



Modelling and Simulation of Vehicle Electric Power Battery System

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Abstract: Electric power battery will continue to play significant role in electrifying of transportation systems as its capacity to store clean energy and provide reliable power is continually being improved through research and development. In a related vein electric power batteries for vehicles and other applications are considered viable alternative energy solutions in the global quest to mitigate the impact of greenhouse effect. Documented evidence shows that enormous resources are being invested by organizations toward development of batteries with higher energy density, longer life, faster and more efficient load. Thus, as national governments continue to legislate on climate change, more productive investments in research and development will ultimately favour the adoption of efficient and sustainable electric battery powered mobility solutions in the long run. Developing efficient and sustainable energy solution requires that electric power batteries' performance should be reliable. Thus, the need to build reliable electric power battery system has provided the impetus to this study. In order to accurately study the performance of electric powered batteries, an equivalent circuit model is usually simulated to analyse dynamic characteristics and contrast with different order models of the battery. Using experiments the parameters of the battery are calibrated for improved efficiency. In this paper, we designed, developed and fitted the electric power battery model through electromagnetic induction and estimated its parameters; the battery will be recharged after a given period of time when the power is exhausted. The reliability model for the electric power battery was determined using the non-parametric method. We developed a simple linear regression model and through it, estimated the parameters of the fitted probability model, the mean time to failure and the reliability of the battery. We determined the mean time to failure of the power battery to be 477.12 hours, which is approximately 20days; and the reliability of the power battery is 3.609×10^{-71} . We found and fitted the probability distribution for the electric power battery system and determined the output for each loop as presented in Table 2 by the Y* column. The developed and fitted regression model is used to forecast for the performance and future output of the power battery system.

Keywords: Electric Power Battery, Electromagnetic Induction, Electric Vehicle, Reliability, Simulation, Probability Distribution

1. Introduction

Decisions about mitigation of impact of climate change effect globally have led to considerations of electric power

batteries for vehicles and other applications as viable alternative energy solutions. Thus, there is a continuous and conscious efforts being made in different fields that will lead to development of batteries with higher energy density, longer life, faster and more efficient load [1]. In the long run,

price is expected to decrease, thus favouring the adoption of efficient and sustainable electric battery powered mobility solutions [1]. The interest of this paper is to develop an electric power battery system that will produce current for vehicular electrical applications and others. The electric power battery will maintain a steady power supply over its life span. Our focus is on the electromagnetic induction, a renewable source of energy. This will be achieved through the electromagnetic induction. It has been discovered that a loop of wire connected to a galvanometer in a magnetic field provided by a bar magnet produces electricity. A relative motion between the coil (loop) and the bar magnet induces current in the coil which is registered by the galvanometer. The induced current is produced by keeping the coil stationary while the bar magnet moves by the help of electric motor. Michael Faraday found that current always flows through the galvanometer in the secondary circuit whenever the current in the primary circuit changes. The induced current arises as a result of an induced Electromotive Force (EMF) in the secondary coil. The induced EMF is produced whenever the magnetic field changes. The magnitude of this field in any region of space is determined by the number of lines of force passing through the given surface- referred to as the magnetic flux. In this paper, we developed electric power battery model and determined the reliability of its output (I), the mean time to failure (MTT) and the quantity of electricity generated at any given time; similar approaches can be found in studies by Amuji and Umelo-Ibemere [2] and Ugwuanyim, Amuji, Bartholomew and Wisdom [3]. In order to develop the proposed electric power battery model, we simulate the following using Minitab software (version 16.0): the number of loops; area of the coil; change in magnetic flux and change in time under a constant resistance of the coil (wire) at 8Ω . The sample size for the data is 32, and since the sample size is large, the data follows a normal distribution. Hence, in simulating the number of loops, we choose $\mu = 125$ and $\sigma = 38$; for Area, we choose $\mu = 18$ and $\sigma = 4$; for change in magnetic flux B, we choose $\mu = 14$ and $\sigma = 4$; for change in time, we choose $\mu = 8$ and $\sigma = 3$; and the results are presented in Table 1.

