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The Extended Adjacency Indices for Several Types of Graph Operations

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Abstract: Let G be a simple graph without multiple edges and any loops. At first, the extended adjacency matrix of a graph was first proposed by Yang et al in 1994, which is explored from the perspective of chemical molecular graph. Later, the spectral radius of graph and graph energy under the extended adjacency matrix was proposed. At the same time, for a simple graph G, the extended adjacency index *EA(G)* is also defined by some researchers. All of them play important roles in mathematics and chemistry. In this work, we show the extended adjacency indices for several types of graph operations such as tensor product, disjunction and strong product. In addition, we also give some examples of different combinations of special graphs, such as complete graphs and cycle graphs, and the classical graph, Cayley graph. By combining the special structure of the graph, it will pave the way for the calculation of some chemical or biological classical molecular structure. We can find that it plays a meaningful role in calculating the structure of complex chemical molecules through the graph operation of EA index on the any simple combined graphs, and it can also play a role in biology, physics, medicine and so on. Finally, we put forward some other related problems that can be further studied in the future.

Keywords: Degree of a Vertex, Extended Adjacency Index, Tensor Product, Disjunction, Strong Product

1. Introduction

In mathematical chemistry, many topological indices have been proposed by researchers, such as Randic index [15, 17], Zagreb index [6, 14], Forgotten index [9], Wiener index [7, 8, 22] and Estrada index [2, 12, 16]. Indeed, degree-based topological molecular descriptors have played a significant role in chemical, biological and physical researches in recent years. Especially, they have been found many applications in QSPR/QSAR researches.

As a class of degree-based topological molecular descriptor, the extended adjacency index of a connected graph was proposed by Yang [21] in 1994, and defined as

$$EA(G)=\sum_{uv\in E(G)}\frac{1}{2}\left(\frac{d_G(u)}{d_G(v)}+\frac{d_G(v)}{d_G(u)}\right),$$

where $d_G(u)$ and $d_G(v)$ are the degrees of vertices u and v, respectively.

Recently, some new applications [10, 11, 21] and properties of the EA indices [4, 20] are found. In particular, some extremal graphs and equienergetic graphs are characterized [1, 4]. In this work, we show the extended adjacency indices for several types of graph operations such as tensor product, disjunction and strong product.

The definitions below will be used.

Definition 1 [21] The extended adjacency index of a connected graph G is defined as

$$EA(G)=\sum_{uv\in E(G)}\frac{1}{2}\left(\frac{d_G(u)}{d_G(v)}+\frac{d_G(v)}{d_G(u)}\right),$$

where $d_G(u)$ is the degree of vertex u in G.

Definition 2 [6] The first Zagreb index defined as,

$$M_1(G) = \sum_{u\in V(G)} d(u)^2 = \sum_{(u,v)\in E(G)}[d(u) + d(v)].$$

Definition 3 [18] The tensor product of two graphs G_1 and G_2 is the graph $G_1 \otimes G_2$ with the vertex set $V(G_1) \times V(G_2)$, and the two vertices (a, b) and (c, d) are adjacent if and only if $ac \in E(G_1)$ and $bd \in E(G_2)$. The degree of a vertex (e, f) in $G_1 \otimes G_2$ is given by $d_{G_1\otimes G_2}(e, f) = d_{G_1}(e)d_{G_2}(f)$.

Definition 4 [5] The disjunction of two graphs G_1 and G_2 is the graph $G_1 \wedge G_2$ with the vertex set $V(G_1) \times V(G_2)$, and the two vertices (a, b) and (c, d) are adjacent either $ac \in E(G_1)$ or $bd \in E(G_2)$. The degree of a vertex (e, f) in $G_1 \wedge G_2$ is given by $d_{G_1\wedge G_2}(e, f) = n_1 d_{G_2}(f) + n_2 d_{G_1}(e) - d_{G_1}(e)d_{G_2}(f)$.

