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Gravitational and Electromagnetic Field of a Non-rotating and Rotating Charged Mass

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Abstract: A new alternative method is presented here to find out a metric for an isolated charged mass situated at the origin in empty space. Since the charged mass has the both gravitational and electromagnetic field, therefore at first a crude line element or metric is considered for the mass, and then another crude line element is considered for the electric charge of the body. The both line elements are the functions of the distance, therefore combined the both line elements and a most general form of line element is found. To solve this metric Einstein's gravitational and Maxwell's electromagnetic (e-m) field equations are used. In the method of solutions e-m field tensor is also used which is found from Maxwell's e-m field equations. After a rigorous derivation the metrics are found for both positively charged and negatively charged massive particles. The new metric for an electron is different as the metric is devised by Reissner and Nordstrom. The metric for a proton is extended for the massive body and which gives some new interesting information about the mass required to stop e-m interaction. This means that above the aforesaid mass there is no electrically charged body in the universe. On the other hand we can say that life cannot survive in those massive planets which masses are greater than 1.21 times of Jupiter mass. The metric found for proton is used to find another new metric for rotating charged massive body.

Keywords: Metric, Line Element, Gravitational Field, e-m Field, e-m Field Tensor

1. Introduction

The purpose of this write up is to present an alternative solution of the gravitational and electromagnetic (e-m) field on the basis of Einstein's gravitational field and Maxwell's electromagnetic field equations for an isolated particle which is charged with spherically symmetry and is situated at the origin in empty space.

Einstein's field equations [1] are very simple and elegant, but difficult to find the exact analytical solution, because the field equations are a set of nonlinear differential equations. The solution of these equations has given by the famous Schwarzschild [2], Kerr [3, 4] and FRW [5-8] solutions for cosmology. The gravitational field due to an electron in empty space was given by Reissner [9, 10] and Gunner Nordstrom [11, 12] in 1921. For the first time Schwarzschild metric was understood to describe a black hole [13] in the year 1958 and then in 1963 Kerr [3, 14] generalised the solution for rotating black hole. Newman [15, 16] try to describe the metric for charged, rotating body in the year 1965 on the basis of Reissner-Nordstrom solution.

Today different authors [17-24] proposed different methods of solution of Einstein field equations time to time. A brief introduction about Schwarzschild, Reissner-Nordstrom, Kerr and Newman metrics are given below:

The Einstein's field equation in empty space is given by

$$R_{\mu\nu} = 0 \tag{1}$$

The exact solution of the above field equations in empty space of an isolated particle continually at rest at the origin was first given by Schwarzschild and the metric is found

$$ds^2 = -\left(1+\frac{B}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) + \left(1+\frac{B}{r}\right)dt^2$$

Here B is a constant of integration and is considered as $B = -2m$. This is done in order to facilitate the physical interpretation of m as the mass of the gravitating particle. Then the above equation becomes,

$$ds^2 = -\left(1-\frac{2m}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) + \left(1-\frac{2m}{r}\right)dt^2 \quad (2)$$

The value of constant m was found as follows: Let the field be weak static, so that at a large distance from the attracting particle

$$g_{44} = 1 + \frac{2\phi}{c^2} \quad (3)$$

Here ϕ is Newtonian potential. On the other hand in (2),

$$g_{44} = 1 - \frac{2m}{r} \quad (4)$$

Hence (3) and (4) gives,

$$\phi = -\frac{mc^2}{r} \quad (5)$$

If M is the mass of the particle and G the gravitational constant, then

$$\frac{\partial\phi}{\partial r} = \frac{GM}{r^2} \quad (6)$$

Differentiating (5) with respect to r and comparing with (6) we get

$$m = \frac{GM}{c^2} \quad (7)$$

Hence (2) becomes,

$$ds^2 = -\left(1-\frac{2GM}{rc^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) + \left(1-\frac{2GM}{rc^2}\right)c^2 dt^2 \quad (8)$$

The metric (8) is spherically symmetric and may be regarded as the gravitational field of a non-rotating point mass M at rest at origin.

The metric for an isolated non-rotating electron was given by Reissner and Gunner Nordstrom as,

$$ds^2 = -\left(1-\frac{2m}{r}+\frac{4\pi\varepsilon^2}{r^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) + \left(1-\frac{2m}{r}+\frac{4\pi\varepsilon^2}{r^2}\right)dt^2 \quad (9)$$

Equation (9) gives the Newtonian potential ϕ as,

$$\phi = \frac{-m}{r} + \frac{2\pi\varepsilon^2}{r^2}$$

Therefore the potential force is,

$$\frac{\partial\phi}{\partial r} = \frac{m}{r^2} - \frac{4\pi\varepsilon^2}{r^3} \quad (10)$$

If we put $m = 0$ in the above (10) then it gives that the force is inversely proportional to the cube of the distance, which is not possible. This means that in the mathematical derivation by Reissner-Nordstrom there may have some discrepancy. This is interesting and compelled me to think this problem seriously. The author [22-24] is trying to solve this problem studiously but still no satisfaction. Again considering the problem a more simple elegant and systematic method of derivation is used to solve the problem in this research article.