2. Literature Review

Most of the papers in extant literature on electric power battery concentrated on improvement of performance parameters on select range of vehicles. Notable ones include: Thombare, Bhatiya and Sapkal [1] that designed battery management systems in electric vehicles. In their work, an electrical equivalent circuit (EEC) model was adopted, with its parameters estimated on the basis of experimental data collected through in-cell tests at different temperatures and charge/discharge current profiles. Using MATLAB software and simulation, a design model of electric battery with optimized input and output parameters was developed. Their findings are also consistent with Bhovi, Ranjith, Sachin and Kariyappa [4]. Hu [5] was more concerned with designing battery management systems for safe operation of lithium-ion batteries in electric

vehicle applications. An improved battery model was designed considering the self-discharging effect, the temperature effect and the fading-capacity effect observed in all batteries. The model was simulated using MATLAB/Simulink. The simulation results showed improvement on the original system by adding a user interface, a thermal management system and a current-monitoring function.

Qin, Li, Wang and Zhang [6] investigated the dynamic characteristics of LiFePO₄ battery in a simulation experiment. The electromotive force, resistance, capacitance and other parameters were calibrated through battery charge and discharge experiments. This model was built by using Modelica- a modeling language for object-oriented multidomain physical systems. The results showed that the third-order RC battery model with hysteretic voltage well reflects the dynamics of a LiFePO₄ battery.

However, Sagaria [7] assessed the performance and range of vehicle powered by battery, fuel cell, ultra-capacitor and combination of the former using relevant parameters on flexible vehicle simulation model. Vehicle performance was evaluated in various test cases based state of charge (SOC), energy consumption/km, overall range and other performance details. The study showed that battery electric vehicle (BEV) has the least energy consumption (23%), followed by fuel cell electric vehicle (FCEV) (65%) compared to internal combustion engine (ICE) vehicles. The performance analysis showed that increasing the battery capacity of BEV by three-fold, the range will be extended by 294%, while the battery with higher energy density helps to reduce 2-4% in energy consumption. The findings are also consistent with Hanifah, Toha and Ahmad [8]. The present work is tangential to existing studies reviewed. In contrast, we see electric power battery development as a reliability problem. Hence, we considered the estimation of reliability and associated parameters as most important consideration in modelling electric power batteries for optimal performance.

The development of the proposed electric power battery model is a reliability problem. Reliability in this case is the probability that the power battery will perform its intended function under a given environmental condition at a given time; this assertion is consistent with Lewis [9], Smith, Crowder, Kimba and Sweeting [10], Irwin and John [11], Billinton and Alan [12] and Barlow & Proschan [13]. By intuition, the stored electricity will naturally run down over time; hence, we say that the power decays exponentially over time. The decay follows exponential curve and therefore exponential distribution. But exponential distributions is memory-less according to Arua, Chigbu, Chukwu, Ezekwem and Okafor [14] and hence cannot be used to model the proposed power battery that needs to retain current for a relatively long time. A more appropriate model for this system is a Weibull model, which is an extension of exponential model and has memory. Weibull distribution accommodates the three characteristics shown by every system under reliability. These characteristics are: the early (wear-in) failure characterized by high failure rate due to inefficiency in manufacturing or production; the second is the period for useful life of the system characterized by chance or random failure, this portion has a constant failure rate and regarded as the period for

the useful life of the system and the last stage is the increasing (wear-out) failure due to aging of the system [15]. These characteristics are presented in figure 1.

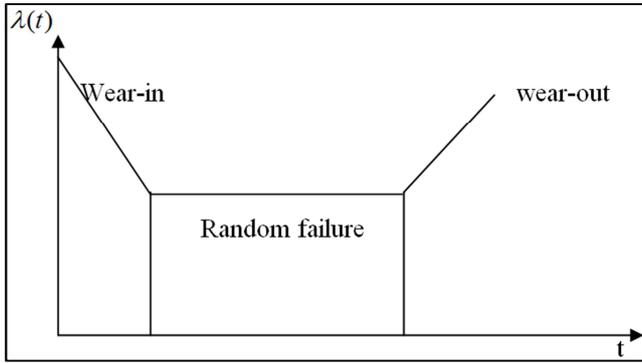


Figure 1. Bathtub Curve.

