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On Normal Process of Diffusion Equation in Monitoring Carbon Monoxide Concentrations in Nigeria

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Abstract: Normal processes produce random variables with a normal distribution, which is the most important model in statistics. Due to the constant speed and direction of the carrier medium, a continuous source releases particles like environmental pollutants in drift be it in the air, water or soil. By differentiating the normal density function, this study used the knowledge of the plume model to build two separate paths of utilizing Gaussian probability density function with mean of zero to show that it meets the diffusion equation from physical principles through the knowledge of a Brownian motion in monitoring emissions of carbon monoxide from different sources in the most populous black country. Carbon monoxide emissions from manufacturing industries and construction (MIC), fugitive emissions from solid fuels (FESO), and agricultural waste burning (AWB) are all higher than other sources in Nigeria, according to this research. Rail transportation (RAIL) is the lowest source of carbon monoxide emissions, and pollution diffusion in the country follows a predictable pattern in form of a normal process. The magnitude of the standard deviations affects the precision of confidence intervals used to estimate mean pollutant concentrations. Decision-makers in the country will know which sectors to focus on in order to reduce carbon monoxide emissions.

Keywords: Gaussian Density Function, Fugitive Emissions, Agricultural Waste, Rail Transportation, Pollutant Concentrations

1. Introduction

As a result of the complexities of the processes at work on a pollutant when released into the environment, a single model of its movement, transformation, and fate cannot be created. Models of a number of very simple environmental processes are preferable to gain insights into these phenomena. The purpose of such models is to illustrate how environmental concentrations develop their statistical properties. It is an attempt to illustrate how basic processes operate under far more complex circumstances in real-life environmental situations using idealized models. Diffusion occurs when a molecule swaps positions with a neighboring molecule as a result of releasing a pollutant into the atmosphere [4, 6, 12, 19, 20, 23]. Pollutant molecules tend to swap places with adjacent carrier medium molecules when released into a carrier medium. The molecules will generally spread out or become dispersed in the carrier medium, which will also result in dilution of the pollutant (number of molecules per unit volume) since the molecules will become diluted. As pollutants becomes dispersed in the environment, it affects the ecosystem. In the marine environment, oil pollution is caused by releases of fluid oil from oil platforms or drilling rigs as well as pipelines caused by human activities. Historically, tankers and drilling platforms have released unrefined petroleum may seriously harm the environment [1, 2, 5, 16, 22, 24, 25]. Bayesian Model Averaging was usually applied to an ensemble of climate model simulations from the Paleo-climate Modelling Inter comparison Project phase 3 (PMIP3) and phase 5 of the Coupled Model Inter-comparison Project (CMIP5). Uncertainties, weights and variances of individual model simulations were estimated from a training period using the

National Centers for Environmental Prediction-National Center for Atmospheric Research (NCEP-NCAR) reanalysis dataset. Their results shows that the selected proxy-based reconstructions and simulations are consistent with BMA estimates regarding climate variability in the past 10 centuries, though differences can be found for some periods. One of the main greenhouse gases in the atmosphere is CO_2. It is emitted to the atmosphere through many ways, but the larger emissions of the gas in the atmosphere leads to higher concentration in the atmosphere thereby altering the global carbon cycle and causing global warming of the earth planet. Emissions from a number of growing economies have been increasing rapidly over the last few decades. Fast-forwarding to annual emissions in 2014, we can see that a number of low to middle income nations are now within the top global emitters. In Nigeria, CO_2 gas is emitted from a lot of sources. As important as CO_2 is, in sustaining a habitable temperature, continuous increase in the emissions can disrupt the global cycle and thereby lead to a planetary warming impact. Since CO_2 emissions is rapidly increasing in Nigeria, and through this study we have been able to discover that the Industry, Agriculture, Resident and Commercial sector plays the most important role in the emission to the environment, there is need for the concerned authorities to restructure these sectors and provide necessary adjustment to reduce carbon emission being released to the environment or to provide ways by which the carbon emitted will be properly sequestered [3, 7, 10, 14, 20]. It is possible to predict the spillage area by solving the mass transport equation that governs the flow field phenomena. There is only one rational solution for the diffusion equation (parabolic) [12, 13, 15, 17, 21]. Ahmad *et al.* [1], modified the diffusion and Allen-Cahn equations to analyze approximate solutions arising from oil pollution. A wide variety of techniques are used to solve the Allen–Cahn (AC) equation; A few Newell-whitehead (NW) and AC equations were approximated using the Legendre wavelet-based approximation method by Hariharan [10]. A fractional Laplacian was used by Gui & Zhao [9] to obtain traveling wave solutions of the AC equation. Using the integrative method, Javeed *et al.* [11] developed a coupled space-time fractional Drinfeld-Sokolov-Wilson system and space-time fractional AC equation. Series of mechanical model, like the wedge machine, can be used to explain diffusion of pollutants by generating probability distributions that are both symmetrical and right skewed. This work try to extend the work of Ott [12], by using the knowledge of Brownian motion in a plume to assume normality as pollutants flows from a source to satisfy the diffusion equation.

