreneur offering his goods on the market.

4.1. The Analysis of Tree-dimensional X-Y-q Coordinate System

The X-Y-q coordinate system is not new in economic dynamics. Even J. Hicks [10] himself used one price axis and two axes for two interacting commodities and built tree-dimensional space, because it is no use to talk about dynamics in general case without space and time description and in particular case of Hicks' dynamics. Hicks wrote about it: 7n principle, it's easy enough but not quite convenient to work with graphs in three dimensions. Fortunately, we don't have to stay in three dimensions once we entered there. We can drop third dimension and turn to two-dimension system

[10]. It had happened because parameters denoting axes were not elementary as price (\$ apiece) and space axes were not homogeneous (first axis is Price, two others are quantities of goods) that implies dependency of axes on each other. It allowed him (by Hicks' own words) not to stay in the third dimension. But our research proved that the axis of speed (and so price axis) cannot describe economic space. All these obviously prevented Hicks from noting the link with force interaction when analyzing the income and substitution effects [10]. When we use axes X, Y, q in a rectangular coordinate system we have completely new economic space illustrated at Figure 5 (you can also find there descriptions of its coordinate planes). Figure 6, Pd - price of demand, Ps - price of supply.

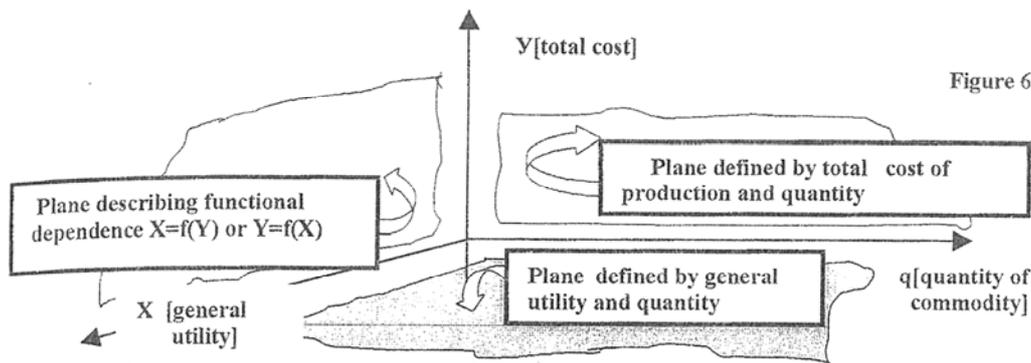


Figure 6. X-Y-q coordinate system of demand and supply.

Further direction of the research is to determine a correlation between proposed economic space and two - dimensional constructions of Hicks' and Marshall's' models. To do this we need to move initial X-Y-q coordinate system (figure to the system of speed parameters of this space (Figure 6, Pd - price of demand, Ps - price of supply).

and supply price equality.

Let us check whether it is possible to plot supply and demand curves in perpendicular planes. If it is possible, we can next transform the space of these perpendicular planes by aligning price axes of demand and supply (Figure 7) into one axis P (it is consistent with typical mathematical technique of expanding or contracting mathematical space when we space ton-dimensions or reduce it to n-1, n-2, n-3 ...).

This is a starting point to compare two models. According to Marshall [8] the equality of commodity quantity demanded and supplied at the equilibrium point originates from demand

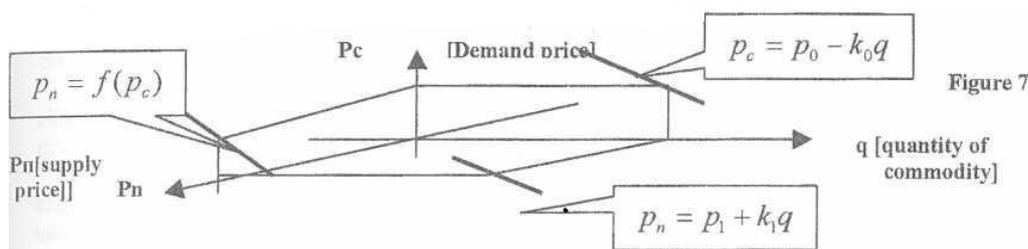


Figure 7. Interaction between demand price and supply price commodity.

If we take demand and supply equations (according to (5) and (14)) and express q we obtain:

$$q = \{p_0 - p_d\}/k_0 \iff q = \{p_s - p_1\}/k_1, \text{ or } (p_0 - p_d)/k_0 = (p_s - p_1)/k_1 \tag{19}$$

At the equilibrium price $p_d = p_s = p \implies k_1 * p_0 - k_1 * p = k_0 * p - k_0 * p_1$ we obtain for the equilibrium price

$$p = (k_1 * p_0 + k_0 * p_1) / (k_0 + k_1) \tag{20}$$

The latest expression of equilibrium price allowed Marshall [8] and later Hicks [10] to analyze successfully the market equilibrium in the vicinity of the intersection point.