Weibull distribution adequately describes the proposed electric power battery model with the failure times when its failure rate either increases or decreases with time. It has the parameters α and β . The Weibull curve is asymptotic to both axes and highly skewed to the right for values of β less than 1; it is identical to that of the exponential density for $\beta = 1$, and it is bell-shaped but skewed for values of $\beta > 1$.

3. Materials and Methods

Flux, according to Weiss [16], is the product of the magnetic field and the area of the surface perpendicular to the magnetic lines of force given as

$$\Phi_B = \int B ds \tag{1}$$

The integration is over the area surrounded by the circuit. Induced EMF generated in a circuit is given as

$$\varepsilon = N \frac{d\Phi}{dt} = NA \frac{dB}{dt} \tag{2}$$

where N is the number of loops in the secondary circuit. Since the induction effect depends on the size of the loops within the field region, a change in the magnetic flux can be produced by a change in the area of the field within the loop. Thus, we can obtain electromagnetic induction with a uniform field by altering the area of the field inside the loop, that is

$$\varepsilon = N \frac{d\Phi}{dt} = NB \frac{dA}{dt} \tag{3}$$

But if a coil is twisted and allowed to rotate in a uniform magnetic field, it can produce a change in flux which can generate an induced current. Therefore, the electric power battery model is given as

$$f(t) = \beta \alpha^{-\beta} (I)^{\beta-1} \exp\{-((I) / \alpha)^\beta\} \tag{4}$$

where $(I) > 0, \alpha$ and $\beta > 0; \alpha$ being a scale parameter and β being a shape parameter.

The mean time to failure (MTTF) of the power battery model in equation (4) is

$$\begin{aligned} \mu &= \int_0^\infty I.f(I)dI \\ \text{but } \Gamma(x) &= \int_0^\infty U^{x-1}e^{-u}du \\ \mu &= \int_0^\infty I. \beta \alpha^{-\beta} I^{\beta-1} \exp\{-(I / \alpha)^\beta\}dI = \int_0^\infty \beta(I / \alpha)^\beta e^{-(I/\alpha)^\beta} dI \end{aligned}$$

$$\text{let } U = (I / \alpha)^\beta \Rightarrow \alpha \frac{1}{\beta} U^{\frac{1}{\beta}-1} du = dI$$

$$= \beta \int_0^\infty U e^{-u} dI$$

$$= \beta \int_0^\infty U e^{-u} \cdot \alpha \frac{1}{\beta} U^{\frac{1}{\beta}-1} du$$

$$\mu = \alpha \int_0^\infty U^{(1+\frac{1}{\beta})-1} \cdot e^{-u} du$$

$$\therefore \mu = \alpha \Gamma\left(1 + \frac{1}{\beta}\right) \tag{5}$$

The reliability function is given as

$$R(t) = \text{Exp}(-\alpha I^\beta) \tag{6}$$

Estimation of the Parameters of the model (α and β): Taking the natural logarithm of equation (6), we have

$$\ln R(I) = -\alpha I^\beta \tag{7}$$

This implies that

$$\ln\left(\frac{1}{R(I)}\right) = \alpha I^\beta \tag{8}$$

Again, taking logarithm of both sides, we have

$$\ln\left[\ln\left(\frac{1}{R(I)}\right)\right] = \ln \alpha + \beta \ln I \tag{9}$$

and it can be seen that the right hand side (RHS) is linear in $\ln I$.

To estimate the parameters, we require estimates of $R(I)$ for various values of I , and the usual procedure is to place n

units of life test and observe their failure times. If i th unit fails at I_i , we estimate

$$F(I_i) = 1 - R(I_i) \quad (10)$$

by using the estimator

$$\hat{F}(I_i) = \frac{i-1/2}{n} \quad (11)$$

This is consistent with Irwin and John [11]. The parameters α and β are estimated by applying the method of least squares to the transformed points $(x(I)_i, y_i)$ where;

$$x(I)_i = \ln I_i \quad (12)$$

$$y_i = \ln \left(\ln \left[\frac{1}{1 - \hat{F}(I_i)} \right] \right) \quad (13)$$

$$\hat{F}(I_i) = \frac{i-1/2}{32}; i = 1, \dots, 32. \quad (14)$$

Given the regression model:

$$y_i = \alpha + \beta x_i + e_i \quad (15)$$

where y_i is the dependent variable (yield or quantity of electricity), α is the overall mean, β is the parameter measuring the effect of the predictor variables $x(I)_i$ and e_i is random error corresponding to the dependent variable (yield) y_i [17]. Our interest is to minimize the error (e_i), which we achieved as follows:

$$e_i = y_i - \alpha - \beta x_i \quad (16)$$

Squaring equation (16) and summing the squares, we have

$$\sum_{i=1}^n e_i^2 = \sum_{i=1}^n (y_i - \alpha - \beta x_i)^2; \text{ let } \sum_{i=1}^n e_i^2 = \varphi \quad (17)$$

$$\varphi = \sum_{i=1}^n (y_i - \alpha - \beta x_i)^2 = \quad (18)$$

Taking the partial derivative of both sides of equation (18) with respect to α and β , we have

$$\frac{\partial \varphi}{\partial \alpha} = -2 \sum_{i=1}^n (y_i - \hat{\alpha} - \beta x_i) \quad (19)$$

$$\frac{\partial \varphi}{\partial \beta} = -2 \sum_{i=1}^n x_i (y_i - \hat{\alpha} - \hat{\beta} x_i) \quad (20)$$

At turning point, $\frac{\partial \varphi}{\partial \alpha}$ and $\frac{\partial \varphi}{\partial \beta} = 0$ respectively

From (19),

$$0 = -2 \sum_{i=1}^n (y_i - \hat{\alpha} - \hat{\beta} x_i) \Rightarrow \sum_{i=1}^n (y_i - \hat{\alpha} - \hat{\beta} x_i) = 0 \quad (21)$$

From (20),

$$-2 \sum_{i=1}^n x_i (y_i - \hat{\alpha} - \hat{\beta} x_i) = 0 \Rightarrow \sum_{i=1}^n x_i (y_i - \hat{\alpha} - \hat{\beta} x_i) = 0 \quad (22)$$

From (21),

$$\sum y_i - n\hat{\alpha} - \hat{\beta} \sum x_i = 0 \quad (23)$$

$$\sum y_i - \hat{\beta} \sum x_i = n\hat{\alpha} \quad (24)$$

$$\hat{\alpha} = \frac{1}{n} (\sum y_i - \hat{\beta} \sum x_i) \quad (25)$$

$$\hat{\alpha} = \bar{y} - \hat{\beta} \bar{x} \quad (26)$$

From (22),

$$\sum x_i y_i - \hat{\alpha} \sum x_i - \hat{\beta} \sum x_i^2 = 0 \quad (27)$$

$$\sum x_i y_i - [\bar{y} - \hat{\beta} \bar{x}] \sum x_i - \hat{\beta} \sum x_i^2 = 0 \quad (28)$$

$$\sum x_i y_i - \sum x_i \bar{y} + \hat{\beta} \sum x_i \bar{x} - \hat{\beta} \sum x_i^2 = 0 \quad (29)$$

$$\sum x_i y_i - \sum x_i \frac{\sum y_i}{n} + \hat{\beta} \sum x_i \frac{\sum x_i}{n} - \hat{\beta} \sum x_i^2 = 0 \quad (30)$$

$$n \sum x_i y_i - \sum x_i \sum y_i + \hat{\beta} (\sum x_i)^2 - n \hat{\beta} \sum x_i^2 = 0 \quad (31)$$

$$n \sum x_i y_i - \sum x_i \sum y_i + \hat{\beta} ((\sum x_i)^2 - n \sum x_i^2) = 0 \quad (32)$$

$$n \sum x_i y_i - \sum x_i \sum y_i = \hat{\beta} (n \sum x_i^2 - (\sum x_i)^2) \quad (33)$$

$$\hat{\beta} = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{(n \sum x_i^2 - (\sum x_i)^2)} \quad (34)$$

The fitted regression model becomes

$$y_i = \hat{\alpha} + \hat{\beta} x_i \quad (35)$$

4. Analysis

Table 1. Induced current (A) and EMF, No. of loop (N); magnetic flux (B), change in time (t).

