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Magnetic Dipole and Quadrupole Interaction Fields of Neutron Star

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Abstract: Neutron stars (NSs) are rapidly rotating entities, spinning at approximately 10^4 Hz, and possess extremely strong magnetic fields, ranging from 10^{13} to 10^{14} Gauss. These compact objects, characterized by a radius of about 10 kilometers and a density of 10^{13-14}, gcm^{-3} are formed as a result of supernova explosions that mark the end of the life cycles of massive stars. Observations of electromagnetic emissions associated with curvature radiation from well-known pulsars, such as the Crab and Vela pulsars, provide compelling evidence that the magnetic field configuration near the surfaces of these neutron stars deviates significantly from the traditionally anticipated pure dipole structure. Researchers now propose that the inclusion of non-dipolar components in the magnetic field may address this longstanding discrepancy. Furthermore, the arrangement of magnetic field lines plays a crucial role in determining the characteristics and geometry of accretion discs surrounding neutron stars in binary systems. This study has focused on elucidating the geometry of the combined dipole and quadrupole magnetic field lines. In idealized scenarios, the magnetic field lines in proximity to these compact objects are typically closed; however, they may become open at greater distances due to interactions with external magnetic fields or the stress energy generated by other sources, including the accretion discs.

Keywords: Pulsar, Neutron Star, Magnetic Field, Dipole, Quadrupole, Supernova, Magnetar

1. Introduction

Neutron stars (NSs) are one of the possible ends for stars. They result from massive stars that have mass greater than 4 to 8 times that of our Sun. After these stars have finished burning their nuclear fuel, they undergo a supernova explosion [2, 18]. This explosion blows off the outer layers of a star into a beautiful supernova remnant. The central region of the star collapses under gravity. It collapses so much that protons and electrons combine to form neutrons according to the reaction

$$e^- + p \rightarrow n + \nu_e$$

The pressure of cold degenerate neutrons supports NS, almost all electrons and protons having been converted into neutrons through the reaction of the neutrinos escaping from the star. The most important correction at higher densities is due to the inverse β-decay. The condition for the inverse β-decay $(e^- + p \rightarrow n + \nu_e)$ is that the kinetic energy of the electrons is larger than or equal to $1.36Mev$. The β-decay of a neutron $(n \rightarrow e^- + p + \nu_e)$ is blocked when the density is so large that all the electron levels in the Fermi sea are filled up to the energy of the emitted electron. Hence the name "Neutron Star" [25]. An NS is about 20 km in diameter and has a mass of about 1.4 solar mass. This means that an NS is so dense that on Earth, one teaspoonful would weigh a billion tons!. Because of its small size and high density, an NS possesses a surface gravitational field about 2×10^{11} times that of Earth [18, 25]. NSs can also have magnetic fields a million times stronger than the strongest magnetic fields produced on Earth. NSs may appear in supernova remnants, as isolated objects, or in binary systems.

When a NS is in a binary system, astronomers are able to measure its mass [18, 28]. If a neutron star is rotating rapidly, as most young NSs are, the strong magnetic fields combined with rapid rotation create an awesome generator that can produce electric potential differences of quadrillions of volts [20, 25]. Such voltages, which are 30 million times greater than those of lightning bolts [27], create deadly blizzards of high energy particles [6]. The neutron star currently "nearest to Earth" is a radio pulsar called J0108-1431. It is within about 100 parsecs away from our planet.

Our universe consists of elementary particles (electron, proton, neutron, positron, neutrino, and photon, etc.) and massive bodies (stars, galaxies, and so on). Stars are formed in molecular clouds in the interstellar medium, which consist mostly of molecular hydrogen (primordial elements made a few minutes after the beginning of the universe) and dust. The dust originates from the cool surfaces of supergiants, massive stars in a late stage of stellar evolution.This dust is an irregularly shaped grains of carbon or silicate measuring a fraction of a micron across which is found between the stars [1]. The night sky contains myriads of stars including those in the Milky Way, which is a side view of our Galaxy looking along the plane of the disc. Our Galaxy includes about 10^{11} stars. Beyond our galaxy are billions of other Galaxies. The nearest star is Proxima Centauri it is about 4 light years away and the nearest large Galaxy is Andromeda is about 2 million light year away. Our Galactic disc has a diameter of about 100,000 light years [24].