Definition 5 [19] The Strong product of two graphs G_1 and G_2 is the graph $G_1 \boxtimes G_2$ with the vertex set $V(G_1) \times V(G_2)$, and the two vertices (a, b) and (c, d) are adjacent whether $[a = c \in V(G_1)$ and bd $\in E(G_2)]$ or $[b = d \in V(G_2)$ and ac$\in E(G_1)]$ or[ac $\in E(G_1)$ and bd $\in E(G_2)]$. The degree of a vertex (e, f) in $G_1 \boxtimes G_2$ is given by

$$d_{(G_1\boxtimes G_2)}(e, f) = d_{G_1}(e) + d_{G_2}(f) + d_{G_1}(e)d_{G_2}(f).$$

For other undefined notations and terminologies, refer to [3]. Examples of graph operations are as follows, see Figures 1-3.

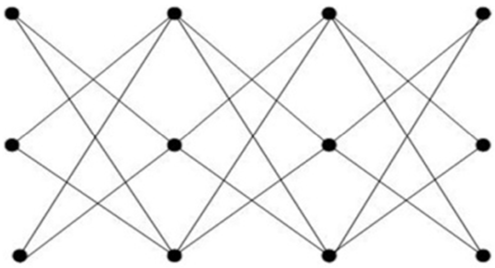

Figure 1. $K_3 \otimes P_4$.

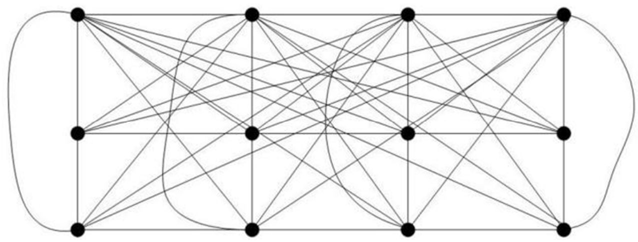

Figure 2. $K_3 \wedge P_4$.

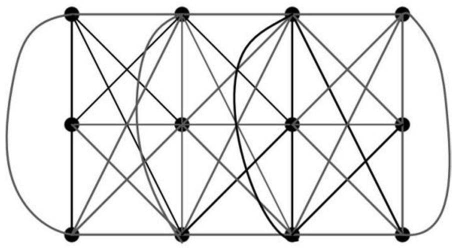

Figure 3. $K_3 \boxtimes P_4$.

2. Main Results

In this section, we calculate the EA indices for the graph operations above.

Theorem 2.1 Let $G_1(n_1, m_1)$ and $G_2(n_2, m_2)$ be two connected graphs. Suppose $|E(G_1)| = m_1, |E(G_2)| = m_2$ and $|V(G_1)| = n_1, |V(G_2)| = n_2$. Then

$$EA(G_1 \otimes G_2) \le 2m_1 m_2 \frac{\Delta_1\Delta_2}{\delta_1\delta_2},$$

where $\Delta_1(\delta_1)$ and $\Delta_2(\delta_2)$ are the maximum (minimum) degrees of G_1 and G_2, respectively. Equality holds if and only if $G_1(n_1, m_1)$ and $G_2(n_2, m_2)$ are all regular.

Proof Let $V(G_1) = \{e_1, e_2, \dots e_{n_1}\}$ and $V(G_2) = \{f_1, f_2, \dots f_{n_2}\}$ be the vertex sets of G_1 and G_2successively. By the above definition 3,$d_{G_1\otimes G_2}(e, f) = d_{G_1}(e)d_{G_2}(f)$. Then