Loop Name	No. of Loops	Area (m ²)	dB (T)	Dt (min)	EMF (V)	EMF*60 (V)	R (Ohmn)	I(A) = V/R
1	126	0.22	0.16	2	2.2	133	8	16.6
2	91	0.15	0.16	4	0.5	33	8	4.1
3	134	0.17	0.8	6	3.0	182	8	22.8
4	94	0.14	0.11	7	0.2	12	8	1.6
5	137	0.19	0.15	6	0.7	39	8	4.9
6	132	0.13	0.16	14	0.2	12	8	1.5
7	105	0.15	0.17	7	0.4	23	8	2.9
8	183	0.24	0.16	7	1.0	60	8	7.5
9	123	0.16	0.14	15	0.2	11	8	1.4
10	93	0.16	0.16	4	0.6	36	8	4.5
11	133	0.22	0.72	14	1.5	90	8	11.3
12	118	0.16	0.12	1	2.3	136	8	17.0
13	104	0.9	0.18	11	1.5	92	8	11.5
14	156	0.21	0.13	8	0.5	32	8	4.0
15	99	0.16	0.15	6	0.4	24	8	3.0
16	190	0.16	0.14	9	0.5	28	8	3.5
17	115	0.16	0.82	3	5.0	302	8	37.7
18	181	0.17	0.12	10	0.4	22	8	2.8
19	143	0.13	0.17	6	0.5	32	8	4.0
20	184	0.17	0.79	9	2.7	165	8	20.6
21	127	0.18	0.14	12	0.3	16	8	2.0
22	108	0.21	0.13	11	0.3	16	8	2.0
23	143	0.24	0.18	9	0.7	41	8	5.1
24	147	0.17	0.1	8	0.3	19	8	2.3
25	122	0.2	0.11	8	0.3	20	8	2.5
26	127	0.17	0.17	3	1.2	73	8	9.2
27	113	0.18	0.15	5	0.6	37	8	4.6
28	121	0.22	0.12	5	0.6	38	8	4.8
29	90	0.15	0.39	4	1.3	79	8	9.9
30	64	0.16	0.18	6	0.3	18	8	2.3
31	91	0.16	0.15	11	0.2	12	8	1.5
32	105	0.13	0.92	6	2.1	126	8	15.7
TOTAL QUANTITY OF ELECTRICITY (current, I) GENERATED								244.9

Having determined the quantity of electricity; we determine how stable the electricity supply from the power battery would be. We determine the parameters of the probability distribution function of the electric power battery of equation (4), the reliability of the system and the mean

time to failure (MTTF) of the system. For computations in Table 2, refer to Table 1 for current (I), equation (12) for computation of the independent variable, $x(I)_i$ and equation (14) for computation of the estimator, $\hat{F}(I)_i$.

Table 2. Presents the values for the computation of the trend of equation (35).

Loop Name	F(I _i)	I(A)	X(I _i)	Y _i	X(I _i)Y _i	X(I _i) ²	Y _i *
1	0.0156	16.6	2.809403	-4.15263	-11.6664	7.892744	3.53612
2	0.0469	4.1	1.410987	-3.03582	-4.2835	1.990884	1.37237
3	0.0781	22.8	3.126761	-2.50938	-7.84623	9.776631	4.60934
4	0.1094	1.6	0.470004	-2.15537	-1.01303	0.220903	0.93962
5	0.1406	4.9	1.589235	-1.88703	-2.99894	2.525669	1.51085
6	0.1719	1.5	0.405465	-1.66801	-0.67632	0.164402	0.92231
7	0.2031	2.9	1.064711	-1.48269	-1.57864	1.133609	1.16465
8	0.2344	7.5	2.014903	-1.32015	-2.65997	4.059834	1.96091
9	0.2656	1.4	0.336472	-1.17538	-0.39548	0.113214	0.9050
10	0.2969	4.5	1.504077	-1.0434	-1.56935	2.262249	1.44161
11	0.3281	11.3	2.424803	-0.92219	-2.23614	5.879668	2.61869
12	0.3594	17	2.833213	-0.80889	-2.29177	8.027098	3.60536
13	0.3906	11.5	2.442347	-0.70263	-1.71607	5.965059	2.65331
14	0.4219	4	1.386294	-0.60146	-0.83381	1.921812	1.35506
15	0.4531	3	1.098612	-0.50503	-0.55483	1.206949	1.18196
16	0.4844	3.5	1.252763	-0.41185	-0.51595	1.569415	1.26851