Basic factors in the formation of stellar objects are gravity, dust, gas pressure, rotation, magnetic fields, winds and radiation from nearby young stars and radiative shock waves. Building a comprehensive physical picture of stellar structure and evolution is one of the greatest triumphs of 20th century Astrophysics. However, many important aspects of the life cycle of stars are still not completely understood [26]. They include: the origin and evolution of stellar magnetic fields, including the large-scale fields, the sun-spot cycle, the evolution of the stellar rotation, the origin and character of differential rotation, high-energy coronal activity, mass loss in massive stars and various aspects of binary evolution [2]. Even more critical questions remain open regarding the end of the life of stars and their afterlife as

Compact objects [4, 22]. These include the mechanisms of core-collapse supernovae (SNe) and gamma-ray bursts (GRBs), the origin of magnetars,the workings of radio pulsar and pulsar-wind nebulae, some aspects of accretion disks and jets in binary systems [9]. All these questions are at the forefront of modern Astrophysics. Importantly, many of them unavoidably involve magnetic fields interacting with the plasma. Therefore, they belong to the realm of Plasma Astrophysics. A common theme in plasma astrophysics is the life cycle of magnetic fields: How are they produced (dynamo)? How do they interact with the plasma? Magnetohydrodynamic (MHD) instabilities such as Magnetic resonance imaging (MRI)? And how are they destroyed (reconnection) [14]?

In this paper the complex magnetic fields of a compact star called neutron star has been considered. Observations and theory suggest that complex multipolar magnetic fields prevail near the surface of neutron stars and play an important role in the physics of rotation powered pulsars [2, 24]. This work focus on driving field line equation for magnetic dipole-quadrupole combination of neutron star, modeling (simulating) the geometry of this interaction field lines in 2-dimension using matlab and finding (showing) the neutral point for this interaction field lines graphically.

2. Theoretical Background

2.1. Magnetic Field Generation of Neutron Star

Magnetized neutron stars serve as the foundational elements for various classes of compact objects, including pulsars, magnetars, isolated neutron stars, and X-ray binaries, among others. [19]. The origin of strong magnetic fields in neutron stars is still a matter of controversy [9, 11]. The present understandings are:

1. The configuration of the magnetic field in the vicinity of neutron stars represents a fundamental inquiry within the realm of pulsar magnetospheric physics. [19]. The standard theory simply related to the magnetic fields of the progenitor main sequence stars frozen during collapse or flux conservation [7]. This model does not deliver the mechanism for the generation of magnetic multipoles. It can not generate field strength of magnitude about 10^{16} G often observed in GRB activities such very strong magnetic fields are required force for supernova bounce since the energy of the shock blast normally gets dissipated half way through the infalling iron core.
2. Naturally if they are generated through a dynamo mechanism driven by turbulent motions. This opposes by several theorems such as Cowley's theorem.
3. The generation of neutron star magnetic fields by thermomagnetic effects. This is a model which requires along time $(\sim 10^3 years)$ to generate a surface magnetic field of $(\sim\ 10^{12}G)$ provided that there is an original field of $(\sim\ 10^8G)$ which could come from flux conservation.This is why too slow and too weak to describe such violent processes as GRBs, etc.
4. By the spinning of separated charges. This model is able to generate field strengths up to $\sim\ 10^{18}$ G enough to meet all requirements of current experimental findings. Most importantly it very clearly indicates that NS surface fields have infinite multipoles. The powerful magnetic fields, when coupled with swift rotational motion, form an extraordinary generator capable of generating electric potential differences reaching quadrillions of volts. Rapidly rotating, distorted neutron stars represent significant prospects for

the generation of continuous gravitational waves (CWs) [21].