2. Methodology

Assume that pollution particles (or molecules) are released from a point source, such as a chimney discharge, into a wind moving in a laminar flow in a constant direction. The plume will be symmetrical, with its center line extending from the source parallel to the flow direction, if the carrier medium is homogeneous and free of interfering barriers. Consider how this method would work in two dimensions. Each particle experiences longitudinal drift after leaving the source, similar to the particle frame analog, but it is subject to countless collisions as it moves along. The particle will experience horizontal displacement d_i on the *ith* impact. If $y(m)$ is the particle's horizontal position after m collisions, then $y(0) = 0$ is the particle's initial location at the source, and the particle's final position is the sum of all horizontal displacements it has undergone since leaving the source:

$$y\ (m) = d_1 + d_2 + - - - + d_m \tag{1}$$

Let Y be a random variable that represents the particle's horizontal location as measured by its positive or negative distance from the center line. Let D be a random variable that represents the horizontal displacement of the object after each impact. The sum of the realizations d_1, $d_2 \ldots {}_{dm}$ obtained from the random variable D will thus equal Y for each unique particle. Assume that the particles are equally likely to be moved to the left or right, and that the expected value of the displacement is zero: E[D] = 0. Because the displacements are independent, the expected value of Y by the criteria for combining expected values will be the sum of the displacements' expected values.

$$E[Y] = E[d_1] + E[d_2] + \ldots + E[d_m] = m\ E[D] = 0 \tag{2}$$

By the criteria for combining variances, the variance of Y will be the sum of the variances of the separate displacements.

$$V[Y] = V[d_1] + V[d_2] + \ldots + V[d_m] = m\ V[D] \tag{3}$$

According to the Central Limit Theorem, Y asymptotically approaches a normal distribution since it is made up of independent random variables with finite variance.

This study shows that after m collisions, the particle's final location Y (or that of many similar particles) will asymptotically approach a normal distribution with a mean at the center line and variance proportional to m. The normal distribution's probability density function with mean 0 and variance $\sigma^2 = m$ is as follows:

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{y^2}{2\sigma^2}} \tag{4}$$

More collisions will occur as time goes on. As a result, it's plausible to infer that the number of collisions is proportionate to the amount of time that has passed. Considering the one-dimensional distance Y of a particle from the center line in a normal PDF with mean 0 and variance t. We demonstrated that this density satisfies the diffusion equation from physical principles by differentiating the normal density function with respect to y.

$$f(y) = \frac{1}{\sqrt{2\pi t}} e^{-\frac{y^2}{2t}} \tag{5}$$

$$\frac{\partial f}{\partial y} = \frac{1}{t^{3/2}\sqrt{2\pi}} e^{-\frac{y^2}{2t}} \tag{6}$$

Differentiating the above equation with respect to y once again.

