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Examining Senior High School Girls' Thinking Levels in Solving Problems on Circle Theorems

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Abstract: The learning of Circle theorems is frequently highlighted as one challenging geometric content area for students at Senior High Schools (SHS). However, it remain unclear how students, especially girls are reasoning when dealing with problems in circle theorems. This descriptive study examined students' ability and reasoning in solving problems involving Circle theorems. Two objectives were set to guide the study, focusing on SHS students' levels of thinking in problem solving involving Circle theorems. A test, covering four basic theorems of geometry of circles in SHS curriculum, was designed and used to collect data from a purposive sample of 80 3rd-year students of Mfantsiman Girls' SHS. The results showed that students' thinking in Circle theorems follows the taxonomy of Uni-structural, Multi-structural, Relational and Extended abstract thinking levels. The study also found that about two-fifth of the students could not cope with problems involving relational and extended abstract reasoning, drawing of conclusions or even stating the four circle theorems. The study concludes that SHS mathematics teachers should reconsider how to teach deductively to improve the relational and abstract thinking levels of the 3rd-year students before they write the SHS final examinations. This could be facilitated through the use of SOLO taxonomy with super-item model as a diagnostic tool to ensure the growth of the students' problem solving and reasoning skills in circle theorems.

Keywords: Circle Theorems, Geometric Thinking, Senior High School Girls, SOLO

1. Introduction

1.1. Background

Geometric thinking is an integral component of the mathematics thinking which provides for the ability to solve complex problems in the real physical world [1, 2]. In secondary school mathematics curriculum, the scope of geometric thinking encompasses the application of skills, knowledge and understanding of plane and solid shapes, their properties and relationships between and within them [3, 4]. In the Ghanaian mathematics curriculum for senior high school, geometric thinking covers coordinate geometry, mensuration and circle theorems [5].

As one key components of Mathematics thinking, geometry thinking plays important role in projecting Science, Technology, Engineering and Mathematics Education (STEM) at the senior high school level in Ghana. First, in studying Geometry, students can develop their intuition and spatial visualization abilities, and reason about relationships within and between geometric figures [5, 6]. According to Ministry of Education [5], students are expected to develop several geometric thinking skills including logical thinking, intuitive reasoning, formal deductions, analysis and appreciation of mathematical arguments. Second, geometry thinking also serves as a prerequisite that enables students to make sense of daily life situations and solve problems in other areas of STEM and art education. Through the study of geometry, students can also understand and appreciate the beauty of the physical world.

Despite the importance attached to the students' geometric thinking, research has shown that various cohorts of students continually perform poorly in the geometry contents in mathematics at national examinations such as those conducted by West African Examination Council [1, 7]. Mathematics researchers and educators have identified some major problems associated with the poor performance namely poor background knowledge of students in mathematics, teachers' methodological styles, students' lack of interest, students' perception that mathematics is a difficult subject, large class size and the psychological state of students [4]. The poor performance are also associated with gender where girls are claimed to be underperforming than boys. Several efforts have so far been taken in an attempt to address these issues including the use of technology and reforms of teacher education and training [1]. However, these efforts are yet to focus on students' thinking levels in geometric problem solving involving circle theorems.

Recently, a plethora of studies suggests that the problem of poor thinking in mathematics and particularly in geometry persists. Recent insight in the chief examiner's annual reports of the West African Secondary School Certificate Examinations (WASSCE) in core Mathematics particularly shows that the Ghanaian high school students are not developing deep thinking skills needed to solve problems in circle theorems [7, 8]. In view of the above, there is need to understand at what level our students are reasoning when engaged in problem solving involving various geometry content domains such as circle theorems.

1.2. Statement of the Problem

The issue of how our senior high school boys and girls are thinking when it comes to proofing and solving theorems related geometry problems remains unresolved in literature. As noted earlier, a plethora of national assessment reports revealed that students graduating senior high school do not exhibit sound geometric knowledge [1, 7, 8]. However, these are national data that fail to define students' thinking behaviour or describe the issue in details at school level so as to direct the right choice of classroom instructions. In Ghana, little is known about the structure of cognitive functioning and degree of thinking in circle theorems demonstrated by students at school level. In fact, so far, not much empirical evidences exist on how students who are preparing to write their final examination at the senior high school level are thinking with respect to theorems in circles. This is a gap that need to be filled with empirical evidence. Such empirical evidence when gathered at school level is admittedly much richer in prescribing appropriate teaching methodology and policy interventions to improve learning than national examinations reports.