2.2. Vector Potential Expansion

Let us consider a charge Q uniformly distributed over the surface of spherical neutron star of radius R_{*}. Assume that a neutron star is spinning with a frequency of Ω about its diameter [12, 13]. To calculate (drive equations) for magnetic multipole fields generated by the spinning charge at the external surface of the neutron star [5, 17]. The vector potential can be written in general form as

$$\vec{A}(\vec{r}) = \int \frac{\vec{J}(r')}{\mid \vec{r} - \vec{r'} \mid} dv' \tag{1}$$

But it is known that

$$\vec{J}(\vec{r'}) = - \mid \sigma \mid V \delta(r' - R_{*}) \hat{e_{\varphi}} \tag{2}$$

Where

$$\sigma = \frac{\mid Q \mid}{4\pi R^2} \ and \ V = \Omega \times R_{*} \tag{3}$$

Therefore

$$\vec{J}(\vec{r'}) = -\frac{\mid Q \mid}{4\pi R_{*}^{2}} [\Omega \times R_{*}] \delta(r' - R_{*}) \hat{e_{\varphi}}$$

$$= -\frac{\mid Q \mid}{4\pi R_{*}} \Omega sin\theta' \delta(r' - R_{*}) \hat{e_{\varphi}}$$

considering that $\vec{J}(r') = \vec{J}(r')_{\varphi'} \hat{e_{\varphi}}$ where

$$\vec{J}(\vec{r'})_{\varphi'} = -\frac{\mid Q \mid}{4\pi R_{*}} \Omega sin\theta' \delta(r' - R_{*}) \tag{4}$$

substituting this in to the expression of $\vec{A}$ we find that

$$\vec{A}(\vec{r}) = \hat{e_{\varphi}} \int \frac{\vec{J}(\vec{r'})\varphi'}{\mid \vec{r} - \vec{r'} \mid} dv' \tag{5}$$

Where R_{*} is radius of the star.The current density $\vec{J}$ has only a component in the φ direction. The vectorial current density can be written as

$$\vec{J} = -\vec{J}_{\varphi} \sin \varphi' \hat{i} + \vec{J}_{\varphi} \cos \varphi' \hat{j} \tag{6}$$

Since the integration of equation (5) is symetric about $\varphi = 0$ the x component of the current does not contribute. This leaves only the y-component which is $\vec{A}_{\varphi}$. The solution of the laplace equation was decomposed in to a product of factors for the three variables r,θ and φ. It is convenient to combine the angular factors and construct orthonormal functions over the unit sphere. The functions $Q_m(\varphi) = exp(im\varphi)$ form a complete set of orthogonal functions in the index m on the interval $0 \leq \varphi \leq 2\pi$. The functions $P_l(cos\theta)$ form a similar set in the index l for each m value in the interval $-1 \leq cos\theta \leq 1$. Therefore their product $Q_m P_l^m$ will form a complete orthogonal set on the surface of a unit sphere in the two indices l, m. From the normalization condition it is clear that the normalization function, denoted by $Y_{lm}(\theta, \varphi)$, is

$$Y_{lm}(\theta, \varphi) = \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} P_l^m(cos\theta) exp(im\varphi) \tag{7}$$