$$\begin{aligned}
EA(G_1 \otimes G_2) &= \frac{1}{2}\sum_{((e_i,f_j),(e_k,f_l))\in E(G_1\otimes G_2),(e_i,f_j)\neq(e_k,f_l)}\left[\frac{d_{G_1\otimes G_2}(e_i,f_j)}{d_{G_1\otimes G_2}(e_k,f_l)} + \frac{d_{G_1\otimes G_2}(e_k,f_l)}{d_{G_1\otimes G_2}(e_i,f_j)}\right]\\
&= \frac{1}{2}\sum_{(e_i,e_k)\in E(G_1)}\sum_{(f_j,f_l)\in E(G_2)}\left[\frac{d_{G_1}(e_i)d_{G_2}(f_j)}{d_{G_1}(e_k)d_{G_{21}}(f_l)} + \frac{d_{G_1}(e_k)d_{G_2}(f_l)}{d_{G_1}(e_i)d_{G_2}(f_j)}\right]\\
&\leqslant \frac{1}{2}\times 2m_1m_2\left(\frac{\Delta_1\Delta_2}{\delta_1\delta_2} + \frac{\Delta_1\Delta_2}{\delta_1\delta_2}\right)\\
&= 2m_1m_2\frac{\Delta_1\Delta_2}{\delta_1\delta_2}
\end{aligned}$$

Corollary 2.2 Let C_n and C_m be two cycles with orders n and m, respectively. Then $EA(G_1 \otimes G_2) = 2nm$.

Corollary 2.3 Let K_n and K_m be two complete graphs with orders n and m, respectively. Then $EA(G_1 \otimes G_2) = \frac{nm(n-1)(m-1)}{2}$.

Corollary 2.4 Let G_1 and G_2 be two cayley graphs of nilpotent matrix group of length one. Then $EA(G_1 \otimes G_2) = 8n^2$, where $n \ge 7$.

Proof: For $n \ge 7$, we know that, $|V(G_1)| = |V(G_2)| = n$, $|E(G_1)| = |E(G_2)| = 2n$, and $d_{G_1}(u) = d_{G_2}(v) = 4$, where $u \in V(G_1), v \in V(G_2)$ [13]. □

Theorem 2.5 Let $G_1(n_1, m_1)$ and $G_2(n_2, m_2)$ be two connected graphs. Set $|E(G_1)| = m_1, |E(G_2)| = m_2$ and $|V(G_1)| = n_1, |V(G_2)| = n_2$. Then

$$EA(G_1 \wedge G_2) \le (n_2^2 m_1 + n_1^2 m_2)\frac{n_1\Delta_1 + n_2\Delta_2 - \delta_1\delta_2}{n_1\delta_1 + n_2\delta_2 - \Delta_1\Delta_2},$$

where $\Delta_1(\delta_1)$ and $\Delta_2(\delta_2)$ are the maximum (minimum) degrees of G_1 and G_2, respectively. Equality holds if and only if $G_1(n_1, m_1)$ and $G_2(n_2, m_2)$ are all regular.

Proof Let $V(G_1) = \{e_1, e, \dots e_{n_1}\}$ and $V(G_2) = \{f_1, f_2, \dots f_{n_2}\}$ be the vertex sets of G_1 and G_2 successively. By the above definition 4, we get $d_{G_1 \wedge G_2}(e, f) = n_1 d_{G_2}(e) + n_2 d_{G_1}(f) - d_{G_1}(e) d_{G_2}(f)$, and the edges of $G_1 \wedge G_2$ are partitioned into two subsets as follows.

$$Q_1(G_1 \wedge G_2) = \{e_i e_k \in E(G_1)\},$$

$$Q_2(G_1 \wedge G_2) = \{f_j f_l \in E(G_2)\}.$$

Then

$$EA(G_1 \wedge G_2) = \frac{1}{2} \sum_{((e_i,f_j),(e_k,f_l)) \in E(G_1 \otimes G_2),(e_i,f_j) \neq (e_k,f_l)} \left[\frac{d_{G_1 \wedge G_2}(e_i,f_j)}{d_{G_1 \wedge G_2}(e_k,f_l)} + \frac{d_{G_1 \wedge G_2}(e_k,f_l)}{d_{G_1 \wedge G_2}(e_i,f_j)} \right] = \frac{1}{2}(Q_1 + Q_2)$$