$$\frac{\partial^2 f}{\partial^2 y} = \left[\frac{y^2}{t^{3/2}} - \frac{1}{t^{3/2}}\right]\frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2t}} \tag{7}$$

Now differentiating the original PDF with respect to t and compare it with the result obtained in equation 7.

$$\frac{\partial f}{\partial y} = \frac{1}{2}\left[\left(\frac{y^2}{t^{5/2}} - \frac{1}{t^{3/2}}\right)\frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2t}}\right] \tag{8}$$

$$\frac{\partial f}{\partial y} = \frac{1}{2}\frac{\partial^2 f}{\partial^2 y} \tag{9}$$

These two independent paths of analysis that yield the same results shows that the Gaussian PDF satisfies the diffusion equation.

The position of a drifting particle can be visualized as a bell-shaped probability distribution symmetrical about the center line, which extends out as the particle drifts along, based on the aforementioned analysis. The distribution asymptotically becomes normal as the number of collisions grows, with the variance proportional to time. If a large number of particles are released from the source at the same time, their "anticipated arrival density" (expected number of particles per unit length) will follow similar normal distributions, with the exception that the quantities will be multiplied by the number of particles released. The area under the normal curve for a given segment of the Y-axis will give the estimated number of particles arriving in that segment. Any process in which particles behave in a normal distribution with respect to space whose variance is proportional to time satisfies the diffusion equation and is a diffusion process.

Each particle in a three-dimensional model has a horizontal (Y-direction) and vertical (Z-direction) coordinate, both of which are normally distributed. Because the mixing qualities of the carrier media may differ in the horizontal and vertical directions, the standard deviations of these two normal distributions will not necessarily be the same in a real plume. More collisions will occur as time passes, and the standard deviations $\sigma_y(t)$ and $\sigma_z(t)$ will become functions of time. The particle's position will have a bivariate normal probability distribution with time-dependent standard deviations.

$$f_{YZ}(y, z) = \frac{1}{2\pi\sigma_y(t)\sigma_z(t)} e^{-\frac{1}{2}\left(\frac{y^2}{2\sigma_y^2(t)} - \frac{z^2}{2\sigma_z^2(t)}\right)} \tag{10}$$

We can get a broad technique with various practical applications by expanding these notions further. The above concepts are limited in their utility because they require knowledge of the mean and variance of the original population from which the samples were taken; with this information, we can then compute the probability that the observed sample mean will fall within a given range of the population mean.

$$P\left\{\mu_0 - z\frac{\sigma_0}{\sqrt{n}} \le \bar{X} \le \mu_0 + z\frac{\sigma_0}{\sqrt{n}}\right\} \tag{11}$$

Although this result is intriguing, its direct utility is restricted because we rarely know μ_0 or σ_0. Rather, we usually only have a set of n observations; we can compute an observed sample mean $\bar{x}$ from these, and then we want to know how near the true population mean μ_0 is to the sample mean. Let's say we start by subtracting μ_0 from all terms in the above inequality, then alter all the signs in the equation and reverse the inequality's direction, then add $\bar{X}$ to all terms in the equation and rearrange it.

$$P\left\{\bar{X} - z\frac{\sigma_0}{\sqrt{n}} \le \mu_0 \le \bar{X} + z\frac{\sigma_0}{\sqrt{n}}\right\} \tag{12}$$

Using the methods just explained, calculating confidence intervals from a collection of observations is simple. Environmental quality data, on the other hand, may have some quirks that prompt the analyst to doubt the assumptions that these methodologies are built on. Hourly water quality or ambient air quality readings, for example, frequently have substantial serial relationships. As a result, the concentration seen in one hour is not reliant on the concentration observed the following hour.

The data used for this study was assessed from the Electronic Data Gathering, Analysis, and Retrieval system (EDGAR). In addition to greenhouse gas emissions, EDGAR also provides air pollution emissions for each sector and country for several years. Carbon monoxide been a toxic pollutant was used for the study. There are various sources of CO emission in Nigeria. They include monthly emissions of CO from public electricity and heat production (PEHP), other energy industries (OEI), manufacturing industries and construction (MIC), rail transportation (RAIL), inland navigation (IN), residential and Other Sectors (ROS), fugitive emissions from solid fuels (FESO) and agricultural waste burning (AWB) between 2000 and 2012.