Is spherical harmonics, spherical harmonics for $l = 1$ dipole,$l = 2$ quadrupole and $l = 3$ octupole are given as

$$Y_{10} = \sqrt{\frac{3}{4\pi}} cos\theta,\ Y_{11} = -\sqrt{\frac{3}{8\pi}}\ sin\theta\ exp(i\varphi)$$

$$Y_{20} = \frac{1}{2}\sqrt{\frac{5}{4\pi}}\ (3cos^2\theta - 1),\ Y_{21} = -\sqrt{\frac{15}{8\pi}}\ sin\theta\ cos\theta\ exp(i\varphi)$$

$$Y_{30} = \frac{1}{2}\sqrt{\frac{7}{4\pi}} sin(5\ cos^3\theta - 3\ cos\theta)$$

$$Y_{31} = -\frac{1}{4}\sqrt{\frac{21}{4\pi}}\ sin\theta(5cos^2\theta - 1)\ exp(i\varphi) \tag{8}$$

Where

$$Y_{l,-m}(\theta, \varphi) = (-1)^m Y^*_{lm}(\theta, \varphi) \tag{9}$$

Which gives a way of generating the unwritten negative m harmonics above. Also, remember, for a given l there are $2l + 1$ different spherical harmonics: $m = l, l-1, ..., 0, ..., -l+1, -l$. Also note, that a spherical harmonic, with m = 0 is just the normalization constant, multiplied by the Legendre polynomial: $Y_{l0}(\theta, \varphi) = \sqrt{\frac{2l+1}{4\pi}} P_l(cos\theta)$. The normalization and orthogonality conditions are:

$$\int_0^{2\pi} d\varphi \int_0^{\pi} sin\theta d\theta Y^*_{l'm'}(\theta, \varphi) Y_{lm}(\theta, \varphi) = \delta_{l'l}\delta_{m'm} \tag{10}$$

the completeness relation, corresponding to $\sum_{n=1}^{\infty} U^*_n(\eta) U_n(\eta) = \delta(\eta' - \eta)$ is

$$\sum_{l=0}^{\infty} \sum_{m=-l}^{l} Y^*_{lm}(\theta', \varphi') Y_{lm}(\theta, \varphi) = \delta(\varphi - \varphi')\delta(cos\theta - cos\theta') \tag{11}$$

considering two coordinate vectors $\vec{x}$ and $\vec{x'}$, with spherical coordinates (r, θ, φ) and (r', θ', φ') respectively and angle ζ between them. From the addition theorem [10] for spherical harmonics as

$$P_l(cos\zeta) = \left(\frac{4\pi}{2l+1}\right) \sum_{m=-l}^{l} Y^*_{lm}(\theta', \varphi') Y_{lm}(\theta, \varphi) \tag{12}$$

The expansion for $\frac{1}{|\vec{r} - \vec{r'}|}$ is

$$\frac{1}{|\vec{r} - \vec{r'}|} = \sum_{l=0}^{\infty} \left(\frac{r'^l}{r^{l+1}}\right) P_l(cos\zeta) \tag{13}$$

which is the potential at r due to a unit charge at r'. By substituting equation (12) in to (13) one can find the explicit form as

$$\frac{1}{|\vec{r} - \vec{r'}|} = 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^{m=l} \left(\frac{1}{2l+1}\right) \left(\frac{r^l_<}{r^{l+1}_>}\right) Y^*_{lm}(\theta', \varphi') Y_{lm}(\theta, \varphi) \tag{14}$$

Inserting equation (14) in (1) yields

$$\vec{A}^{(l)}_\varphi = \hat{e}_\varphi 4\pi \sum_{l=0}^{\infty} \left(\frac{1}{2l+1}\right) \left(\frac{r^l_<}{r^{l+1}_>}\right) \tag{15}$$

$$* \sum_{m=-l}^{m=l} Y_{lm}(\theta, \varphi) \int_{r'} \int_{\theta'} \int_{\varphi'} \vec{J}(r')_{\varphi'} Y^*_{lm}(\theta', \phi') r'^2 sin\theta' dr' d\phi' d\theta'$$

The presence of $exp(i\varphi')$ means that only $m = +1$ will contribute to the sum.