In view of this, the purpose of this current study is to descriptively analyse the thinking levels of final year senior high school students who have been prepared to sit in the WASSCE. In particular, the research question that guided the study is ''How and at what levels are senior high school students reasoning when solving problems in Circle Theorems?''. Also, despite the rising argument that girls often underperform in mathematics and geometry, research is still inconclusive about the difference. Therefore, of interest in this study is to fill the gap regarding the dearth of literature on how girls are reasoning in circle theorems.

2. Literature Review

2.1. Geometry of Circles

Geometry of circles is a branch of mathematics that deals with logical arguments about properties of lines and angles within, on and outside the circle. It is a part of Euclidean geometry that incorporates the use of theorems, converses, corollaries and axioms that promotes theoretical deductions and practical applications to real problems [2]. In the Ghanaian mathematics curriculum, circle geometry deals with the use of theorems and it is placed under the Plane geometry II, which is taught to SHS 2 students [5]. The mathematics curriculum has identified the following under circle theorems.

1) Theorem 1: Angle subtended from a diameter is 90°.
2) Theorem 2: Angles subtended by the same arc or chord in the same segment are equal.
3) Theorem 3: Angles in alternate segments are equal.
4) Theorem 4: Angle formed between a tangent and radius (diameter) is 90° [5].

Research has revealed that circle theorem is underrepresented in literature [2], yet how students learn to reason intuitively in theorems of circle geometry and how such thinking enables them solve related problems remain uncertain. In this study, the structure and levels of students' geometric thinking in relation to circle theorems are examined through the Structure of the Observed Learning Outcomes (SOLO) taxonomy.

2.2. Geometric Thinking of Students

There are several frameworks used to study students' geometric thinking including Piaget 1985 cognitive model, Duval's 1995 theory of figural apprehensions and van Hieles' 1985 model [9]. However, over the years, the Van Hiele's model which is largely in line with Piagetian theory of cognitive development, has been widely accepted and utilized to assess the thinking ability of students' geometrical thinking levels. According to the Dutch couple, Pierre and his wife Dina Van Hiele, geometric thinking is categorized into five hierarchical levels of visualization, analysis, abstraction, deduction and rigor. The model is framed such that a student must go through these levels sequentially to gain highest order geometry understanding.

By the above categorization, each level has its own language and symbols and a student cannot reach a higher level without going through an adjacent lower one. For instance, a student cannot perform analysis of a geometric object without visualization. Chimuka [10] asserted that the Van Hiele's model could be applied to any group of students regardless of their age or gender. He further proposed that the model is not a developmental model where students need to reach a certain age before advancing through the levels but rather students' experiences and activities are necessary and sufficient. For this to happen, the learning environment or instructional strategies must provide experiences that can advance students from visualization through to the rigor stage. Unfortunately, research suggests that most geometry teachers have not been able to employ appropriate methodology, experiences and resources that could advance students' geometric thinking. This has resulted in poor reasoning and performance problems in geometry.

The van Hiele's framework is often viewed as suitable for examining elementary geometric thinking levels. However, as learning progress, one encounters more complex situations, where observable learning outcomes tend to much better assessed in terms of its quality and degree of cognitive functioning. Research has therefore maintained that the Structure of the Observed Learning Outcomes (SOLO) taxonomy appears apposite in categorizing existence of certain thinking levels and explaining more clearly a learner's learning outcomes in terms of problem solving.

2.3. Structure of the Observed Learning Outcome (SOLO) Taxonomy

Closely related to Van Hiele's model of geometric thinking is the Structure of the Observed Learning Outcomes (SOLO) taxonomy which characterizes students' in-depth knowledge and skills in solving problems into levels of cognitive functioning [11-13]. These different levels are described in Table 1.