Loop Name	F(I _i)	I(A)	X(I _i)	Y _i	X(I _i)Y _i	X(I _i) ²	Y _i *
17	0.5156	37.7	3.62966	-0.3218	-1.16802	13.17443	7.18853
18	0.5469	2.8	1.029619	-0.23365	-0.24057	1.060116	1.14734
19	0.5781	4	1.386294	-0.14736	-0.20428	1.921812	1.35506
20	0.6094	20.6	3.025291	-0.0618	-0.18696	9.152386	4.22852
21	0.6406	2	0.693147	0.023052	0.015978	0.480453	1.00886
22	0.6719	2	0.693147	0.108349	0.075102	0.480453	1.00886
23	0.7031	5.1	1.629241	0.194217	0.316426	2.654425	1.54547
24	0.7344	2.3	0.832909	0.281989	0.234871	0.693738	1.06079
25	0.7656	2.5	0.916291	0.372064	0.340919	0.839589	1.09541
26	0.7969	9.2	2.219203	0.466282	1.034775	4.924864	2.25518
27	0.8281	4.6	1.526056	0.565792	0.863431	2.328848	1.45892
28	0.8594	4.8	1.568616	0.673881	1.05706	2.460556	1.49354
29	0.8906	9.9	2.292535	0.794234	1.820808	5.255716	2.37635
30	0.9219	2.3	0.832909	0.936001	0.779604	0.693738	1.06079
31	0.9531	1.5	0.405465	1.118329	0.453443	0.164402	0.92231
32	0.9844	15.7	2.753661	1.425632	3.925705	7.582647	3.38033
			51.6041	-18.1867	-33.7182	108.5783	

$$\sum x(I)_i = 51.6041; \sum y_i = -18.1867; \sum x(I)_i y_i = -33.7182; \sum (x(I)_i)^2 = 108.5783$$

$$\bar{x}(I) = 1.61483; \bar{y} = -0.69102$$

From (34),

$$\hat{\beta} = \frac{32(-33.7182) - (51.6041)(-18.1867)}{32(108.5783) - (51.6041)^2} = -0.1731$$

From (26),

$$\hat{\alpha} = -0.69102 - (-0.1731)(1.61483) = -0.41149$$

$$\hat{\alpha}^* = \text{Exp}(-0.41149) = 0.66266$$

Therefore, the fitted regression model or the trend from (35) is

$$y_i^* = 0.66266 - 0.1731x(I)_i \quad (36)$$

For the mean time to failure (MTTF) from (5), we have

$$\hat{\mu} = (0.66266)\Gamma\left(1 - \frac{1}{0.1731}\right) = 0.66266(6!) = 477.12 \text{hrs} = 20 \text{days} \quad (37)$$

For Reliability function from (7), we have

$$R(t) = \text{Exp}(-0.66266(I)^{-0.1731}) = 3.609E-71 \quad (38)$$

For the probability distribution function of the electric power battery from (4), we have

$$f(t) = -0.1731(0.66266)^{0.1731}(I)^{-1.1731} \exp\{-((I)/0.66266)^{-0.1731}\} \quad (39)$$

Since $\hat{\beta} < 1$, the failure rate is decreasing with time, this assertion agrees with McAtee [18], Tobias and Trindade [19], Ruggeri and Siva [20] and Amuji *et al.* [21].

5. Conclusion

We have designed and modeled an electric power battery. We developed the electric power battery model; estimated its parameters and fitted the model. We also determined the reliability of the developed electric power

battery and fitted a probability distribution to the electric power battery model. We developed a linear regression model and estimated their parameter which forms the parameters of the electric power battery model. We determined that the mean time it will take the developed electric power battery to fail or discharge its current was 477.12 hours, that is, approximately 20days. We found that the reliability of the power battery was 3.609×10^{-71} . We determined the output for each loop, this was presented in Table 2 by the Y* column. We can also use

the fitted regression model from equation (36) to forecast the performance and future output of the power battery. Since the system has a decreasing failure rate, it implies that it can retain current for a relatively long time.

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