But here we should draw attention to the third coordinate plane Ps-0-Pd (figure 6).

We can obtain equations for this plane directly from (20) i.e.

$$p_0 \cdot k_1 - p_1 \cdot k_0 = p_0 \cdot k_0 - p_1 \cdot k_1 \Rightarrow p_0 \cdot k_0 = (p_0 \cdot k_1 + p_1 \cdot k_0) - k_1 \cdot p_1 \tag{21}$$

$$\text{Then } p_0 = (p_1 + k_1 / k_0 \cdot p_0) - k_1 / k_0 \cdot p_1 \Rightarrow p_1 = (p_1 + k_0 / k_1 \cdot p_1) - k_0 / k_1 \cdot p_0 \tag{22}$$

Expressions (21) represent straight lines missing point of the origin. The analysis of these expressions using the model introduced below creates many other concepts and insights.

But it is even more significant result of three-dimensional analysis that we now have the possibility to represent force interaction between factors of economic models that will later allow us to introduce fourth (the last) dimension.

4.2. Price as a Vector Quantity and Path of the Point Denoting Commodity

Many prominent economists have long made the assumptions about force interaction in the economic life of a society. Over hundred years ago Bohm-Baverk wrote: "In my opinion the easiest, natural and most fruitful point of view on exchange and price is when we consider price formation as a result of people's estimations. It is not a fairy-tale but a reality itself. Firstly, the real forces act in price formation process. Certainly, not physical but psychological. These forces are desires: desire of buyer to get commodity and desire of entrepreneur to get money. The intensity of this force is determined by utility which buyer derives from consuming desirable thing". Only hundred years later we read about further development of the idea that force interaction is apparently connected with price fluctuation on the market. It allows us to realize the principle of such interaction for infinitely divisible commodity. Price as a vector quantity and path of the point denoting commodity.

If we check the supply and demand curves for vector properties, we shall find lack of these, because a price of a commodity depends on its quantity that is not a vector. But if we use the path of utility we derived from general dynamics principles, tire influence of the very same price P on X ordinate will also depend on a direction of these changes. If the direction of price change coincides with the positive direction of X-axis, the general utility will rise relatively fast. On the other hand, if price change coincides with the negative direction of X-axis, the general utility will rise much slower.

The economic sense of this property is following. The consumer aspires to buy more commodity in advance if die price is rising. The same situation is for a tendency of price lowering. The consumer will delay his purchases waiting for even lower price.

The presence of vector properties of price can be found everywhere. For example, advertising usually contributes to the direction of a price vector to an increase of the general utility function.

Figure 8 shows price vector pd (positively directed along X-axis) contributing to an increase of general utility by stimulating customers to buy the next portion of commodity. On die contrary, price vector that has opposite direction along X-axis slows down general utility growth. Similar arguments may be applied to the total cost function when only the tendency of price increase stimulates entrepreneurs to produce

more goods. The market cannot wait long since there is a possibility to substitute this commodity for another one if the price vector for the latest does not change Figure 9 show's price vector P_s , positively directed on Y-axis and so contributing to an increase of total costs, contrarily, oppositely directed price vector leads to lowering of total costs.

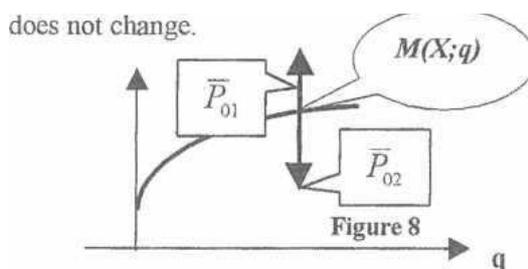


Figure 8. Price as a vector quantity and path of the point denoting commodity.

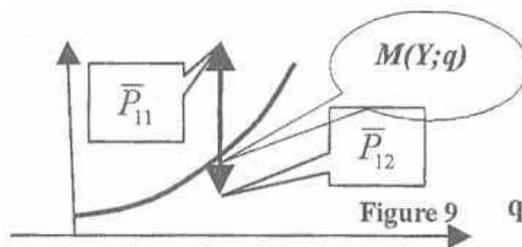


Figure 9. The reflection of price as a vector quantity and path of the point denoting commodity.