where,

$$Q_1 = \sum_{((e_i,f_j),(e_k,f_l)) \in E(G_1 \wedge G_2), i \neq k} \left[\frac{d_{G_1 \wedge G_2}(e_i,f_j)}{d_{G_1 \wedge G_2}(e_k,f_l)} + \frac{d_{G_1 \wedge G_2}(e_k,f_l)}{d_{G_1 \wedge G_2}(e_i,f_j)} \right] = \sum_{f_j \in V(G_2)} \sum_{f_l \in V(G_2)} \sum_{(e_i,e_k) \in E(G_1)} \left[\frac{d_{G_1 \wedge G_2}(e_i,f_j)}{d_{G_1 \wedge G_2}(e_k,f_l)} + \frac{d_{G_1 \wedge G_2}(e_k,f_l)}{d_{G_1 \wedge G_2}(e_i,f_j)} \right].$$

$$Q_2 = \sum_{((e_i,f_j),(e_k,f_l)) \in E(G_1 \wedge G_2), j \neq l} \left[\frac{d_{G_1 \wedge G_2}(e_i,f_j)}{d_{G_1 \wedge G_2}(e_k,f_l)} + \frac{d_{G_1 \wedge G_2}(e_k,f_l)}{d_{G_1 \wedge G_2}(e_i,f_j)} \right] = \sum_{e_i \in V(G_1)} \sum_{e_k \in V(G_1)} \sum_{(f_j,f_l) \in E(G_2)} \left[\frac{d_{G_1 \wedge G_2}(e_i,f_j)}{d_{G_1 \wedge G_2}(e_k,f_l)} + \frac{d_{G_1 \wedge G_2}(e_k,f_l)}{d_{G_1 \wedge G_2}(e_i,f_j)} \right].$$

According to the definition of disjunction for two graphs, we can easily get

$$\begin{aligned} \frac{d_{G_1 \wedge G_2}(e_i,f_j)}{d_{G_1 \wedge G_2}(e_k,f_l)} + \frac{d_{G_1 \wedge G_2}(e_k,f_l)}{d_{G_1 \wedge G_2}(e_i,f_j)} &= \frac{n_1 d_{G_2}(f_j) + n_2 d_{G_1}(e_i) - d_{G_1}(e_i) d_{G_2}(f_j)}{n_1 d_{G_2}(f_l) + n_2 d_{G_1}(e_k) - d_{G_1}(e_k) d_{G_2}(f_l)} + \frac{n_1 d_{G_2}(f_l) + n_2 d_{G_1}(e_k) - d_{G_1}(e_k) d_{G_2}(f_l)}{n_1 d_{G_2}(f_j) + n_2 d_{G_1}(e_i) - d_{G_1}(e_i) d_{G_2}(f_j)} \\ &\leq \left[\frac{n_1 \Delta_2 + n_2 \Delta_1 - \delta_1 \delta_2}{n_1 \delta_2 + n_2 \delta_1 - \Delta_1 \Delta_2} + \frac{n_1 \Delta_2 + n_2 \Delta_1 - \delta_1 \delta_2}{n_1 \delta_2 + n_2 \delta_1 - \Delta_1 \Delta_2} \right] \\ &= 2 \frac{n_1 \Delta_2 + n_2 \Delta_1 - \delta_1 \delta_2}{n_1 \delta_2 + n_2 \delta_1 - \Delta_1 \Delta_2} \end{aligned}$$

Therefore,

$$\begin{aligned} EA(G_1 \wedge G_2) &\leq \frac{1}{2}(n_2^2 m_1 (2 \frac{n_1 \Delta_1 + n_2 \Delta_2 - \delta_1 \delta_2}{n_1 \delta_1 + n_2 \delta_2 - \Delta_1 \Delta_2}) + n_1^2 m_2 (2 \frac{n_1 \Delta_1 + n_2 \Delta_2 - \delta_1 \delta_2}{n_1 \delta_1 + n_2 \delta_2 - \Delta_1 \Delta_2})) \\ &= (n_2^2 m_1 + n_1^2 m_2) \frac{n_1 \Delta_1 + n_2 \Delta_2 - \delta_1 \delta_2}{n_1 \delta_1 + n_2 \delta_2 - \Delta_1 \Delta_2} \end{aligned} \quad \square$$

Corollary 2.6 Let C_n and C_m be two cycles with orders n and m, respectively. Then $EA(G_1 \wedge G_2) = mn(m+n)$.