The Kerr [3, 14, 16] solution for rotating body is

$$\begin{aligned} ds^2 = &-\left(1-\frac{2mr}{\rho^2}\right)dt^2 - \frac{4mra\sin^2\theta}{\rho^2}d\varphi\, dt + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 \\ &+\left(r^2+a^2+\frac{2mra^2\sin^2\theta}{\rho^2}\right)\sin^2\theta\, d\varphi^2 \end{aligned} \quad (11)$$

Where we write,

$$\rho^2 = r^2 + a^2\cos^2\theta, \quad \Delta = r^2 + a^2 - 2mr \text{ and } c = 1 \quad (12)$$

The metric for rotating charged particle according to Newman [16] is

$$\begin{aligned} ds^2 = &-\left(1-\frac{2mr}{\rho^2}\right)dt^2 - \frac{4mra\sin^2\theta}{\rho^2}d\varphi\, dt + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 \\ &+\left(r^2+a^2+\frac{2mra^2\sin^2\theta}{\rho^2}\right)\sin^2\theta\, d\varphi^2 \end{aligned} \quad (13)$$

Where $\rho^2 = r^2 + a^2\cos^2\theta$, $c = 1$ are same as given by (12), but $\Delta = r^2 + a^2 - 2mr + q^2$. The symbol q is taken as the charge of the particle.

2. Derivation of Metric for a Charged Mass

For simplicity let us consider an isolated proton which is positively charged is situated at origin at rest in empty space. The proton has mass and electric charge, therefore produces both gravitational and e-m field. Both gravitational and e-m fields of the proton are assumed to be spherically symmetric. The range of interaction of the gravitational field is from infinity to 10^{-33} cm and for e-m field it is from infinity to 10^{-8} cm. This means both gravitational and e-m fields have combined interaction range from infinity to 10^{-8} cm. Hence our time coordinate is valid for the range from infinity to

10^{-8} cm . The surroundings of proton are under the influence of both gravitational and e-m fields, therefore Einstein field equation (1) is applicable.

The fundamental metric is given by,

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu \tag{14}$$

The isolated proton is assumed to be spherically symmetric and hence the gravitational field will depend on r alone and not on θ, φ. Therefore the line element or metric in most general possible form may be taken as

$$ds^2 = -A(r)dr^2 - B(r)\, r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) + C(r)dt^2 \tag{15}$$

The equation (15) can be expressed in another most general form which is

$$ds^2 = -e^\lambda dr^2 - r^2(d\theta^2 - \sin^2\theta\, d\varphi^2) + e^\nu dt^2$$

Here λ and ν are the functions of r only.

Similarly the most general form of the metric for the e-m field of the isolated proton situated at origin at rest in empty space satisfying the condition of spherically symmetric is given by,

$$ds^2 = -D(r)dr^2 - E(r)\, r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) + F(r)dt^2 \tag{16}$$

The equation (16) can be expressed in another most general form which is

$$ds^2 = -e^a dr^2 - r^2(d\theta^2 - \sin^2\theta\, d\varphi^2) + e^b dt^2 \tag{17}$$

Here a and b are the functions of r only.

The equations (15) and (16) are individual metric or line element for gravitational field and e-m field of the isolated proton respectively. Actually the isolated proton at rest at origin produces both gravitational and electromagnetic fields therefore the most general form of metric is the combination of (15) and (16), which gives,

$$ds^2 = -\left\{\frac{A(r)+D(r)}{2}\right\}dr^2 - \left\{\frac{B(r)+E(r)}{2}\right\}r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) + \left\{\frac{C(r)+F(r)}{2}\right\}dt^2 \tag{18}$$

Let us consider,

$$\frac{A(r)+D(r)}{2} = X(r),\ \frac{B(r)+E(r)}{2} = Y(r) \text{ and}$$

$$\frac{C(r)+F(r)}{2} = Z(r)$$

The equation (18) becomes,

$$ds^2 = -X(r)dr^2 - Y(r)\, r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) + Z(r)dt^2$$

Taking $r^2 Y(r) = r_1^2$, then we get

$$ds^2 = -X_1(r_1)dr_1^2 - r_1^2(d\theta^2 + \sin^2\theta\, d\varphi^2) + Z_1(r_1)dt^2$$

Dropping the suffix '1' in r_1, we get

$$ds^2 = -X_1(r)dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) + Z_1(r)dt^2$$

A line element or metric equivalent to this line element can be expressible as

$$ds^2 = -e^\rho dr^2 - r^2(d\theta^2 - \sin^2\theta\, d\varphi^2) + e^\sigma dt^2 \tag{19}$$

Here ρ and σ are the functions of r only.