4.2.1. The Concept of Acceleration in Microeconomics

The concept of acceleration as a change in a commodity price is long in the air. The best economics textbooks while explaining interaction of supply and demand talk about interactions linked with price changes. McConnell and Bru's textbook [3] describes these interactions in following words: "push", "push the price down", "market forces"; Samuelson's [1] - "pressure", "price will push..."; Heine and Porter [12] explicitly talk about forces moving both demand and supply. Following the principles of dynamic interaction, we can formulate the concept of force interaction more rigorously. Since the price is a speed parameter and it describes die path of a motion, the limit of die change in price vector $\Delta P(\text{middle})$ per unit of commodity determines the vector of acceleration that characterize die force interaction for infinitely divisible commodity:

$$a_{mdl} = \lim \Delta p_{mdl} / \Delta q \text{ or } a_{mdl} = dp_{mdl} / dq, \tag{23}$$

where dp_{mdl} - the infinitely small change in the price vector per infinitely small change in quantity of commodity on die market.

The concept of acceleration or rather acceleration vector is applicable as for demand so for supply. Since acceleration in economics is vector quantity it has absolute value ($|a_{mdl}|$) and direction $\pm a$.

+a - positive direction of acceleration vector when it is directed along price vector or what is absolutely the same when a motion of the point denoting commodity along general utility or total cost function is positively directed against X, Y axes receptively (Figure 7, Figure 8);

-a - negative direction of acceleration vector, when it is oppositely directed against price vector or what is absolutely the same when a motion of the point denoting commodity along general utility or total cost function is negatively directed against X, Y axes receptively (Figure 7, Figure 8);

Example #1. Find out the absolute value and direction of acceleration vector of the point denoting commodity moving along general utility function. If demand function is $p_d = p_0 - k_0 * q$. Since $a = d p_d / dq$, then (24)

$$a = d(p_0 - k_0 * q) / dq \tag{24}$$

The absolute value of acceleration equals the slope of demand curve (in our case is constant). Negative sign reflects the fact that acceleration vector has opposite direction to price vector.

Example #2. Find out the absolute value and direction of acceleration vector of the point denoting commodity moving along total cost function.

If supply function is $p_s = p_x + k_1 * q$.

Since $a = dp_s / dq$, then $a = d(p_x + k_1 * q) / dq = +k_1 = \text{const.}$ (25)

The absolute value of acceleration equals the slope of supply line (in our case is constant). Positive sign reflects the fact that acceleration vector has the same direction with price vector.

4.2.2. The Factors That Make the Point of Market Interaction Move Along General Utility or Total Cost Functions

Using (23) we can select parameters of a motion for the market interaction for infinitely divisible commodity:

- 1) speed parameter p (price vector), its direction depends on a tendency of price change;
- 2) acceleration vector a expressed as a change in price related to a change in commodity quantity.

Let us look at the contribution of each parameter. Look at the market demand. If price vector p is constant then purchases of a commodity may be stable for relatively long time. It means, that a consumer purchases tire next portion of a commodity regularly. Since price is not changing, acceleration is equal 0. In this case we have a motion along the general utility function with constant price vector.

Another case when price vector changes the absolute value and/or direction. In this case there is a price change that causes purchases of additional portion of a commodity. This case as stated above is linked with acceleration.

The same situation is for the supply. If the price vector p is constant the supply will not change for quite a long time (but certainly limited time). Total cost depending on supply of commodity will rise. Since the price is not changing, acceleration along price vector p is zero. In this case we have tire path along total cost function with constant price vector.

Another case when price vector changes the absolute value and/or direction. In this case there is a price change which causes producer to supply additional portion of commodity. This change is due to acceleration of price.

Thus, on the one hand the probability of constant supply price and on the other, the probability of price changes for sold or unsold amount of commodity related to these changes, require unified research technique that takes into account both properties of a motion. This technique should include the "history" of the process i.e. parameters of a motion which existed before analyzed period on the time axis (in our case axis q - quantity of commodity). Such a technique is typical for mathematical sciences. It includes initial speed parameter. In our case it is the vector of initial price. The further analysis is reduced to the study of force interaction and movement along general utility or total costs path.

4.2.3. The link Between Force Interaction and Acceleration Vector

Newton was tire first who had found and explained the link between force interaction and acceleration. From the economic dynamics point of view the relation between force interaction and acceleration does not differ from the concepts of natural philosophy i.e. die vector of force interaction between consumer and seller, between a producer of a commodity and the market of this commodity is directly proportional to acceleration vector (check 3.2.1).