Table 1. *Levels and Criteria of the Structure of the Observed Learning Outcomes (SOLO) taxonomy.*

SOLO levels	Criteria
Pre-structural	Students have just partial or no knowledge of the concept (s). No evidence of working or responses are shown
Uni-structural	Students recognize unrelated isolated parts of the concept (s) but the responses are often inconsistent with the given concept (s). For example, a student is able to recognize the diameter of a circle but fails to think of the existence of a semi-circle
Multi-structural	Students are able to integrate related part of the concept to see another concept. For example, a student can state the sum of interior angles of a triangle but cannot connect it to solve related problems leading to incorrect conclusions.
Relational	Students are able to integrate related parts of the concept (s) into a systematic structure or have an abstract concept of it. Students apply concepts or ideas to only familiar task or work-based situation. For example, a student can recognize that two angles are equal because of the theorem (angles subtended by the same arc or chord in the same segment are equal), and hence, substituting them fully in the question.
Extended Abstract	Students are able to use the generalized structure to solve problems in new situations and/or formulate extended structures. They can transfer or apply learning to other fields of study. For example, a student is able to devise and solve an equation, check back by substituting the results into the equation and see if the results are true within the context. [11-13]

The SOLO Taxonomy describes a hierarchy of increasing complexities of reasoning, understanding, and problem-solving that a learner exhibits in the mastery of academic tasks [13]. Unlike van Hiele model, the advantages of using SOLO to characteristics students' levels of thinking lie on the fact that, as students' thinking levels increase, their skills of making consistent explanations, creating relations and thinking in single to multiple situation also increase [14]. Korkmaz and Unsal [15] noted that the quality and structure of thinking to a question can be determined by SOLO taxonomy. The SOLO taxonomy is therefore applied in this study to analyse students' reasoning capabilities when solving problems involving circle theorems.

A review of 62 studies on SOLO taxonomy by Adeniji, et al [16] concluded that most of the studies were focused on algebra and statistics with only 23% related to geometry. They also indicated that in many cases, the super-item test model were widely utilized test [15, 17, 18] used a super-item test based on the SOLO taxonomy as an alternative assessment tool for examining the growth of learners' cognitive ability in solving mathematics problems. Also, Lian, et al [20] employed the super-item model based on SOLO taxonomy and found that most students' algebraic working processes were lower at each SOLO levels. A super-item test consists of a problem situation and different questions. The problem situation is often represented by text, diagram or graphic, while the questions represent the four levels of cognitive reasoning defined by SOLO model. The test format for super-item graduates from simple questions to the more complex ones [18, 19]. Apawu, et al [18] admitted that the use of super-item, based on SOLO teaching technique is better than conventional assessment modes as it ensures appropriate categorization of students' ability and reasoning configuration at each level of SOLO.

Based on the literature reviewed, it can be concluded that the super-item test model is suitable for assessing the different thinking levels of students regarding theorems of circles which is the focus of this study.

3. Methodology

The study employed a descriptive research design, using test as the instrument for data collection. This design was suitable as it allowed for in-depth description of how students at the senior high school are thinking to solve problems involving circle theorems and at what levels their thought processes could be clustered within the structure of cognition taxonomy.

3.1. Participants

Of interest in this study is to examine how girls at the senior high school reason to solve problems in circle theorem. Therefore, purposive sampling technique was used to select 80 third year students at Mfantsiman Girls' Senior High School in the Central Region of Ghana. The ages of the students who were offering General Science, General Art and Business programmes ranged from 16 to 20.

3.2. Instrument

Test was used to collect data for the study. The test followed a super-item model designed in line with SOLO taxonomy [15, 17, 18, 20] and within the scope of circle theorems in the SHS core mathematics curriculum [5]. The super-item test was modelled as scenario where an architect designs a cylindrical storey building with each floor utilizing one of the four circle theorems. The items were conceived within the four different levels of the SOLO taxonomy. Four sub-items on basic facts and definitions of circle theorems tested uni-structural thinking while another four sub-items focused on ability to integrate parts of the theorems tested multi-structural thinking. Another four sub-items which sought for students' ability to integrate theorem properties, previous knowledge and visual analysis to present formal solution structures, tested relational thinking. Finally, sub-items which focused on assessing valid conclusions and reasons tested the extended abstract thinking. The items were open-ended to minimize or eliminate guessing and also to avoid awarding marks to learners on aspects they did not understand. Figure 1 illustrates the scenario and the third item of the super-item in the test.