Or we may get the same metric as shown in (19) taking as follows:

$$e^\lambda e^a = e^{\lambda+a} = e^\rho \text{ and } e^\nu e^b = e^{\nu+b} = e^\sigma$$

Now the solution of (19) is found as,

$$ds^2 = -\left(1+\frac{D}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) + \left(1+\frac{D}{r}\right) dt^2 \tag{20}$$

Here '*D*' is an integrating constant. The solution is same as given by Schwarzschild, but the constant of integration *D* is not same as interpreted by Schwarzschild which is shown in (7). Because the isolated massive particle produces both gravitational and e-m fields, but in Schwarzschild solution only gravitational field is considered.

The value of constant D is unknown to us; hence our attempt is to find out the value of constant *D*. Now to find out the value of *D* we proceed step by step as given below:

2.1. Equation of Motion of a Particle in a Static Field

Suppose a particle which is in motion with very low velocity in a static field. The geodesic equation for a non-relativistic particle is

$$\frac{d^2x^\alpha}{ds^2} + \Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0 \tag{21}$$

The fundamental metric in Riemannian space is given by the (14) and we assume that in this metric $g_{\mu\nu}$ are not constant, but differ from the values in flat space by infinitesimal amount. Therefore we can write,

$$g_{\mu\nu} = \xi_{\mu\nu} + \eta_{\mu\nu} \tag{22}$$

Here $\eta_{\mu\nu}$ are very small quantities and functions of x, y, z; but independent of time *t*. This gives that,

$$\frac{\partial \eta_{\mu\nu}}{\partial x^4} = \frac{\partial g_{\mu\nu}}{\partial x^4} = 0 \tag{23}$$

Furthermore,

$$\Gamma^{\alpha}_{\mu\nu} = g^{\rho\alpha}\Gamma_{\rho,\mu\nu}$$

So we can write,

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}\left(\xi^{\rho\alpha} + \eta^{\rho\alpha}\right)\left(\frac{\partial \eta_{\alpha\mu}}{\partial x^{\nu}} + \frac{\partial \eta_{\alpha\nu}}{\partial x^{\mu}} - \frac{\partial \eta_{\mu\nu}}{\partial x^{\alpha}}\right)$$

Here the term $\eta^{\rho\alpha}$ is neglected, because it is very small quantity and considered as $\rho = \alpha$, we obtain,

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}\left(\frac{\partial \eta_{\alpha\mu}}{\partial x^{\nu}} + \frac{\partial \eta_{\alpha\nu}}{\partial x^{\mu}} - \frac{\partial \eta_{\mu\nu}}{\partial x^{\alpha}}\right) \tag{24}$$

Let us taking $x^1 = x,\ x^2 = y,\ x^3 = z$ and $x^4 = ct$ we can write,

$$ds^2 = -dx^2 - dy^2 - dz^2 + c^2 dt^2$$

From above equation one can write,

$$ds^2 = -\mathrm{v}^2 dt^2 + c^2 dt^2 = c^2 dt^2\left(1 - \frac{\mathrm{v}^2}{c^2}\right),$$

Since the velocity v is non-relativistic, i.e. $\mathrm{v} << c$ therefore we can write,

$$ds = cdt = dx^4 \tag{25}$$

Hence we can write,

$$\frac{dx^1}{ds} = \frac{dx^2}{ds} = \frac{dx^3}{ds} = 0 \text{ and } \frac{dx^4}{ds} = 1 \tag{26}$$

By the virtue of (21) and using (24) the above equation yields

$$\frac{d^2 x^{\alpha}}{ds^2} = -\Gamma^{\alpha}_{44} = -\frac{1}{2}\left(\frac{\partial \eta_{44}}{\partial x^{\alpha}}\right) \text{ for } \alpha = 1,2,3 \tag{27}$$

Using the (25),

$$\frac{d^2 x^{\alpha}}{dt^2} = -\frac{\partial}{\partial x^{\alpha}}\left(\frac{1}{2}c^2 g_{44}\right) \tag{28}$$

This is the equation of motion of a particle in a static field.

2.2. The e-m Field Tensor

Now our next attempt is to find out the e-m field tensor of a charged particle. The e-m field is considered symmetrical and our assumption implies that the field is purely electrostatic. Therefore the magnetic field intensities are,

$$H_x, H_y, H_z = 0 \tag{29}$$

The general potential K^{μ} is defined in terms of the e-m potential A and scalar potential ψ is given below:

$$K^{\mu} = \left(-A_x\ ,\ -A_y\ ,\ -A_z\ ,\ \psi\right)$$

The e-m field tensor $F_{\mu\nu}$ is defined now as,

$$F^{\mu\nu} = K_{\mu,\nu} - K_{\nu,\mu} = \frac{\partial K_{\mu}}{\partial x^{\nu}} - \frac{\partial K_{\nu}}{\partial x^{\mu}} \tag{30}$$

And $\vec{H} = \nabla \times \vec{A}$

In the view of above equation, the (29) gives us,

$$A_x,\ A_y,\ A_z = 0$$

This means that the scalar potential ψ is a function of r only.