Certainly, above-mentioned relation in verbal description can be found in the best economics textbooks and researches. This relation between the force interaction and acceleration may be stated mathematically as follows:

$$F_{mdl} = m * a_{mdl} \tag{26}$$

where F_{mdl} - tire vector of force (force interaction); a - the vector of acceleration (see (23));

m- coefficient of proportionality between force interaction vector and acceleration vector.

Example #3. Find out the force interaction vector applied to the point denoting commodity on the general utility path (demand function is taken from example #1).

Since the absolute value of acceleration from p. 3.2.1 example is equal

$|a| = k_0$, the magnitude of force interaction vector is:

$$F_{mdl} = m * k_0 \tag{27}$$

This vector will be directed along acceleration vector k_0 , which direction is opposite to the price vector P_d according to (24).

Let us denote F as a force interaction of demand.

Example #4. Find out the force interaction vector applied to tire point denoting commodity on tire total cost path (supply function is taken from example #2 part 3).

Since absolute value of acceleration from example #2, part 3 is equal $|a_{mdl}| = k_1$, the magnitude of force interaction vector is:

$$R_{mdl} = m * k_1 (mdl) \tag{28}$$

This vector will be directed along acceleration vector $k_1(\text{mdl})$, which direction is opposite to the price vector p_s according to (25).

Let us denote R as a force interaction of supply.

4.2.4. The Testing of Applicability of the Equation of Dynamics to Demand

The process of testing is following:

1. In this research experimentally obtained demand function. ($p_d = p_0 - k_0 * q$ in “ $p_d - q$ ” coordinates) and supply function ($p_s = p_1 + k_1 * q$) are used. These functional relations were confirmed by experimental data.
2. As a result of integration we plotted the general utility function in “ $X - q$ ” coordinates and total cost function in “ $Y - q$ ” coordinates. These functions are indirect results of the experimental research.
3. The further step of the testing is to express four curves from part (2) using the basic equation of dynamics (mainly by projection on to proper axis). The accelerations are taken from tire examples 1, 2 that enables us to calculate the values of force interaction.

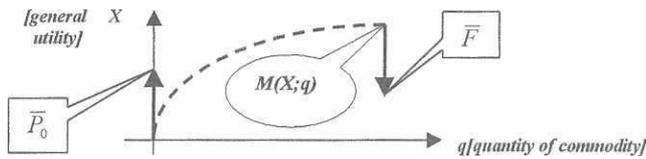


Figure 10. The testing of applicability of the equation of dynamics to demand.

Example #5. Find out the initial demand and the general utility function. The vector of initial price P_0 is positively directed with respect to X and P axes. Accordingly, the vector of force interaction F will be directed oppositely to the vector of initial price (see example 1 in 24). Figure 10 shows the vectors $P_{0(\text{mdl})}$ and $F_{(\text{mdl})}$ “ $X - q$ ” coordinates. Let us formulate the four-dimensional dynamics equation in tire differential form and in the projection onto X, q axes:

In projection onto X -axis

$$m * d^2X/dq^2 = F_x, \tag{29}$$

where F_x - is the projection of the force interaction of demand on to X -axis.

Similarly projecting into q -axis:

$$m * d^2X/dq^2 = F_0 \tag{30}$$

where 0 -is the projection of the force interaction of demand on to Y -axis The projection of the force interaction of demand onto X -axis is $F_x = -F$.

Therefore

$$m * d^2X/dq^2 = -F \text{ or } m * d^2X/dq^2 = -m * k_0 \tag{31}$$

Then $= dp/dq = k_0$ or $dp = -k_0 * dq$. Integrating

$$\int dp = -\int k_0 * dq \text{ or } p = -k_0 * q + C, \tag{32}$$

where C - constant of integration. From Figure 9 we have, that

if $q=0$ in projection onto X -axis.

$$p = p_0 \tag{33}$$

Substituting (33) into (32):

$$p_0 = k_0 * 0 + C \Rightarrow C = p_0 \tag{34}$$

Taking into account (34) the expression (32) will be:

$$p = p_0 - k_0 * q \tag{35}$$

So we obtained experimentally proved demand line in the form of (5).

$$\text{Substituting (3) into (35)} \Rightarrow dX/dq = p_0 - k_0 - q \tag{36}$$

$$\text{or } \int dX = \int p_0 * dq - \int k_0 * q * dq \Rightarrow X = p_0 * q - k_0 * (q^2/2) + C, \tag{37}$$

where C -constant of integration.

According to Figure 9 general utility starts from the origin point.