3. Discussion of Results

Although the plume model is most commonly used to solve air pollution problems [12], it might also be used to solve plumes caused by water contaminants discharged into streams or lakes [8]. The model might potentially be used to simulate the spread of pollutants as they sink downhill through porous soils. This model is applicable whenever one chemical is conveyed by a carrier medium and encounters random molecule collisions as it drifts. Many variables in nature are the consequence of the addition of a number of unconnected elements. As the number of components in the total grows larger, the final sum tends toward normality when the individual components are sufficiently unrelated and complex. The accumulation of multiple continuous random variables, as well as the independence of these random variables, are two fundamental requirements for a normal process. In this study, we used the concentration of monthly emissions of carbon monoxide in Nigeria from 2000 to 2012. The sources of emissions include public electricity and heat production (PEHP), other energy industries (OEI), manufacturing industries and construction (MIC), rail transportation (RAIL), inland navigation (IN), residential and Other Sectors (ROS), fugitive emissions from solid fuels (FESO) and agricultural waste burning (AWB). From figure 1, it shows that majority of the emission sources are positively skewed. As the emission increases over time, it will asymptotically approach a normal distribution in accordance with the central limit theorem.

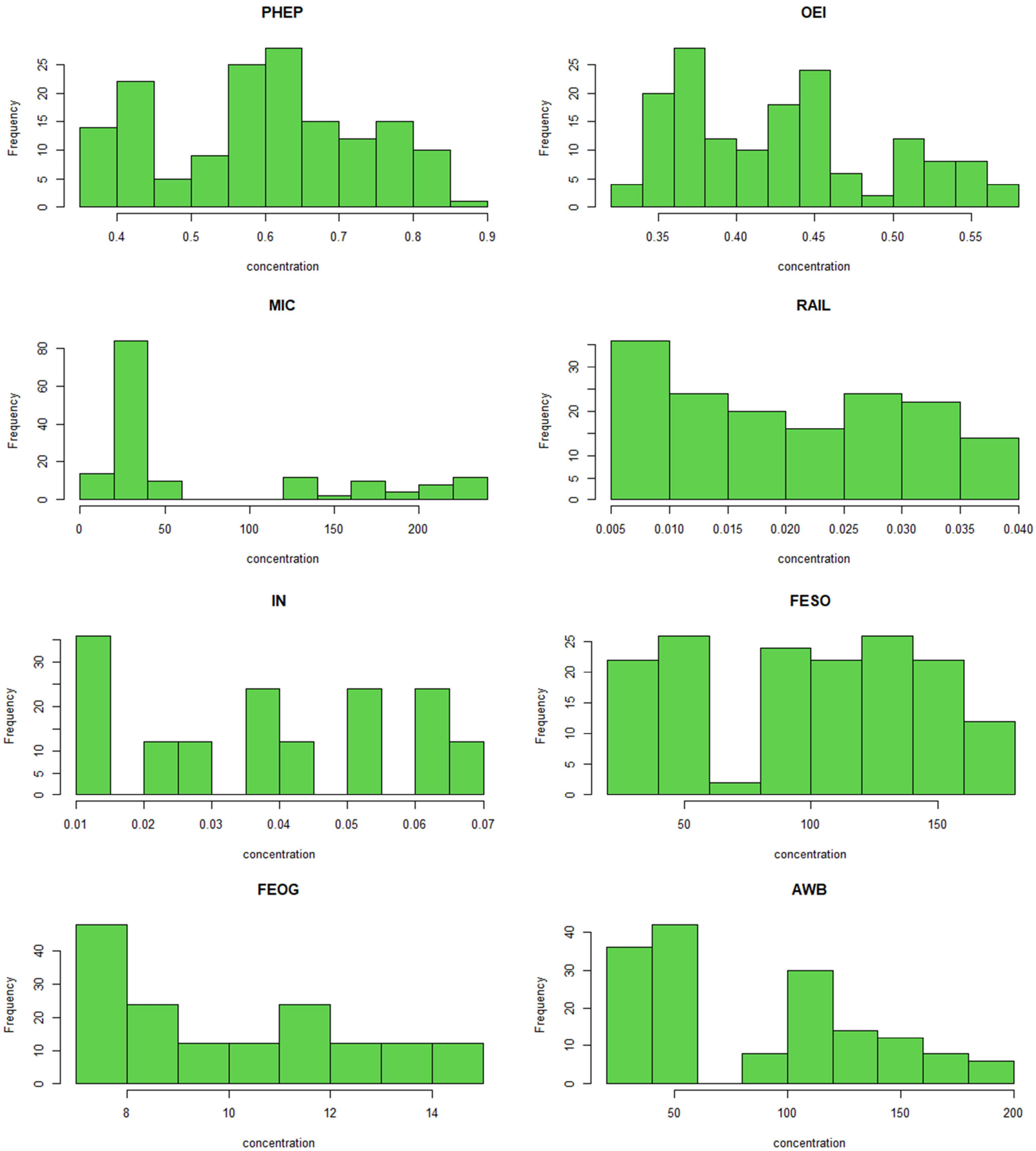

Figure 1. *Histogram of monthly CO emissions in Nigeria.*

Table 1 below shows the mean and standard deviations of monthly carbon monoxide concentrations (in Gg) in Nigeria from 2000 to 2012. On average the emissions are higher from manufacturing industries and construction (MIC), fugitive emissions from solid fuels (FESO) and agricultural waste burning (AWB) than other sources. Rail transportation (RAIL) is the lowest source of emission of carbon monoxide in Nigeria.