Using the (30) one can write

$$F_{12},\ F_{23},\ F_{31},\ F_{24},\ F_{34} = 0 \text{ and } F_{14} = -\frac{\partial \psi}{\partial r} \tag{31}$$

This means that the non-vanishing components of $F_{\mu\nu}$ is only F_{14} and $F_{14} = -F_{41}$.

The current density J^{μ} is,

$$J^{\mu} = F^{\mu\nu}_{\nu} = \frac{\partial F^{\mu\nu}}{\partial x^{\nu}} + F^{\alpha\nu}\Gamma^{\mu}_{\alpha\nu} + F^{\mu\alpha}\Gamma^{\nu}_{\alpha\nu} \tag{32}$$

The value of $F^{\alpha\nu}\Gamma^{\mu}_{\alpha\nu} = 0$ and now we have taken $J^{\mu} = q$, where q is considered as charge, this yield

$$\sqrt{-g}\ q = \frac{\partial(\sqrt{-g}\ F^{41})}{\partial r}$$

But there is no charge and no current in the space surrounding the charged particle which is situated at origin in empty space. This gives us,

$$\frac{\partial(\sqrt{-g}\ F^{41})}{\partial r} = 0 \tag{33}$$

Furthermore we know,

$$F^{41} = -F_{41} = -F^{14} = F_{14} = -\frac{\partial \psi}{\partial r}$$

From these equations the (33) can be written as,

$$\frac{\partial}{\partial r}\left[e^{(a+b)/2} r^2 \sin\theta\ (-e^{-(a+b)})F_{41}\right] = 0$$

Integrating we get,

$$F_{14} = \frac{\varepsilon}{r^2} e^{(a+b)/2} \quad (34)$$

Here the symbol ε is considered as an absolute constant. The constant ε is related with the charge of the isolated particle which is situated at origin in empty space. This means $4\pi\varepsilon = q$, here q is taken as the charge of the particle.

After some rigorous calculations of the (17) gives us $b = -a$, putting this in (34) and compare with (31) we get,

$$-\frac{\partial \psi}{\partial r} = \frac{\varepsilon}{r^2} \quad (35)$$

2.3. The Value of Constant D

Now the equation of motion of a charged particle in the influence of both combined gravitational and e-m field is given by,

$$\frac{d^2 x^\alpha}{dt^2} = -\frac{\partial \phi}{\partial x^\alpha} - \frac{\partial \psi}{\partial r} \quad (36)$$

Where α = 1, 2, 3

Putting $M = m_p$ in (6), m_p is mass for proton then (6) becomes

$$\frac{\partial \phi}{\partial r} = \frac{G m_p}{r^2} \quad (37)$$

Hence from (35), (37) and (28)

$$-\frac{\partial}{\partial r}\left(\frac{1}{2} c^2 g_{44}\right) = -\frac{G m_p}{r^2} + \frac{\varepsilon}{r^2} \quad (38)$$

Integrating,

$$g_{44} = -\frac{2 G m_p}{c^2 r} + \frac{2\varepsilon}{c^2 r} + k$$

In the above equation k is considered as an integrating constant.

At $r \to \infty$, $g_{44} = 1$ gives $k = 1$, hence

$$g_{44} = 1 - \frac{2 G m_p}{c^2 r} + \frac{2\varepsilon}{c^2 r} \quad (39)$$

Considering $4\pi\varepsilon = q$ and $\frac{1}{4\pi} = K$ then we can write the (39) as,

$$g_{44} = 1 - \frac{2 G m_p}{c^2 r} + \frac{2Kq}{c^2 r} \quad (40)$$

Comparing g_{44} of (20) with (40) we get the value of D as,

$$D = \left(-\frac{2 G m_p}{c^2} + \frac{2Kq}{c^2}\right) \quad (41)$$

2.4. The Metric for Non-rotating Charged Particle

Putting the value of D in (20) gives

$$ds^2 = -\left(1 - \frac{2 G m_p}{c^2 r} + \frac{2Kq}{c^2 r}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2) + \left(1 - \frac{2 G m_p}{c^2 r} + \frac{2Kq}{c^2 r}\right) dt^2 \quad (42)$$

Also we can write as

$$ds^2 = -\left\{1 - \frac{2(G m_p - Kq)}{c^2 r}\right\}^{-1} dr^2 - r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2) + \left\{1 - \frac{2(G m_p - Kq)}{c^2 r}\right\} dt^2 \quad (43)$$