Therefore if

$$q = 0 \Rightarrow X = 0 \tag{38}$$

Substituting (38) in (37),

$$0 = p_0 * 0 - k_0 * (0^2/2) + C \Rightarrow C = 0 \tag{39}$$

Substituting for condition (38), we have general utility curve (39)

$$X = p_0 * q - k_0 * q^2/2 \tag{40}$$

Let consider projection of the force onto q -axis that in dynamic equation equals second derivative d^2q/dq^2 . Since first derivative is $dq/dq=1$, then

$$d^2q/dq^2 = d(1)/dq = 0 \tag{41}$$

Therefore equation (41) is an identity and we do not consider it any further.

As a result of the testing in example 1 following conclusion can be made:

1. In the space of economic force interactions are subject to tire basic equation of dynamics and as in many other sciences these interactions take the form of the motion along certain path.
2. The fact that theoretical models agree with experimentally proved ones allows us to determine the directions of vectors of force interactions between demand and supply and initial vectors of demand and supply prices. All four vectors are parallel to X, Y axes and perpendicular to q axis.

That the value q is scalar stresses its “temporal” nature.

4.3. The Attributes of Three-dimensional space In Microeconomics (Except Time Axis)

J. Hicks [10] understood more than anyone that in the analysis of microeconomic models four dimensional space (including time axis) should be considered. Unfortunately, having denoted two axes as commodities and one as a time

axis he found two axes to be interdependent that allowed him to turn to two-axes coordinate system. Such a conversion is only possible if initial space is heterogeneous. In reality, however, if price is a vector quantity it should act as a vector in any spatial dimension. As for the time axis is concerned it is definitely defined by scalar property. The nature of phenomena and substance we experience in our sensations is like this. Doubtfully, that market relations which had begun in human society from the time immemorial, are not specified by these properties. The consequence of homogeneity' of the space is that we are able to analyze free motion of the point-denoting commodity in the presence of force interaction in three-dimensional space.

Free motion of the point-denoting commodity means the lack of any kind of forces that limit its trajectory in any direction. The motion of the point is defined as explicitly stated periodical fluctuations of each elementary axis parameter depending on time q . That is why in our reasoning we proceeded from the fact that economic space is homogeneous. Later, using the principles of special theory of relativity we shall be able to set relationship between different coordinate systems of homogeneous space in economics modeling.

The only purpose of this article is to find out the third previously unknown axis in economics. The technique used to find its meaning is following. Consider the nature of interaction linked with general utility of commodity' or with total costs using time axis of commodity' quantity in XOY coordinate system. Then taking into account the nature of force interaction the projection of force interaction or rather projection of a resultant of a system of force interactions should be found. The later projections on to previously unknown axis Z will clarify its economic meaning.

4.3.1. Price Vector in XOY Coordinate System

From [8] it is known that the price vector as any other vector is characterized by the velocity of parameter change and relation of its direction with the motion trajectory. Therefore, it is a necessary to find out die direction of the vector of initial price in the starting point of the path. Since a trajectory' in XOY coordinate system is obtained by merging the parameters of general utility' and total cost which express accordingly demand and supply, the direction of the initial price vector can be found by adding the vectors of initial demand price and initial supply price.

$$p_{midl} = p_{0midl} + p_{lmidl}. \tag{42}$$

Then the slope of initial price vector is

$$\text{tg } X = p_1/p_0, \tag{43}$$

where p_1 -the absolute value of initial supply price; p_0 -the absolute value of initial demand price;

On the other hand, if we have equation describing the path and coordinates of the starting point $0(0,0)$ we can calculate the slope in any point of this path. Since the slope of the path is equal to the derivative of function in this Point; for $0(0,0)$ we have,

$$\text{tg } X_{1i} = Y'(X)_{X_0=0} \tag{44}$$

motion at $0,0$ point is linked with the direction of the initial price vector, which is directed along the tangent line at 0 point. where X_{1i} - the slope of the tangent line at $0(0,0)$. Comparing $0(43)$ with (44) we realize that the vector of initial price is directed along the tangent line (through the equality of (43) and (44)). Similar transformation can be made for any point of the curve. The vector of price is always directed along the tangent line because the beginning of the motion at $O(0,0)$ point is linked with the direction of the initial price vector, which is directed along the tangent line at O point.

4.3.2. Vector of Price Acceleration in XOY Coordinate System

Assume the price vector in at M of the commodity q path is represented by vector. By selling additional commodity price vector P_{midl} will be changed by ΔP_{midl}

(Figures 11 a, b, c):

$$P_1 = P_{midl} + \Delta P_{midl} \tag{45}$$

Then the change in the price at point M (Figure 11b) can be expressed as

$$\Delta P_{midl} = P_{midl} \cdot 1 + P_{midl} \tag{46}$$

Let divide vector ΔP_{midl} by Δq at M .