The Scenario

An architect wants to design a circular tower with three floors for an institution. From the entrance of the tower there are two pathways which are tangential to the ground floor of the circular tower. On the first floor she wants to partition the floor into two equal parts from the centre to the circumference of the floor. On the second floor she will partition the floor with two intersecting inscribed angles. On the last floor she wants to have major and minor segments with an inscribed triangle whose vertex will be protruded to develop a monument tangential to the wall.

3. The figure shows a circular floor (second floor) which has points P, Q, R and S on the circumference. If she wants that $\angle PQR = 54^0$ and $\angle STR = 76°$, use the information to help her construct the floor by

Defining a theorem that will be used to find the angle at S

Calculating the value of α.

Stating what is the relationship between the angles at Q and at S?

Giving reason for your solution

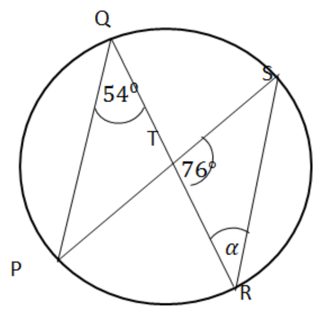


Figure 1. *Scenario of circle theorem application and item 3 of the Super-item.*

The entire super-item test was validated by a university mathematics educator and core mathematics teacher with 10 years teaching experience a senior high school in Ghana. Both experts found the items well-constructed and consistent with the SOLO taxonomy and well-aligned with the scope of circle theorems ascribed in the senior high school mathematics curriculum. In addition, all items have been displayed and integrated into the presentation of results to enable readers examine and make valid judgment about the quality and dependability of the test.

3.3. Data Collection

Approval was granted by the authority of District Education office, Mfantsiman, for us to collect data in the school. The test was administered to the selected students in their classrooms with the assistance of two mathematics teachers of the school. The students were first informed of the purpose of the study and how to answer the test. Each student was asked to submit their test paper at any time they wish within 1 hour or opt to withdraw from participation without being forced to give an explanation. Participants were handled with respect and anonymity. The return rate was 100% as all students who collected the test papers returned them to the researcher.

3.4. Data Analysis

The written test responses of students were first categorized based on accurate or incorrect reasoning in utilizing each theorem. This was summarized into frequency counts and percentage and interpreted descriptively based on each theorem. The written scripts of each of the 80 students in this study were examined to determine how accurate they dealt with uni-structural items, multi-structural items, relational items and extended abstract items in the super-item. The categorization of students' thinking in circle theorem were presented in bar chart and described according to SOLO taxonomy.

4. Results and Findings

4.1. How are Students' Reasoning in Solving Problems Involving Circle Theorems

The first objective of the study was to examine senior high school students' thinking ability in applying four circle theorems to solve problems at the uni-structural, multi-structural, relational and extended abstract levels of SOLO taxonomy. A theorem by theorem analysis of how students are reasoning in their solution structures to circle theorem problems is as follows.

4.1.1. Theorem 1: Angle Subtended from a Diameter Is 90°

Figure 2 shows item 1 i)-iv) which was responded to by participants to examine how students are thinking when solving problems in circle theorem 1 at each level of SOLO taxonomy. Item 1 i) was at uni-structural level and required students to state the theorem that can be used to solve for the angle $< ABC$ in the diagram, while item 1 ii) extended students thinking to multi-structural level and required students to recognize that $< ABC$ is subtended at the circumference by the diameter. Item iii) was to find out the relational thinking of students and required students to apply the angle theorem which states that "*A chord or an arc subtends at the centre is twice the angle at the circumference*". The extended abstract thinking level was elicited by asking students to draw conclusion from the circle theorem provided in item 1 iv).

From the first floor, she wants chord AC to divide the first floor into two equal parts.

State the theorem she can use to solve for the angle$< ABC$.

Using the theorem given in (i), what measure of angle is formed at B in the first floor?

Find $< BAC$

Drawn your conclusion and give reason for your solution?