From (43),

$$g_{\mu\nu} = \begin{bmatrix} -\left\{1 - \frac{2(G m_p - Kq)}{c^2 r}\right\}^{-1} & 0 & 0 & 0 \\ 0 & -r^2 & 0 & 0 \\ 0 & 0 & -r^2 \sin^2\theta & 0 \\ 0 & 0 & 0 & \left\{1 - \frac{2(G m_p - Kq)}{c^2 r}\right\} \end{bmatrix} \quad (44)$$

For negatively charged particle or electron,

$$ds^2 = -\left\{1 - \frac{2(G m_p + Kq)}{c^2 r}\right\}^{-1} dr^2 - r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2) + \left\{1 - \frac{2(G m_p + Kq)}{c^2 r}\right\} dt^2 \quad (45)$$

In above equation m_e is the mass of electron.

2.5. The Metric for Rotating Charged Particle

Now we proceed to find out a metric for rotating charged mass as done by B. K. Borah [24]. Consider $m_p = M$ and $q = Q$ for a massive body in (42), then the metric for non-rotating charged particle

$$ds^2 = -\left(1-\frac{2GM}{c^2 r}+\frac{2KQ}{c^2 r}\right)^{-1} dr^2 - r^2(d\theta^2+\sin^2\theta\, d\varphi^2)+\left(1-\frac{2GM}{c^2 r}+\frac{2KQ}{c^2 r}\right)dt^2 \tag{46}$$

Equation (46) can be written in another form as

$$ds^2 = -\left(1-\frac{r_S}{r}+\frac{r_Q}{r}\right)dt^2 + \left(1-\frac{r_S}{r}+\frac{r_Q}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{47}$$

Here $r_S = \frac{2GM}{c^2}$, $r_Q = \frac{2KQ}{c^2}$ and $d\Omega^2 = (d\theta^2 + \sin^2\theta\, d\varphi^2)$

We consider the constants are simply in the natural units as $G = K = c = 1$ and which gives $r_S = 2M$, $r_Q = 2Q$ then (47) becomes,

$$ds^2 = -\left\{1-\frac{2(M-Q)}{r}\right\}dt^2 + \left\{1-\frac{2(M-Q)}{r}\right\}^{-1} dr^2 + r^2 d\Omega^2 \tag{48}$$

Now we can transform (48) in to a null co-ordinate system where the time co-ordinate t is replaced by the null time co-ordinate u as,

$$u = t + r + 2(M-Q)\ln\left\{\frac{r}{2(M-Q)}-1\right\} \tag{49}$$

Therefore

$$du = dt + \left\{1-\frac{2(M-Q)}{r}\right\}^{-1} dr \tag{50}$$

The equation (50) putting in (48) we get

$$ds^2 = -\left\{1-\frac{2(M-Q)}{r}\right\}du^2 + 2du dr + r^2 d\Omega^2 \tag{51}$$

Therefore,

$$g_{\mu\nu} = \begin{pmatrix} -\left\{1-\frac{2(M-Q)}{r}\right\} & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix} \tag{52}$$

The contra-variant form of $g_{\mu\nu}$ is

$$g^{\mu\nu} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & \left\{1-\frac{2(M-Q)}{r}\right\} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{1}{r^2\sin^2\theta} \end{pmatrix} \tag{53}$$

Now the metric can be expressed in terms of null tetrad $\{\, l, n, m, \bar{m}\}$. The vectors l, n are real and $m, \bar{m}$ are complex conjugates. The world space metric is

$$g_{\mu\nu} = -l_\mu n_\nu - l_\nu n_\mu + m_\mu \bar{m}_\nu + m_\nu \bar{m}_\mu \tag{54}$$

The contra-variant form of (54) is

$$g^{\mu\nu} = -l^\mu n^\nu - l^\nu n^\mu + m^\mu \bar{m}^\nu + m^\nu \bar{m}^\mu \tag{55}$$

The equivalent null tetrads of above are

$$\left.\begin{aligned} l^\mu &= (0,1,0,0) = \partial r \\ n^\mu &= \left(-1, -\frac{1}{2}\left\{1-\frac{2(M-Q)}{r}\right\}, 0, 0\right) = -\partial u - \frac{1}{2}\left\{1-\frac{2(M-Q)}{r}\right\}\partial r \\ m^\mu &= \frac{1}{r\sqrt{2}}\left(0,0,1,\frac{i}{\sin\theta}\right) = \frac{1}{r\sqrt{2}}(\partial\theta + i\,\mathrm{cosec}\theta\,\partial\varphi) \\ \bar{m}^\mu &= \frac{1}{r\sqrt{2}}\left(0,0,1,-\frac{i}{\sin\theta}\right) = \frac{1}{r\sqrt{2}}(\partial\theta - i\,\mathrm{cosec}\theta\,\partial\varphi) \end{aligned}\right\} \tag{56}$$