Corollary 2.7 Let K_n and K_m be two complete graphs with orders n and m, respectively. Then $EA(G_1 \wedge G_2) = \frac{nm(2nm-n-m)}{2}$.

Corollary 2.8 Let G_1 and G_2 be two cayley graphs of nilpotent matrix group of length one. Then $EA(G_1 \wedge G_2) = 4n^3$.

Theorem 2.9 Let $G_1(n_1, m_1)$ and $G_2(n_2, m_2)$ be two connected graphs. Suppose $|E(G_1)| = m_1, |E(G_2)| = m_2$ and $|V(G_1)| = n_1, |V(G_2)| = n_2$. Then

$$EA(G_1 \boxtimes G_2) \leq \frac{1}{2}[(\frac{2\Delta_1 m_2 n_1}{\delta_1 + \delta_2 + \delta_1 \delta_2} + \frac{(1+\Delta_1) n_1}{\delta_1 + \delta_2 + \delta_1 \delta_2} M_1(G_2)) + \frac{2\Delta_2 m_1 n_2}{\delta_1 + \delta_2 + \delta_1 \delta_2} + \frac{(1+\Delta_2) n_2}{\delta_1 + \delta_2 + \delta_1 \delta_2} M_1(G_1) + 4 m_1 m_2 \frac{\Delta_1 + \Delta_2 + \Delta_1 \Delta_2}{\delta_1 + \delta_2 + \delta_1 \delta_2}],$$

where $\Delta_1(\delta_1)$ and $\Delta_2(\delta_2)$ are the maximum (minimum) degrees of G_1 and G_2, respectively. Equality holds if and only if $G_1(n_1, m_1)$ and $G_2(n_2, m_2)$ are all regular.

Proof Let $V(G_1) = \{e_1, e_2, \dots e_{n_1}\}$ and $V(G_2) = \{f_1, f_2, \dots f_{n_2}\}$ be the vertex sets of G_1 and G_2successively. By the above definition of 5, we get $d_{(G_1 \boxtimes G_2)}(e, f) = d_{G_1}(e) + d_{G_2}(f) + d_{G_1}(e) d_{G_2}(f)$, and the edges of $G_1 \boxtimes G_2$ are partitioned into three subsets S_1, S_2 and S_3 as follows.

$$S_1(G_1 \boxtimes G_2) = \{e_i = e_k \in V(G_1) \text{ and } f_j f_l \in E(G_2)\},$$

$$S_2(G_1 \boxtimes G_2) = \{f_j = f_l \in V(G_2) \text{ and } e_i e_k \in E(G_1)\},$$

$$S_3(G_1 \boxtimes G_2) = \{e_i e_k \in E(G_1) \text{ and } f_j f_l \in E(G_2)\}.$$

Then

$$EA(G_1 \boxtimes G_2) = \frac{1}{2} \sum_{((e_i,f_j),(e_k,f_l)) \in E(G_1 \boxtimes G_2),(e_i,f_j) \neq (e_k,f_l)} \left[\frac{d_{G_1 \boxtimes G_2}(e_i,f_j)}{d_{G_1 \boxtimes G_2}(e_k,f_l)} + \frac{d_{G_1 \boxtimes G_2}(e_k,f_l)}{d_{G_1 \boxtimes G_2}(e_i,f_j)} \right]$$
$$= \frac{1}{2}(S_1 + S_2 + S_3)$$