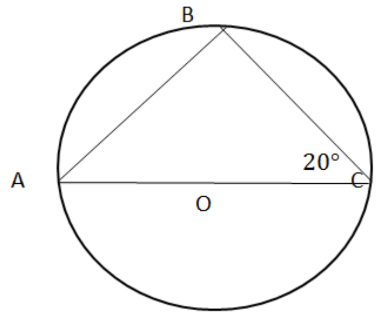


Figure 2. *Test item 1 i-iv) on application of Circle theorem 1.*

The result of analysis of students' responses to item 1 i) in Figure 2 shows that 77 students representing 96.3% correctly identified the chord as a diameter and stated the theorem. Also for item 1 ii), 70 students representing 87.5% recognized that the angle subtended at the circumference by the diameter, is twice the angle at the centre, hence $< ABC = 90°$. Surprisingly, for item 1 iii), measuring multi-structural thinking, only 53 students representing 66.3% recognized that the angle at A can be got by adding all the interior angles of ΔACB and equating to 90°. Furthermore, as low as 47 students representing 58.8% were able to justify why $< BAC = 70°$.

For circle theorem 1, the overall responses show that more than 85% of the students could think at the uni-structural and multi-structural levels but less than one-half of the students could think correctly at relational and extended abstract levels.

4.1.2. Theorem 2: Angles Subtended by the Same Arc or Chord in the Same Segment Are Equal

Figure 3 shows item 2 i)-iv) which was used to examine how students are thinking in circle theorem 2 at each level of SOLO taxonomy. Item 2 i) elicited uni-structural thinking where the students should be able to define correctly the theorem required to find the angle at S while 2 ii) required multi-structural thinking to calculate the value of α. Items 2 iii) and iv) were at relational and extended abstract thinking levels and required deduction of the relationship between the angles at Q and at S and explanation of why both angles are equal. Students were expected to use the theorem to give reasons for their approach.

The figure shows a circular floor (second floor) which has points P, Q, R and S on the circumference. If she wants that $\angle PQR = 54^0$ and $\angle STR = 76°$, use the information to help her construct the floor by

Defining a theorem that will be used to find the angle at S

Calculating the value of α.

Stating what is the relationship between the angles at Q and at S?

Giving reason for your solution.

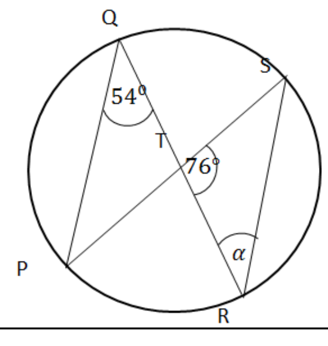


Figure 3. *Test item 2 i-iv) on application of Circle theorem 2.*

The result of analysis of students' responses to item 2 i) shows that 46 students representing 57.5% correctly defined theorem 2. Also, for item 2 ii), 61 students representing 76.3% found the solution value for the unknown angle α. Again for item 2 iii), only 54 students representing 67.5% deduced the relationship between the angles at Q and at S while 49 students representing 61.3% were able to provide the right reason why theorem 2 was applicable in the problem.

The overall results for theorem 2 show that even though, most students provided reasons for their correct answers, up to one-third of the students could not think uni-structurally, multi-structurally, relationally and in extended abstract regarding theorem 2.

4.1.3. Theorem 3: Angles in Alternate Segments Are Equal

Figure 4 shows item 3 i)-iv) which was used to examine how students are thinking in circle theorem 3 at each level of SOLO taxonomy. Item 3 i) elicited uni-structural thinking where the students should be able to identify correctly all equal angles in the circle provided. Item 3 ii) required multi-structural thinking to explain why certain angles are equal in the circle. Items 3 iii) required the use of certain angular relationships in the circle to compute the value of angle b. Finally, item 3 iv) was an extended abstract item which required students to provide reasons or justification for arriving at the correct answers in their solution structure in item 3 iii).

From the third floor, she wants DEF to form a triangle with PDQ as a tangent to the circle at D. O is the centre of the circle, $< EDF = 46°$ and $\angle DFE = 82°$.
Identify the equal angles in the diagram.
Explain why they are equal.
Calculate the value of angle b.
Give reasons for your answer in (iii).