We can easily found the followings:

$$\left.\begin{aligned} & g^{00} = 0, \quad g^{01} = g^{10} = 1 \\ & g^{11} = \left\{1-\frac{2(M-Q)}{r}\right\}, \quad g^{22} = \frac{1}{r^2} \\ & g^{33} = \frac{1}{r^2\sin^2\theta} \quad \text{and } g^{12} = g^{13} = g^{23} = g^{31} = \dots = 0 \end{aligned}\right\} \tag{57}$$

Now we have used the method used by Newman and Janis [4], to make r complex (with $\bar{r}$ its complex conjugate) and replaced the above tetrad by,

$$\left.\begin{aligned} l^{\mu} &= (0,\ 1,\ 0,\ 0) = \partial r \\ n^{\mu} &= \left(-1,\ -\frac{1}{2}+\frac{(M-Q)}{2}\left(\frac{1}{r}+\frac{1}{\bar{r}}\right),\ 0,\ 0\right) = -\partial u\ -\frac{1}{2}M'\,\partial r \\ m^{\mu} &= \frac{1}{\bar{r}\sqrt{2}}\left(0,\ 0,\ 1,\ \frac{i}{\sin\theta}\right) = \frac{1}{\bar{r}\sqrt{2}}\left(\partial\theta + i\cos ec\theta\,\partial\varphi\right) \\ \bar{m}^{\mu} &= \frac{1}{r\sqrt{2}}\left(0,\ 0,\ 1,\ -\frac{i}{\sin\theta}\right) = \frac{1}{r\sqrt{2}}\left(\partial\theta - i\cos ec\theta\,\partial\varphi\right) \end{aligned}\right\} \tag{58}$$

Where $M' = \left\{1 - \frac{(M-Q)}{2}\left(\frac{1}{r}+\frac{1}{\bar{r}}\right)\right\}$

Still the vectors l^{μ}, n^{μ} are real and $m^{\mu},\ \bar{m}^{\mu}$ are complex conjugates of each other. Now perform the transformations as

$$\left.\begin{aligned} r &= r' - ia\cos\theta, \\ u &= u' - ia\cos\theta \\ \theta &= \theta' \\ \varphi &= \varphi' \end{aligned}\right\} \tag{59}$$

In equation (59) r', u' are real and *'a'* is a parameter. Later we shall justify the interpretation of *'a'* as the angular momentum of the body.

For the determination of values of m'^{μ} and $\bar{m}'^{\mu}$ now we have used the usual formula as given below:

$$V'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\nu}} V^{\nu}$$

Using the above formula the values of,

$$m'^{0} = \frac{-ia\sin\theta}{\bar{r}\sqrt{2}},\ m'^{1} = \frac{-ia\sin\theta}{\bar{r}\sqrt{2}},\ m'^{2} = \frac{1}{\bar{r}\sqrt{2}} \text{ and}$$

$$m'^{3} = \frac{i}{\bar{r}\sqrt{2}\ \sin\theta} \tag{60}$$

$$\bar{m}'^{0} = \frac{ia\sin\theta}{r\sqrt{2}},\ \bar{m}'^{1} = \frac{ia\sin\theta}{r\sqrt{2}},\ \bar{m}'^{2} = \frac{1}{r\sqrt{2}} \text{ and}$$

$$\bar{m}'^{3} = -\frac{i}{r\sqrt{2}\ \sin\theta} \tag{61}$$

Using (59), (60) and (61) the tetrad becomes,

$$\left.\begin{aligned} l'^{\mu} &= (0,\ 1,\ 0,\ 0) \\ n'^{\mu} &= \left(-1,\ -\frac{1}{2}+\frac{(M-Q)\,r'}{(r'^2+a^2\cos^2\theta)},\ 0,\ 0\right) \\ m'^{\mu} &= \frac{1}{(r'+ia\cos\theta)\sqrt{2}}\left(-ia\sin\theta, -ia\sin\theta,\ 1,\ \frac{i}{\sin\theta}\right) \\ \bar{m}'^{\mu} &= \frac{1}{(r'-ia\cos\theta)\sqrt{2}}\left(ia\sin\theta,\ ia\sin\theta,\ 1,\ -\frac{i}{\sin\theta}\right) \end{aligned}\right\} \tag{62}$$

Now the contravariant components of the metric using (55), (62) and dropping the primes we get,

$$\left.\begin{aligned} & g^{00} = \frac{a^2\sin^2\theta}{\rho^2},\ g^{01} = g^{10} = \frac{r^2+a^2}{\rho^2},\ g^{03} = g^{30} = -\frac{a}{\rho^2} \\ & g^{13} = g^{31} = -\frac{a}{\rho^2},\ g^{11} = \frac{r^2+a^2-2(M-Q)r}{\rho^2} \\ & g^{22} = \frac{1}{\rho^2},\ g^{33} = \frac{1}{\rho^2\sin^2\theta} \\ & g^{20} = g^{02} = g^{21} = g^{12} = g^{23} = g^{32} = 0 \end{aligned}\right\} \tag{63}$$