Then die quotient $\Delta P_{midl} / \Delta q$ will be changing and approaching to its limit (some vector).

$$a_{midl} = \lim \Delta P_{midl} / \Delta q \tag{47}$$

$\Delta q \Rightarrow 0$

Similarly, if we diminish commodity quantity Δq (with the beginning of the interval at M_1 point) the quotient will tend to the limit, and exactly to the vector a_{midl} , since the triangle of vectors does not change.

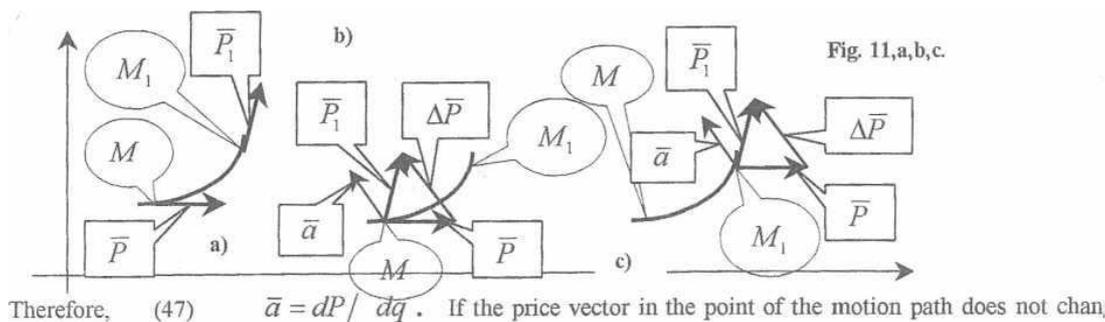


Figure 11. Vector of price acceleration in XOY coordinate system.

Therefore Figure 11

$$amidl = \Delta P / \Delta q \tag{48}$$

If price vector in the point of the motion path does not change, its absolute value moving from point M to M1 as a result of additional sale of a commodity, we also have similar acceleration because it changes its direction from point M to M1.

Analyzing figures 11 a, b, c we can conclude that all triangles of vectors, which are built in point M or M1 have die direction of dpmidl vector an so does vector amidl (in the direction of path curvature). This important property of acceleration allows to resolve the vector of price acceleration, firstly, into directions of coordinate axes and, secondly, into directions of tangent line and normal to tire path of the point denoting commodity M.

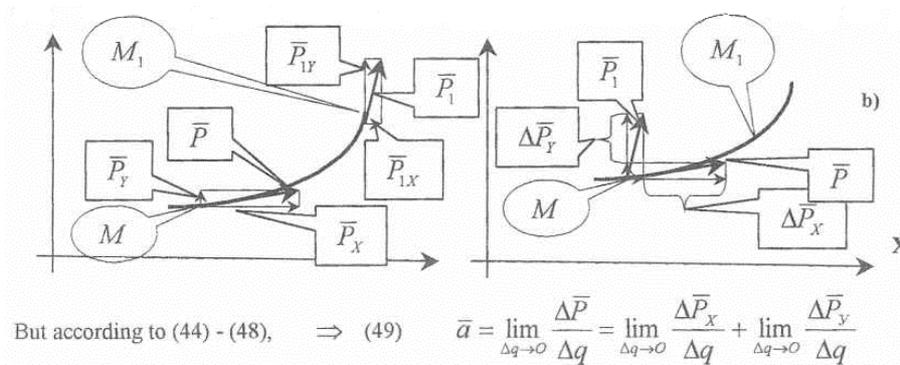


Figure 12. Change price due demand and supply.

where $a_{x, midl}$ - the acceleration vector by changing demand price or acceleration vector by the change of price into direction of X-axis (axis of general utility);

$a_{y, midl}$ - the acceleration vector by changing supply price or acceleration vector by the change of price into direction of Y-axis (axis of total cost).

Based upon (50) the absolute value of acceleration is equal

$$|amidl| = (a^2_x + a^2_y) \tag{50}$$

where a_x ; a_y - the absolute values of acceleration vectors by the changes in demand and supply prices receptively.

The second, even more important method of decomposition of the acceleration vector is based on such property as perpendicularity between a tangent line and normal at each point of the curve - rider M. A. Aizerman. In this case we have coordinate system moving along $Y=f(X)$ curve or as in p. 2 it is a motion about coordinate system: general utility X and total cost Y ('X-Y' coordinates are considered to be stationary).