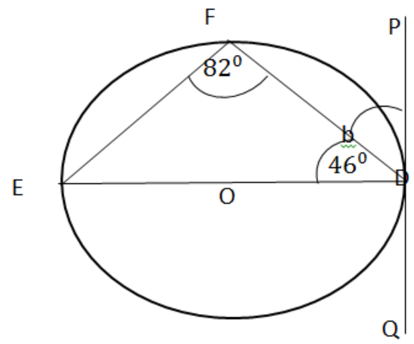


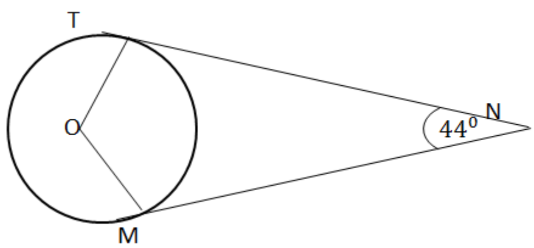
Figure 4. *Test item 3 i-iv) on application of Circle theorem 3.*

The result of analysis of students' responses to item 3 i) in Figure 4 shows that 44 students representing 55.0% correctly identified equal angles in the circle. Also, for item 3 ii), 43 students representing 53.3% were able to correctly explain that $< DEF$ and $< EDF$ are equal. Again, for item 3 iii), only 40 students representing 50.0% correctly applied the relationship between tangent line at point D to solve for the angle value of b. Finally for item 3 iv), only a few (37 students) representing 46.3% provided reasons for the correct value of angle b using the theorem that < EDF and < PDF are formed between a tangent and radius hence their sum equals 90°**.**

The overall results for theorem 3 show that, very few (about 50%) students could think uni-structurally and multi-structurally while fewer (less than 50%) could think at relational and extended abstract levels.

4.1.4. Theorem 4: Angle Formed Between a Tangent and Radius (Diameter) Is 90°

Figure 5 shows item 4 i)-iv) examined how students are thinking in circle theorem 4 at each level of SOLO taxonomy. Item 4 i)-iv) were designed to examine students' SOLO in theorem 4 stated above. Item 4 i) was uni-structural item and required students to state a theorem relating tangent and diameter. Item 4 ii) was multi-structural and relational item and required students to use theorem 4 and previous knowledge on properties of quadrilaterals to calculate value of $< MOT$. Finally item 4 iii) was an extended abstract item requiring students to provide reasons for their solutions.



The ground floor of the building will form a circle with centre O, where M and T are points on the circle. MN and TN are tangents to the circle, $< MNT = 44°$.
Name one theorem that can be found in the diagram of the ground floor.
Calculate the value of $< MOT$.
Give reasons for your answer.

Figure 5. *Test item 1 i-iv) on application of Circle theorem 4.*

The result of analysis of responses to item 4 i) shows that only 23 representing 28.8% were able to relate theorem 4 to the angle formed when a tangent line and diameter meet at the circumference of a circle. For item 4 ii), only 39 students representing 48.8% applied multi-structural and relational reasoning to obtain the correct value of $< MOT$. However, 9 students used alternate ways to arrive at the answer, without applying theorem 4 directly. Finally for item 4 iii), only 21 students representing 26.3% provided correct reasons for their solutions.

The overall results from theorem 4 however indicate that substantial number of students did not understand the theorem involving tangents to a circle as less that 50% were at uni-structural, multi-structural, relational and extended abstract thinking levels.

4.2. Levels of Students' Thinking According to SOLO Taxonomy

The second objective of the study was to determine at what level of SOLO taxonomy are students thinking with respect to problem solving in circle theorems. The written test scripts of each of the 80 students in this study were examined to determine how accurate they were solving uni-structural items, multi-structural items, relational items and extended abstract items in the super-item. Table 2 shows the item by item level frequency and percentage distributions of participants at each level of the SOLO taxonomy.

Table 2. *Frequency and percentage distributions of participants at each level of SOLO taxonomy based on item by item analysis.*

Theorem	Uni-structural			Multi-structural			Relational			Extended abstract		
	T	N	%	T	N	%	T	N	%	T	N	%
1	1 (i)	77	96.3	1 (ii)	70	87.5	1 (iii)	47	58.8	1 (iv)	47	58.8
2	2 (i)	62	77.5	2 (ii)	61	76.3	2 (iii)	54	67.5	2 (iv)	49	61.3
3	3 (i)	44	56.3	3 (ii)	43	53.3	3 (iii)	40	50.0	3 (iv)	37	46.3
4	4 (i)	23	28.8	4 (ii)	39	48.3	4 (iii)	21	26.3	4 (iii)	21	26.3
Average %			64.8			66.3			50.7			48.7

T=Test item, N=number of students, %=Percent.