Where we take

$$\rho^2 = r^2 + a^2\cos^2\theta \tag{64}$$

Therefore we can write

$$g^{\mu\nu}=\begin{pmatrix} \frac{a^2\sin^2\theta}{\rho^2} & \frac{r^2+a^2}{\rho^2} & 0 & -\frac{a}{\rho^2} \\ \frac{r^2+a^2}{\rho^2} & \frac{r^2+a^2-2(M-Q)r}{\rho^2} & 0 & -\frac{a}{\rho^2} \\ 0 & 0 & \frac{1}{\rho^2} & 0 \\ -\frac{a}{\rho^2} & -\frac{a}{\rho^2} & 0 & \frac{1}{\rho^2\sin^2\theta} \end{pmatrix} \tag{65}$$

The covariant component is

$$g_{\mu\nu}=\begin{pmatrix} -\left\{1-\frac{2(M-Q)r}{\rho^2}\right\} & 1 & 0 & -\frac{2(M-Q)ra\sin^2\theta}{\rho^2} \\ 1 & 0 & 0 & -a\sin^2\theta \\ 0 & 0 & \rho^2 & 0 \\ -\frac{2(M-Q)ra\sin^2\theta}{\rho^2} & -a\sin^2\theta & 0 & \left\{r^2+a^2+\frac{2(M-Q)ra^2\sin^2\theta}{\rho^2}\right\}\sin^2\theta \end{pmatrix} \tag{66}$$

Then we can write,

$$\begin{aligned} ds^2 &= -\left\{1-\frac{2(M-Q)r}{\rho^2}\right\}du^2+2dudr-\frac{4(M-Q)ra\sin^2\theta}{\rho^2}\,dud\varphi-2a\sin^2\theta\,drd\varphi \\ &\quad+\rho^2d\theta^2+\left\{r^2+a^2+\frac{2(M-Q)ra^2\sin^2\theta}{\rho^2}\right\}\sin^2\theta\,d\varphi^2 \end{aligned} \tag{67}$$

Now we perform the coordinate transformations as

$$du=dt+\frac{r^2+a^2}{\Delta}dr,\quad d\varphi=d\psi+\frac{a}{\Delta}dr \tag{68}$$

Where we take,

$$\Delta=r^2+a^2-2(M-Q)r \tag{69}$$

After doing some algebra and relabeling $\psi\to\varphi$ we can write the metric as,

$$\begin{aligned} ds^2 &= -\left\{1-\frac{2(M-Q)r}{\rho^2}\right\}dt^2-\frac{4(M-Q)ra\sin^2\theta}{\rho^2}d\varphi\,dt+\frac{\rho^2}{\Delta}dr^2 \\ &\quad+\rho^2d\theta^2+\left\{r^2+a^2+\frac{2(M-Q)ra^2\sin^2\theta}{\rho^2}\right\}\sin^2\theta\,d\varphi^2 \end{aligned} \tag{70}$$

This is the required metric for an isolated rotating positively charged massive body.
The metric tensor is

$$g_{\mu\nu} = \begin{pmatrix} -\left\{1-\frac{2(M-Q)r}{\rho^2}\right\} & 0 & 0 & -\frac{2(M-Q)ra\sin^2\theta}{\rho^2} \\ 0 & \frac{\rho^2}{\Delta} & 0 & 0 \\ 0 & 0 & \rho^2 & 0 \\ -\frac{2(M-Q)ra\sin^2\theta}{\rho^2} & 0 & 0 & \left\{r^2+a^2+\frac{2(M-Q)ra^2\sin^2\theta}{\rho^2}\right\}\sin^2\theta \end{pmatrix} \quad (71)$$

Here $a=(J/M)$ is the angular momentum per unit mass of the source.

3. Results and Discussion

If we consider $Q=0$ in (70) then it takes the form (11) which is Kerr solution. Again if we consider $Q=0,\ a=0$ then it takes the form Schwarzschild metric.

Considering g_{44} in (42) the Newtonian-like potential is

$$1-\frac{2Gm_p}{rc^2}+\frac{2Kq}{rc^2}=1+2\phi$$

$$\phi=\frac{-Gm_p}{rc^2}+\frac{Kq}{rc^2} \quad (72)$$

This gives,

$$\frac{\partial\phi}{\partial r}=\frac{Gm_p}{r^2c^2}-\frac{Kq}{r^2c^2} \quad (73)$$

If $m_p=0$, then the terms of r. h. s. gives,

$$\text{Force}=-\frac{Kq}{r^2c^2}.$$

i.e. force $\propto \frac{1}{r^2}$, this means (73) satisfied the inverse square law whereas in (10) given by Reissner-Nordstrom does not satisfy the inverse square law.