$$\vec{A}^{(l)}_\varphi = \hat{e}_\varphi 4\pi \sum_{l=0}^{\infty} \left(\frac{1}{2l+1}\right) \left(\frac{r^l_<}{r^{l+1}_>}\right) \tag{16}$$

$$*Y_{l1}(\theta,\varphi)\int_{r'}\int_{\theta'}\int_{\varphi'}\vec{J}(r')_{\varphi'}Y_{l1}^{*}(\theta',\varphi')r'^{2}sin\theta'dr'cos\varphi'd\varphi'd\theta'$$

using equation (4) instead of $\vec{J}(r')_{\varphi'} = Jcos\varphi'$ follows

$$\vec{A}_{\varphi}^{(l)} = \hat{e}_{\varphi}\left(-\frac{|Q|\omega}{4\pi R}\right)4\pi\sum_{l=0}^{\infty}\left(\frac{1}{2l+1}\right)\left(\frac{r_{<}^{l}}{r_{>}^{l+1}}\right) \tag{17}$$

$$*Y_{l1}(\theta,\varphi)\int_{r'}\int_{\theta'}\int_{\varphi'}Y_{l1}^{*}(\theta',\varphi')r'^{2}sin^{2}\theta'dr'cos\varphi'd\varphi'd\theta'\delta(r'-R)$$

Here, the calculation is interested to the exterior point, so $r_{<} = R, r' \to R$ and $r_{>} = r$ refers to the inner and outer region of the neutron star respectively. as $r' \to R \quad \int dr'r'^{2}\delta(r'-R) = R^{2}$

$$\vec{A}_{\varphi}^{(l)} = \hat{e}_{\varphi}\left(-\frac{|Q|\Omega}{4\pi R}\right)4\pi R^{2}\sum_{l=0}^{\infty}\left(\frac{1}{2l+1}\right)\left(\frac{R^{l}}{r^{l+1}}\right) \tag{18}$$

$$*Y_{l1}\int_{\theta'}\int_{\varphi'}Y_{l1}^{*}\sin^{2}\theta'cos\varphi'd\varphi'd\theta'$$

Different poles are derived by assigning different values for l. By picking $l = 1, l = 2, l = 3$ for dipole,quadrupole and octupole respectively.

$$\vec{A}_{\varphi}^{(1)} = \hat{e}_{\varphi}\left(-\frac{|Q|\Omega}{12\pi R}\right)\left(\frac{4\pi R^{3}}{r^{2}}\right)Y_{11}\int_{\theta'}\int_{\varphi'}Y_{11}^{*}\sin^{2}\theta'cos\varphi'd\varphi'd\theta' \tag{19}$$

Now, inserting the value of Y_{11} from above

$$\vec{A}_{\varphi}^{(1)} = \hat{e}_{\varphi}\left(-\frac{|Q|\Omega}{8\pi}\right)\left(\frac{R^{2}}{r^{2}}\right)sin\theta\int_{0}^{\pi}sin^{3}\theta'd\theta'\int_{0}^{2\pi}cos\varphi'exp(i\varphi')d\varphi' \tag{20}$$

$$\vec{A}_{\varphi}^{(1)} = \hat{e}_{\varphi}\left(-\frac{|Q|\Omega}{3}\right)\left(\frac{R^{2}}{r^{2}}\right)sin\theta \tag{21}$$

Applying the same method and inserting the value of Y_{21} then

$$\vec{A}_{\varphi}^{(2)} = \hat{e}_{\varphi}\left(-\frac{|Q|\Omega}{4\pi R}\right)4\pi R^{2}\left(\frac{1}{5}\right)\left(\frac{R^{2}}{r^{3}}\right)Y_{21} \quad \times\int_{\theta'}\int_{\varphi'}Y_{21}^{*}\sin^{2}\theta'cos\phi'd\varphi'd\theta' \tag{22}$$

$$\vec{A}_{\varphi}^{(2)} = \frac{3|Q|\Omega R^{3}}{4r^{3}}sin\theta cos\theta \tag{23}$$

$$\vec{A}_{\varphi}^{(3)} = \hat{e}_{\varphi}((|Q|\Omega)\left(\frac{R^{4}}{7r^{4}}\right)Y_{31}\int_{\theta'}\int_{\varphi'}Y_{31}^{*}\sin^{2}\theta'cos\varphi'd\varphi'd\theta' \tag{24}$$