where,

$$S_1 = \sum_{((e_i,f_j),(e_i,f_l)) \in E(G_1 \boxtimes G_2), j \neq l} \frac{d_{G_1 \boxtimes G_2}(e_i,f_j)}{d_{G_1 \boxtimes G_2}(e_i,f_l)} + \frac{d_{G_1 \boxtimes G_2}(e_i,f_l)}{d_{G_1 \boxtimes G_2}(e_i,f_j)}$$
$$= \sum_{e_i \in V(G_1)} \sum_{(f_j,f_l) \in E(G_2)} \left[\frac{d_{G_1}(e_i) + d_{G_2}(f_j) + d_{G_1}(e_i) d_{G_2}(f_j)}{d_{G_1}(e_i) + d_{G_2}(f_l) + d_{G_1}(e_i) d_{G_2}(f_l)} + \frac{d_{G_1}(e_i) + d_{G_2}(f_l) + d_{G_1}(e_i) d_{G_2}(f_l)}{d_{G_1}(e_i) + d_{G_2}(f_j) + d_{G_1}(e_i) d_{G_2}(f_j)} \right]$$
$$\leq n_1 \sum_{(f_j,f_l) \in E(G_2)} \left[\frac{\Delta_1 + d_{G_2}(f_j) + \Delta_1 d_{G_2}(f_j)}{\delta_1 + \delta_2 + \delta_1 \delta_2} + \frac{\Delta_1 + d_{G_2}(f_l) + \Delta_1 d_{G_2}(f_l)}{\delta_1 + \delta_2 + \delta_1 \delta_2} \right]$$
$$= \frac{n_1}{\delta_1 + \delta_2 + \delta_1 \delta_2} \sum_{(f_j,f_l) \in E(G_2)} \left[2\Delta_1 + (1+\Delta_1)(d_{G_2}(f_j) + d_{G_2}(f_l)) \right]$$
$$= \frac{2\Delta_1 m_2 n_1}{\delta_1 + \delta_2 + \delta_1 \delta_2} + \frac{(1+\Delta_1) n_1}{\delta_1 + \delta_2 + \delta_1 \delta_2} M_1(G_2)$$

$$S_2 = \sum_{((e_i,f_j),(e_k,f_j)) \in E(G_1 \boxtimes G_2), i \neq k} \frac{d_{G_1 \boxtimes G_2}(f_j,e_i)}{d_{G_1 \boxtimes G_2}(f_j,e_k)} + \frac{d_{G_1 \boxtimes G_2}(f_j,e_k)}{d_{G_1 \boxtimes G_2}(f_j,e_i)}$$
$$= \sum_{f_j \in V(G_2)} \sum_{(e_i,e_k) \in E(G_1)} \left[\frac{d_{G_2}(f_j) + d_{G_1}(e_i) + d_{G_2}(f_j) d_{G_1}(e_i)}{d_{G_2}(f_j) + d_{G_1}(e_k) + d_{G_2}(f_j) d_{G_1}(e_k)} + \frac{d_{G_2}(f_j) + d_{G_1}(e_k) + d_{G_2}(f_j) d_{G_1}(e_k)}{d_{G_2}(f_j) + d_{G_1}(e_i) + d_{G_2}(f_j) d_{G_1}(e_i)} \right]$$
$$\leq n_2 \sum_{(e_i,e_k) \in E(G_1)} \left[\frac{\Delta_2 + d_{G_1}(e_i) + \Delta_2 d_{G_1}(e_i))}{\delta_1 + \delta_2 + \delta_1 \delta_2} + \frac{\Delta_2 + d_{G_1}(e_k) + \Delta_2 d_{G_1}(e_k)}{\delta_1 + \delta_2 + \delta_1 \delta_2} \right]$$
$$= \frac{n_2}{\delta_1 + \delta_2 + \delta_1 \delta_2} \sum_{(e_i,e_k) \in E(G_1)} \left[2\Delta_2 + (1+\Delta_2)(d_{G_1}(e_i) + d_{G_1}(e_k)) \right]$$
$$= \frac{2\Delta_2 m_1 n_2}{\delta_1 + \delta_2 + \delta_1 \delta_2} + \frac{(1+\Delta_2) n_2}{\delta_1 + \delta_2 + \delta_1 \delta_2} M_1(G_1)$$