Let us consider another proton which is positively charged comes near to the original particle up to distance '*r*' and interacts both electrically and gravitationally. Then we can write,

$$g_{44}=\left(1-\frac{Gm_pm_p}{rc^2}+\frac{Kq^2}{rc^2}\right) \quad (74)$$

In equations (70) the gravitational potential energy is very weak then e-m potential energy. Let we consider the isolated particle at rest in origin is a massive body having mass $M=Nm_p,\ (N=1,2,3,\ .....\infty)$ which is nothing but the combination of protons. As number of proton increases the mass of the body increases and gravitational force increases, since all massive particles gravitationally interacts with all massive particles but one charged particle electromagnetically interacts with only one charged particle. Hence the (74) can be written as,

$$g_{44}=\left\{1-\frac{1}{rc^2}(GMm_p-Kq^2)\right\} \quad (75)$$

Now for flat space-time

$$(Kq^2-GMm_p)=0 \quad (76)$$

This gives,

$$M=\frac{Kq^2}{Gm_p} \quad (77)$$

Let consider the values of the constants,

$$\left.\begin{aligned} &G=6.670\times10^{-8}\,dyne\text{-}cm^2/gm^2\\ &K=9\times10^{18}\,dyne\text{-}cm^2/C^2\\ &m_p=1.67265\times10^{-24}\,gm\\ &q=1.6\times10^{-19}\,C \end{aligned}\right\} \quad (78)$$

Putting the values of constants from (78) the mass required for flat space time or to stop e-m interaction is equal to $M'=2.0667735\times10^{12}\,gm$.

The value of this M' is so large that cannot exist within the range $r\ (=10^{-8}\text{ cm})$. Density of the body will be very high; hence cannot consider such massive particle. Therefore we consider a heavy body of mass $M_1(=\sum_1^{N_1}m_p=N_1m_p)$, where M_1 is required to stop the e-m interaction and R_1 is considered as the radius of the massive body.

Now to determine the values of M_1 one can write,

$$\frac{GM}{r}=\frac{GM_1}{R_1}$$

This gives,

$$M_1=M\left(\frac{R_1}{r}\right) \quad (79)$$

Number of proton contains in mass M is $N\ (=M/m_p)$ and r is interacting range or atomic radius then volume for N atoms is

$$V = \frac{M}{m_p} \times \frac{4}{3} \pi r^3 \tag{80}$$

Therefore density is

$$\sigma = \frac{M}{V} = \frac{m_p}{(4/3)\pi r^3} \tag{81}$$

$$\text{Now } M_1 = \frac{4}{3}\pi R_1^3 \sigma = m_p \left(\frac{R_1}{r}\right)^3 \tag{82}$$

Equating (79) with (82)

$$\frac{R_1}{r} = \left(\frac{M}{m_p}\right)^{\frac{1}{2}} \tag{83}$$

Putting (83) in (82) and putting the value of M from (77) we get,

$$M_1 = \frac{q^3}{m_p{}^2} \left(\frac{K}{G}\right)^{(3/2)} \tag{84}$$

Using the values of G, K, m_p, q from (78) in (84) to stop e-m interaction between two protons putting as $M_1 = M_{em}$,

$$M_{em} = 2.29701\times 10^{30}\,\text{gms} = 0.00116\, M_* \tag{85}$$

Here $M_* = 1.\ 99\times 10^{33}$ gms, the mass of sun.

4. Conclusions

The equation (43) is the metric for proton and (46) is any positively charged particle. For electron or negatively charged particle the metric is given by (45). For flat space-time or to stop electromagnetic interaction in metric (42) the required mass is 2.29701×10^{30} gms. Jupiter is the largest planet in our solar system and the mass of Jupiter planet is 1.898×10^{30} gms. The mass required to produce gravitational field to stop electromagnetic interaction at the surface of the celestial body is just 1.21 times greater than Jupiter's mass. The following conclusions are given below in case of the celestial body which mass is just 1.21 times greater than Jupiter's mass.

i. There is no electrical charged body greater than the aforesaid mass.
ii. Electronic device does not work at the surface of the planet which mass is just greater than above said mass.
iii. It would be impossible for life to exist on the surface of the planet which mass is greater than the above mass. Because gravity stop the e-m interaction, since the life is nothing but the low energy level electromagnetic interaction.
iv. The mass of a cloud of protons dust which is greater than the above said mass then there will start nuclear interaction and a star will appear.

Hence the metric given by the (70) is valid less than the aforesaid mass 2.29701×10^{30} gms. The (70) is the metric for charged rotating body and which is same as given by Kerr metric (13) if we put $(M-Q) \rightarrow m$ in (70). The (43) is the metric for both gravitational and e-m fields those have the interaction range from infinity to 10^{-8} cm. This gives us the probability to find out the metrics for gravitational and strong field or gravitational and weak field.

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