Inserting the value of Y_{31}

$$\vec{A}_{\varphi}^{(3)} = \hat{e}_{\varphi}(|Q|\Omega)\left(\frac{7R^{4}}{20r^{4}}\right)sin\theta(5cos^{2}\theta-1) \tag{25}$$

3. Magnetic Multipolar Fields of Neutron Star

Considering system of spherical coordinates (r,θ,φ) where r is measured from stellar center [8, 14], θ is the polar angle (in radians) measured from the z axis and φ is azimuthal angle (in radians) measured from an arbitrary origin. The z-axis is directed along the dipolar momentum of the star $\vec{\mu}$. It is known that magnetic field is the curl of vector potential i.e,

$$\vec{B} = \nabla\times\vec{A} = \frac{1}{rsin\theta}\begin{pmatrix}\hat{r} & r_{\hat{\theta}} & rsin\theta_{\hat{\varphi}}\\ \partial_{r} & \partial_{\theta} & \partial_{\varphi}\\ A_{r} & rA_{\theta} & rsin\theta A_{\varphi}\end{pmatrix}$$

$$\vec{B} = \frac{1}{rsin\theta}\left[\frac{\partial}{\partial\theta}(sin\theta A_{\varphi}) - \frac{\partial}{\partial\varphi}(A_{\theta})\right]\hat{r} + \frac{1}{r}\left[\frac{1}{sin\theta}\frac{\partial}{\partial\varphi}(A_r) - \frac{\partial}{\partial r}(rA_{\varphi})\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_{\theta}) - \frac{\partial}{\partial\theta}(A_r)\right]\hat{\varphi} \tag{26}$$

In general we can write multipolar fields as

$$\vec{B}^{lm}(r,\theta) = \nabla \times \vec{A}_{\varphi}^{lm} \tag{27}$$

In our case $\vec{A}$ has only an azimuthal component. Therefore, only $\vec{B}_r^{lm}(r,\theta)$ and $\vec{B}_{\theta}^{lm}(r,\theta)$ survive, so that our equation will be

$$\vec{B}_r^{lm}(r,\theta) = \frac{\hat{e}_r}{rsin\theta}\frac{\partial}{\partial\theta}\left(\vec{A}_{\phi}^{lm}(r,\theta)sin\theta\right) \tag{28}$$

$$\vec{B}_{\theta}^{lm}(r,\theta) = -\frac{\hat{e}_{\theta}}{r}\frac{\partial}{\partial r}\left(r\vec{A}_{\varphi}^{lm}(r,\theta)\right) \tag{29}$$

As usual using $l = 1$, $l = 2$ and $l = 3$ for the dipole,quadrupole and octupole field respectively. If m = 0 and l is arbitrary, then one obtains axially symmetric and uniform multipolar components.

3.1. Magnetic Dipole Filed of Neutron Star

Based on the above relation between magnetic field and vector potential to find the dipole field components as shown below