$$S_3 = \sum_{((e_i,f_j),(e_k,f_l)) \in E(G_1 \boxtimes G_2),(e_i,f_j) \neq (e_k,f_l)} \left[\frac{d_{G_1 \boxtimes G_2}(e_i,f_j)}{d_{G_1 \boxtimes G_2}(e_k,f_l)} + \frac{d_{G_1 \boxtimes G_2}(e_k,f_l)}{d_{G_1 \boxtimes G_2}(e_i,f_j)} \right]$$
$$= \sum_{(e_i,e_k) \in E(G_1)} \sum_{(f_j,f_l) \in E(G_2)} \left[\frac{d_{G_1}(e_i) + d_{G_2}(f_j) + d_{G_1}(e_i) d_{G_2}(f_j)}{d_{G_1}(e_k) + d_{G_2}(f_l) + d_{G_1}(e_k) d_{G_2}(f_l)} + \frac{d_{G_1}(e_k) + d_{G_2}(f_l) + d_{G_1}(e_k) d_{G_2}(f_l)}{d_{G_1}(e_i) + d_{G_2}(f_j) + d_{G_1}(e_i) d_{G_2}(f_j)} \right]$$
$$= 2m_1 m_2 \left[\frac{\Delta_1 + \Delta_2 + \Delta_1 \Delta_2}{\delta_1 + \delta_2 + \delta_1 \delta_2} + \frac{\Delta_1 + \Delta_2 + \Delta_1 \Delta_2}{\delta_1 + \delta_2 + \delta_1 \delta_2} \right]$$
$$= 4m_1 m_2 \frac{\Delta_1 + \Delta_2 + \Delta_1 \Delta_2}{\delta_1 + \delta_2 + \delta_1 \delta_2}$$

Then

$$EA(G_1 \boxtimes G_2) \leq \frac{1}{2}[\frac{2\Delta_1 m_2 n_1}{\delta_1 + \delta_2 + \delta_1 \delta_2} + \frac{(1+\Delta_1) n_1}{\delta_1 + \delta_2 + \delta_1 \delta_2} M_1(G_2) + \frac{2\Delta_2 m_1 n_2}{\delta_1 + \delta_2 + \delta_1 \delta_2} + \frac{(1+\Delta_2) n_2}{\delta_1 + \delta_2 + \delta_1 \delta_2} M_1(G_1) + 4m_1 m_2 \frac{\Delta_1 + \Delta_2 + \Delta_1 \Delta_2}{\delta_1 + \delta_2 + \delta_1 \delta_2}]$$

Corollary 2.10 Let C_n and C_m be two cycles with orders n and m, respectively. Then $EA(G_1 \boxtimes G_2) = 4mn$.

Corollary 2.11 Let K_n and K_m be two complete graphs with orders n and m, respectively. Then

$$EA(G_1 \boxtimes G_2) = \frac{1}{2}\left(\frac{(m+n)[(n-1)(m-1)] + m^3(m-1)^2 + n^3(n-1)^2}{(n-1)+(m-1)+(n-1)(m-1)} + mn(n-1)(m-1)\right).$$

Corollary 2.12 Let G_1 and G_2 be two cayley graphs of nilpotent matrix group of length one. Then $EA(G_1 \boxtimes G_2) = 12n^2$.

3. Conclusion

In this work, we compute the EA indices for three graph operations, and use the results to calculate some special graphs. Except that, more other graph operations for EA index and combinations of general graphs can be considered in further researches. Furthermore, it is a meaningful and challenging problem to generalize the EA index of ordinary graph to hypergraph.

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