Table 1. *Mean and Standard Deviation of monthly CO concentrations in Nigeria.*

	Mean	Standard Deviation
PHEP	0.5966	0.1329
OEI	0.4298	0.0649
MIC	76.21	74.2133
RAIL	0.020458	0.0104
IN	0.0382	0.0195
FESO	99.37	46.0751
FEOG	10.081	2.3667
AWB	84.9	48.6899

Table 2. *Confidence Intervals for mean CO concentrations in Nigeria.*

Emission Source	95% C.I. for μ	
	Low	High
PHEP	0.58	0.62
OEI	0.42	0.44
MIC	64.51	87.91
RAIL	0.02	0.02
IN	0.04	0.04
FESO	92.11	106.63
FEOG	9.71	10.45
AWB	77.22	92.58

Table 2 shows the 95% confidence level of estimates where the true mean of carbon monoxide emissions will fall from different sources. The results show that the samples required to provide precise estimate of the mean. The high precision from small sample sizes is caused by relatively small standard deviations.

Figure 2 shows the likelihood of the concentrations of carbon monoxide emissions from all sources. The likelihood plot displays the distribution of the emissions on average using the mean as a single observation while the maximum value observed in the data set is displayed as the maximum likelihood. The likelihood function contains more information about the data and the parameters than some summary measures of the data. Plots of the likelihood, whenever possible, throw more light on random phenomenon and should be employed in as many cases as they permit. The likelihood function of the maximum of the sample is given by

$$L(\mu) = n\{\Phi(x_{(n)} - \mu)\}.^{(n-1)}\, \emptyset(x_{(n)} - \mu) \tag{13}$$

where $x_{(n)}$ denotes the maximum value of the n data points $x_1, \dots, x_n$and $\Phi(.)$ is the cumulative distribution function of the standard normal random variable.