$$B^{(1)}(r,\theta) = \nabla \times \vec{A}_{\varphi}^{(1)} \tag{30}$$

$$\begin{aligned} B^{(1)}(r,\theta) &= \frac{\hat{e_r}}{rsin\theta}\left[\frac{\partial}{\partial\theta}(sin\theta A_{\varphi}^{1})\right] - \frac{\hat{e}_{\theta}}{r}\left[\frac{\partial}{\partial r}(rA_{\varphi}^{1})\right] \\ &= \frac{\hat{e_r}}{rsin\theta}\frac{|Q|\Omega}{3}\left(\frac{R}{r}\right)^2\left[\frac{\partial}{\partial\theta}(sin^2\theta)\right] + \frac{\hat{e_{\theta}}}{r}\frac{|Q|\Omega}{3}sin\theta R^2\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\right)\right] \\ &= \frac{\hat{e_r}}{rsin\theta}\frac{|Q|\Omega}{3}\left(\frac{R}{r}\right)^2 2sin\theta cos\theta - \frac{\hat{e_{\theta}}}{r}\frac{-|Q|\Omega}{3}sin\theta\left(\frac{R}{r}\right)^2 \\ &= \frac{Q\Omega R^2}{3r^3}cos\theta\hat{e_r} + \frac{Q\Omega R^2}{3r^3}sin\theta\hat{e_{\theta}} \end{aligned} \tag{31}$$

The radial and polar components of the dipolar field is written as

$$B_r^{(1)} = \frac{2\mu_d}{3r^3}\,cos\theta\,\hat{e_r}\ \ ,and \tag{32}$$

$$B_{\theta}^{(1)} = \frac{\mu_d}{3r^3}\,sin\theta\,\hat{e_{\theta}} \tag{33}$$

Where $\vec{\mu_d} = \mu_d\hat{\mu} = |Q|\Omega R_*^2$ is the dipole moment. A neutron star total magnetic field is given by: $B_* = B_d + B_q + B_o + ...$ Hence the dipole moment μ_d can be written as

$$\mu_d = B_{*(d)}R_*^3 = |Q|\Omega R_*^2 \tag{34}$$

In general form magnetic multipole moment is

$$\mu^{lm} = B_*^{lm}R_*^{l+2}$$

3.2. Magnetic Quadrupolar Field of Neutron Star

Under this topic magnetic fields that are in "potential state" have been used. This terminology, which is common in the solar/stellar physics literature, refers to a magnetic field in which the current density $\vec{J} = 0$ everywhere within the stellar magnetosphere, and therefore the field $\vec{B}$ can be written in terms of the gradient of a magnetostatic scalar potential [13, 28] so that the intrinsic magnetic field of the star can be written as $\mathbf{B} = -\nabla\varphi$ [3, 14], where the scalar potential of the magnetic field is $\varphi(r) = \sum ma/ \mid \vec{r} - \vec{r}_a \mid$, and m_a is an analogy of the magnetic "charge", $\vec{r}$ and $\vec{r}_a$ are the positions of the observer and the magnetic "charges" respectively. The scalar potential can be represented as a multipole expansion in powers of $1/r$. In the near zone limit $(kr \ll 1)$, the magnetic field vector $B^{lm}(r,\theta,\varphi)$, associated with a given magnetic multipole (lm) [29], can be expressed in terms of spherical harmonics $Y_{lm}(\theta,\varphi)$ (ex.see Jackson 1975).

$$B^{lm}(r,\theta,\varphi) = \nabla\left(\frac{Y_{lm}(\theta,\varphi)}{r^{l+1}}\right) \tag{35}$$

Spherical harmonics are written in terms of the associated Legendre functions as

$$Y_{lm}(\theta,\varphi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(cos\theta)e^{im\varphi}$$

where $P_l^m(x)$, is the associated Legendre function,is defined as

$$P_l^m(x) = \frac{(-1)^m}{2^l l!}(1-x^2)^{m/2}\frac{d^{l+m}}{dx^{l+m}}(x^2-1)$$

In a spherical geometry, the components of an individual multipolar magnetic field vectors are written as

$$B_r^{lm}(r,\theta,\varphi) = -4\pi\,\frac{l+1}{2l+1}\,\frac{q_{lm}}{r^{l+2}}\,Y_{lm}(\theta,\varphi) \tag{36}$$

$$B_\theta^{lm}(r,\theta,\varphi) = \frac{4\pi}{2l+1}\,\frac{q_{lm}}{r^{l+2}}\,Y_{lm}^{(1,0)}(\theta,\varphi) \tag{37}$$

$$B_\varphi^{lm}(r,\theta,\varphi) = \frac{4\pi}{2l+1}\,\frac{q_{lm}}{r^{l+2}}\,im\;sin\theta\,Y_{lm}(\theta,\varphi) \tag{38}$$

where 1 is for m and 0 is for φ it is polar component.