The decomposition of the acceleration vector into normal and tangent line is well known from classical and theoretical mechanics. For the economic space, into which we transform common economic concepts, should be stated drat:

$amidl$ - the vector of total (resultant) acceleration is a sum of tangential acceleration $a_{r, midl}$ and normal, as acceleration.

$$amidl = (a_{r, midl} + a_{n, midl}) \tag{51}$$

Let us analyze decomposition of the vector of the point denoting commodity into axes of general utility X and total cost Y. It was proved earlier that directions of axes of demand price and general utility (axes Pd and X, Figures 12 a, b) as well as of supply prices and total cost (axes ps and Y, Figures 12 a, b) coincide. Therefore, the vector of demand price is parallel to X-axis; the vector of supply price is parallel to Y-axis. Axes X and Y are perpendicular to each other. If we use Figure 11 a and resolve the vectors into X, Y axes (Figure 12 a) and then use rectangular to resolve them into the components of vectors (Figure 12 b), we realize that it is possible to resolve the vector into X, Y axes.

$$\Delta P_{midl} = \Delta P_{midl x} + \Delta P_{midl y}, \tag{49}$$

$\Delta P_{midl x}$ - price change due to demand;
 $\Delta P_{midl y}$ - price change due to supply.

Since total acceleration is always directed along the curve, normal acceleration is always directed to the center (centripetal), absolute value of the total acceleration is

$$|amidl| = (a_r^2 + a_n^2)^{1/2} \tag{52}$$

The tangential acceleration is $a_{r, midl} = dP_{midl} / dq$ \tag{53}

the infinitely small change in the price vector per infinitely small change in commodity quantity on the market; the direction of a tangential acceleration is positive if it coincides with price vector and negative if it is opposite (price vector is always directed along tangent line). By constant price (constant absolute value) acceleration is equal to zero i.e. the point denoting commodity moves without tangential acceleration. The vector of normal acceleration is always directed to the center of curve and equals

$$a_n = (v^2 / \rho) * n_{midl} \tag{54}$$

where ρ - the radius of the curve in a given point.

By the constant absolute value of price and circular movement normal acceleration is constant.

4.3.3. Force Interaction in XOY Coordinate System

Based upon demand and supply laws, there are (he force of demand $F_{d, midl}$ and the force of supply $F_{s, midl}$, that can be found from the fundamental equation of dynamics:

$$F_d \text{ midl} = m * a_d \text{ midl} = m * a_x \text{ midl}; F_s \text{ midl} = m * a_s \text{ midl} = m * a_y \text{ midl}, \quad (55)$$

where $a_d \text{ midl}$ - the vector of demand acceleration;
 $a_s \text{ midl}$ - the vector of supply acceleration; $a_x \text{ midl}$ - the component of the total acceleration vector resolved into X-axis (axis of general utility);
 $a_y \text{ midl}$ - the component of total acceleration vector resolved into Y-axis (axis of total cost).

From,

$$a_d \text{ midl} = a_x; a_s \text{ midl} = a_y \quad (56)$$

Analyzing the total acceleration of the path in XOY coordinates we come to the conclusion that in fact one, common acceleration is directed along the path of a point. So we consider not two forces of supply and demand but one force of market interaction which correlates with total acceleration of the same direction. Then several variants of force interaction and its decomposition into coordinate axes can be used.

Above-mentioned decomposition of interaction force $F_s \text{ midl}$ into X, Y axes does not characterize motion of the point denoting commodity because it does not derive the components of force interaction. This component is obviously linked with direction of price vector $p \text{ midl}$, which unambiguously characterizes motion of this point. But as it was previously shown the direction of the vector is linked with tangential acceleration $a_t \text{ midl}$. Therefore, the force causing irregular motion of the point will be so called tangential force - rider M. A. Aizerman:

$$F_t \text{ midl} = m * a_t \text{ midl}. \quad (57)$$

The force F_{midl} will never be zero given the non-linear path; tangential acceleration may be 0 by the way $a_t \text{ midl} = 0$. Then force of normal acceleration is

$$F_s \text{ midl} = m * a_s \text{ midl}. \quad (58)$$

that will never be zero. Taking into account the fact that force interaction is resolved into mutually perpendicular directions: tangential and normal

$$F_s \text{ midl} = F_{midl} - F_t \text{ midl} \neq 0. \quad (59)$$

Where force $F_s \text{ midl}$ (centripetal) is directed to the center of the curve.

4.3.4. Dynamics of General Utility Path

Experimental economics considers plotting general utility curve as a static procedure of general utility accumulation in the form of acquired commodity. In the reality the procedure of general utility accumulation will be more complex if we take into account two factors:

1. The "history" of evaluation of general utility for previously bought goods;
2. The change in current price of commodity as compared to prices of previously bought goods.