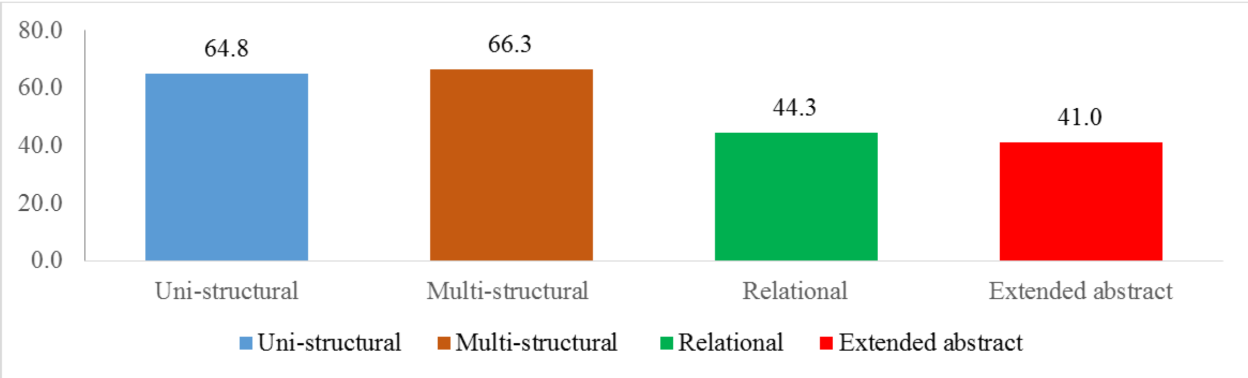


Figure 6. *Proportions of participants at each level of SOLO taxonomy.*

The overall results in Table 2 show that more than one-half of the students are reasoning accurately with respect to solution of problems involving theorems 1, 2 and 3. However, on the average, only about one-third of the students are able to obtain accurate answers to the problems on theorem 4. Cumulatively, the results show that majority of the students tend to think at multi-structural level (66.3%) followed by uni-structural level (64.8%), relational level (50.7%) and extended abstract level (48.7%) when dealing with problems in circle theorems. Conversely, more than one-third of the students tend to demonstrate inaccurate higher order thinking when solving problems involving circle theorems. Figure 6 displays the overall proportions of participants' thinking at each level of SOLO taxonomy.

The result in Figure 6 shows that approximately 65% of the students in the study has demonstrated uni-structural and multi-structural problem solving abilities regarding circle theorems. On the contrary, fewer number of the students were thinking at the relational and extended abstract levels in problem solving involving circle theorems.

5. Discussion

The focus of this study was to understand how senior high school girl students are reasoning to solve problems in circle theorems. First, the study found that the majority of the students were inclined to uni-structural and multi-structural thinking with only a few thinking at relational and extended abstract levels. This finding corroborates earlier findings that majority of the students in geometry are operating at the multi-structural levels of the SOLO taxonomy [17, 19-21]. Per the SOLO taxonomy, students at uni-structural and multi-structural levels are said to be operating at the lower cognitive levels which according to Özdemir and Yildiz [14] are comparable to the visualization and analysis levels of van Hiele's geometric thinking levels. These are lower levels of thinking which perhaps, suggests that the students have not yet moved beyond the superficial knowledge in terms of their structure of the observed learning outcomes in circle theorems. The objective of the SHS mathematics curriculum for teaching circle theorems is for students to develop logical, abstract and deductive thinking abilities for solving problems on geometry of circles. These abilities occur with students at the relational and extended abstract thinking levels of SOLO taxonomy. Unfortunately, the present finding where only few students are operating at relational and extended abstract levels implies that the levels of thinking of most students in this study are lower than the curriculum expectation [5]. The reasons for this may be far fetch. However, research suggests that the direct teaching approaches adopted at the SHS may be attributed to poor mathematical thinking among students [1, 22] as students' thinking levels increase, their skills of making consistent explanations, creating relations and thinking in single to multiple situation also increase. Perhaps, a change in teaching methodology towards multiple representations with dynamic geometric software could enhance students' ability to reason logically, abstractly and deductively to solve problems on geometry of circles.