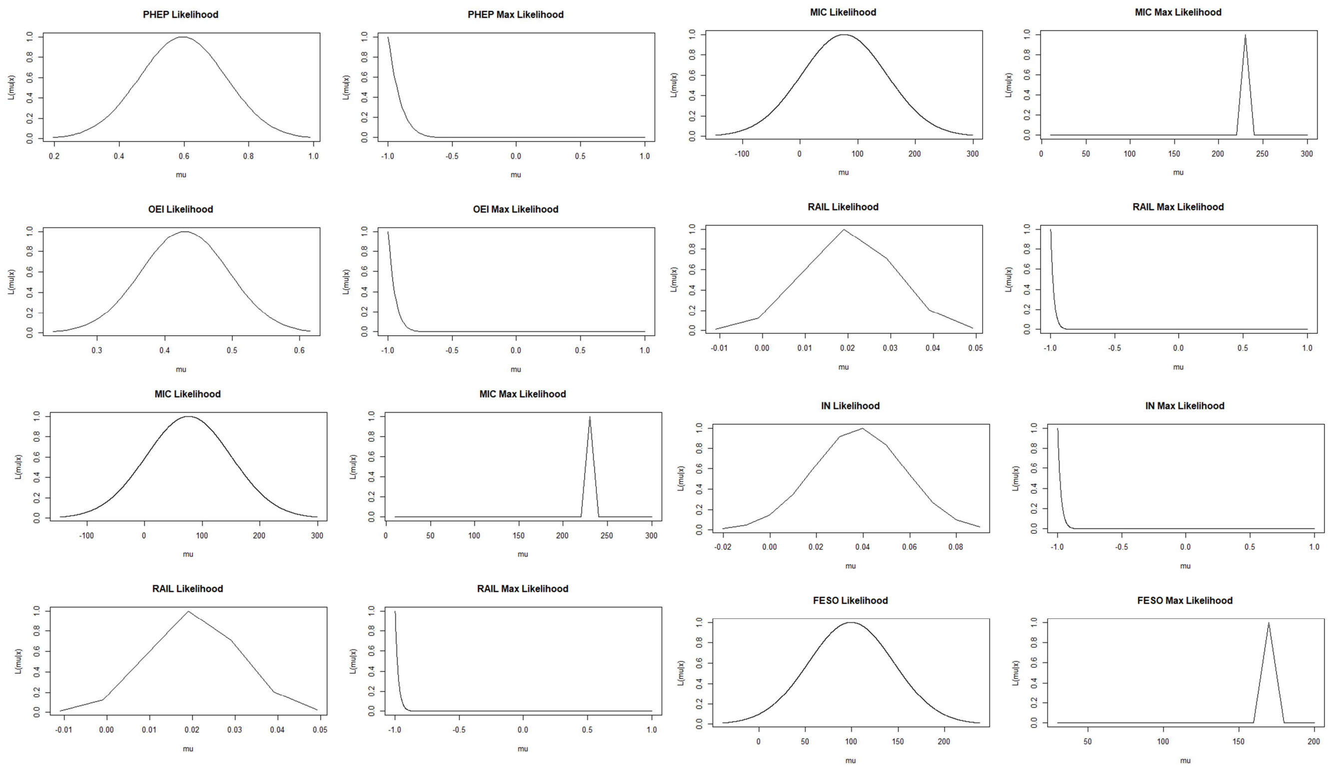

Figure 2. *Likelihoods of CO Emissions in Nigeria.*

4. Conclusion

This study had been able to explain the diffusion process of particles via the principle of differentiation. It was established that diffusion of pollutants in the environment assumes normality through the central limit theorem. Carbon monoxide emissions from different sources in Nigeria was used as case study, and it was discovered that:

1) Carbon monoxide emitted from manufacturing industries and construction (MIC), fugitive emissions from solid fuels (FESO) and agricultural waste burning (AWB) are higher than other sources in the country.
2) Rail transportation (RAIL) is the lowest source of emitting carbon monoxide.

3) Diffusion of pollutants in the country follows a normal process.
4) The precision of confidence intervals estimate of means of pollutant concentrations depend on the magnitude of the standard deviations.

Conflict of Interest

The authors declare no conflict of interest for this study.

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