These two factors bring in additional correlation into dynamics of interaction between the marginal price and

general utility. Thus, commodities at hand influence the general utility of a new commodity being bought at present moment. Essential point in our research is the difference made by dynamics between the curve (functional dependence) and the path of a motion. Only if we analyze the path of the point denoting commodity under force interactions can these factors be taken into account. Nevertheless, the history has two completely different properties; static and dynamic. According to static characteristics the purchase of the next portion of commodity for a given time period can be stopped. In this case the path is fixed (discrete) that is dynamically characteristic for so called "flexible line". Dynamic characteristic is linked with continual motion or with above-mentioned motion along "flexible line". Moreover, we contend that the actual motion along the general utility curve is discrete i.e. general utility curve is represented by quantity of commodity at hands that makes certain effect on additional purchases.

Indeed, after additional purchase the consumer has static (discrete) general utility of last purchase and forms static system of "flexible line". This static system is a marginal case of dynamic system (any dynamic system). This leads to the change in general utility of almost all commodities the consumer owns. It is important that general utility of last bought item of a commodity also changes. It serves as a measure of utility for the next item of commodity. Finally, from the dynamic point of view (or from economic dynamics point of view) general utility of the last acquired item of a commodity is initial general utility for this commodity. This initial general utility is static (discrete) and depends on time period between purchases. Using the concept of motion path and the model of "flexible line" we may consider the path of general utility of last unit of commodity' as boundary point. On the one hand it characterizes end-point of "flexible line", on the other hand initial price of the next path of the point-denoting commodity, which is a path of consumer. There are two meanings of force interaction at the point: firstly, as static force that is keeping stable position of "flexible line" for a given consumer; secondly as initial point of the next motion path of the point denoting commodity on the market with dynamic force interaction.

Estimating static force applied to point (as an end-point of "flexible line") we conclude that it should be related to that component of force interaction which does not cause motion of the point. Such force interaction can only relate to normal acceleration. Noting that when analyzing static system of force interaction, we have to find the force reaction (resultant force) in the form of:

$$N_{midl} = -F_H \text{ midl} - m * a_s \quad (60)$$

Force reaction is in XOY plane and a projection of the force at the initial point of the path, which influence point without dynamic effect. Thus, we obtain component of static force that produces total effect on the point P. This is an elementary case

but it allows us to formulate economic meaning of Z -axis of the three-dimensional economic space: in order to ensure market interaction in the form of perfect interaction between demand and supply laws (stimulating trustful relationships of buyer and seller, production growth and investments) certain expenditures should be done to organize proper market relations (rules).

It can be said that these expenditures are needed to deal with legal, administrative, and even moral aspects. Without talking into account such expenditures in " P - Q " coordinates it is not possible to form economic space. The nature of human society is so that these expenditures (for market infrastructure) are necessary and unavoidable. If government does not regulate necessary market structures, they will come under control of other forces presumably by Mafia.

The analysis of obtained projections into Z -axis is not the goal of this research. It is important to stress the existence of the third axis of the coordinate economic space, which stands for its third independent dimension. These three dimensions of economic space correlate with each other by one common property of economic processes, that allows: to consider interactions between three economic factors: supply, demand, and organization of market interaction (market infrastructure); to derive factors in the form of interaction from interaction between demand and supply.

5. Conclusion

1. Through application of dynamic principles to objective, experimentally proved laws of demand and supply, their market interaction, market equilibrium we have built four-dimensional economic space related to microeconomic processes. Quantity of goods supplied or bought serve as time axis; three spatial axes serve as follows: general utility of commodity, total costs of goods supplied, costs of market infrastructure (costs needed to organize confident, lawful relationships between buyer and seller).
2. The construction of economic micro-space brought out motion factor in the form of the price which is a speed factor from the dynamic point of view.
3. Speed parameter in the form of price vector allows us to introduce (for infinitely divisible commodity) two fundamental concepts of economic dynamics, which previously were intuitively described in economic literature:
 - a. the path of a commodity where the position of each point denoting commodity is described by three coordinates (general utility, total costs, costs of market infrastructure);
 - b. The acceleration of the point-denoting commodity expressed as a change in price vector per change in "temporal" parameter.

4. The acceleration at the motion of the point denoting commodity clearly shows vector properties and determines proportional force interaction. The coefficient between force interaction and acceleration defines market liquidity.

In further article from general conceptions of the nature law, will look at (introduce to) the price conception and its change of price, as the speed parameter, the acceleration parameter.

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