Second, from the theorem by theorem analysis, it was found that most students reasoned accurately with the first three theorems than with the fourth theorem. For instance, most students solved problems involving theorem 1 correctly with few procedural errors. The students also showed more consistent abilities in the statement of theorems and calculation parts of the problems than in drawing of conclusions and giving reasons to their solutions. Particularly, for theorem three and four, it was found in this study that majority were at uni-structural and multi-structural levels with only 29% of the students achieving relational and extended abstract levels of the SOLS taxonomy. That is most students were unable to inter-relate the theorems and their properties in their own calculations. This present finding agrees with the finding of Yulian [19], where students at low order domains were only able to use the visual and qualitative analysis consistently but failed to make connections to the contextual object of analysis.

Lastly, per the item by item analysis, the study result was clear that most students could not cope with questions involving reasons or drawing conclusions or sometimes stating the circle theorem. This finding implies that the students' knowledge of circle theorems was quite limited. The main difficulty of students was their inability to give reasons for each step of their solutions. It was also clear during the analysis of the students' solutions that, either they misrelated the theorems to their equations, miss out key terms or fail to indicate the theorem applied in the solutions. It also appeared that students attempted to memorize and reproduce the theorems or algorithms without understanding. This resulted into a situation where most students scored low marks. Such situation may have been accountable for the poor performance in circle theorems recently reported by chief examiners at national examination such as the West African Secondary School Examinations Certificate (WASSCE).

Of significant importance is the finding that up to two-fifth of the girls in this study operated at relational and extended abstract levels. This present finding, though not generalizable, provides empirical evidence that Ghanaian girls at the senior high school are capable of engaging high order mathematical reasoning needed for problem solving and pursuing mathematics related or STEM programmes. These girls are therefore highly likely to perform well in circle theorems and by extension succeed in the WASSCE examinations as well as in future STEM workplace.

6. Conclusion and Recommendation

A wide range of research argues that geometric thinking remains an essential part of mathematics responsible for the competencies of students in solving problems. It is therefore expected that students will be taught in a manner that ensures the development of higher order thinking abilities in geometry. In this study, it is quite evident that majority of the third year girls senior high school were not successful in solving problems pertaining to higher cognitive demand, i.e. at relational and extended abstract levels of SOLO taxonomy. The findings of this study suggest that students' level of thinking in solving problems in circle theorems has not yet reached the highest cognitive domain to be successful at West African Secondary School Certificate Examinations and will therefore require remediation.

The study therefore recommends that mathematics teachers need to teach circle theorems targeting meaning making, relational thinking and logical abstraction. This can be done by using real life scenarios or dynamic geometric construction of circle theorems in software such as GeoGebra. Mathematics teachers can also use SOLO taxonomy as a diagnostic assessment tool to obtain data on the structure of students' observed learning outcomes and reasoning ability in circle theorems. They can then use such data to devise appropriate teaching methodology or intervention measures to improve learning process.

7. Limitation

This study is quite unique as it deliberately focused on a purposive sample of only girls at one senior high school to contribute to dearth of literature on how girls reason to solve problem in geometry and mathematics in general. However, the study was limited in scope since finding may not be generalizable to mixed sex school settings. It is therefore suggested that further study should focus on a comparative analysis of the thinking levels of males and females in problem solving involving circle theorems to provide better lens for generalization.

Data Availability Statement

The data is integrated into the study. However, for further inquiry, contact the corresponding author.

Ethics Statement

Informed consent and permission were sought in writing and granted by authority at the District Education Office and head of the senior high school. Verbal consent was also sought from the girls who participated in the study. We ensured anonymity of participants and confidentiality of data collected.

Author Contributions

Both authors contributed to the study conception and design. The first author reviewed literature, wrote the background and collected the data for the study. The second author constructed the super-item used for the data collection, developed the methodology and devised the analytic procedure. Both authors contributed to the interpretation of data, revision and refining of the manuscript. Both read and approved the submitted manuscript.

Conflict of Interest

The authors declare that they have no competing interests.

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