$$q_{lm} \equiv \int d^3r' r'^l\;Y_{lm}^*(\theta,\varphi)\rho(\vec{r'}) \tag{39}$$

$$= \int_0^\infty r'^2 dr' \int_0^{2\pi} d\varphi' \int_0^\pi sin\theta' d\theta' \rho(r',\theta',\varphi')r'^l Y_{lm}^*(\theta',\varphi') \tag{40}$$

q_{lm} is known as multipole moment of the charge distribution. This moment satisfy the identity $q_{lm} = (-1)^m\;q_{l,-m}^*$.The first few moments are as follows

$$q_{00} = \frac{q}{\sqrt{4\pi}}, \quad q_{10} = \sqrt{\frac{3}{4\pi}}P_z$$

where q is precisely the total charge of the system and P is the electric dipole moment, when expressed in terms of spherical polar coordinates, $x = rsin\theta cos\varphi$, $y = rsin\theta sin\varphi$ and $z = rcos\theta$ using these

$$q_{11} = -\sqrt{\frac{3}{8\pi}}\int d^3r'\;\rho(\vec{r'})r'sin\theta e^{-i\varphi} = -\sqrt{\frac{3}{8\pi}}\int d^3r'\rho(\vec{r'})(x-iy) = \sqrt{\frac{3}{8\pi}}(p_x - ip_y) \tag{41}$$

$$q_{20} = \frac{1}{2}\sqrt{\frac{5}{4\pi}}Q_{33}\;\;,\;\; q_{21} = -\frac{1}{3}\sqrt{\frac{15}{8\pi}}(Q_{13} - iQ_{23})$$

$$q_{22} = \frac{1}{12}\sqrt{\frac{15}{2\pi}}(Q_{11} - 2iQ_{12} - Q_{22}) \tag{42}$$

where, Q_{ij} is the traceless quadrupole moment tensor,defined by

$$Q_{ij} \equiv \int d^3r'\rho(\vec{r'})(3r'_i r'_j - r'^2\delta_{ij}) \tag{43}$$

i and j stand for cartesian components x, y and z or 1, 2, 3

$$Tr(Q) = \int d^3r'\rho(\vec{r'})(3r'^2 - 3r'^2) = 0 \tag{44}$$

Since, $\sum_i^{1,2,3} r'^2\delta_{ii} = r'^2(\delta_{11}+\delta_{22}+\delta_{33}) = 3r'^2$ and $\sum_i r'_i r'_i = 3r'^2$. The quadrupole moment tensor is symmetric i.e, $Q_{ij} = Q_{ji}$ this reduces the number of possible independent components to six. As its name suggests it has zero trace so that $Q_{33} = -Q_{11} - Q_{22}$ and only two of the diagonal components are independent. Thus the tensor can have at most five independent components [23].

$$Q_{ij} = \begin{pmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{32} & Q_{33} \end{pmatrix}$$

If$m = 0$ and l is arbitrary, then one obtains axially symmetric and uniform multipolar components [14, 15, 29]. The magnetic moment associated with the strength of the magnetic multipole component (lm) at the surface of the star is defined as $\mu^{lm} = B_s^{lm} R_s^{l+2}$ [11**?**]. i.e $\mu^{lm} = B_{*0} R_*^{l+2}$, where B_{*0} is the reference magnetic field on the surface of the star and R_* is the radius of the star. Now applying the above expressions from equations (36) and (37) then obtain.

$$B_r = 4\pi\,\frac{2+1}{4+1}\,\frac{q_{20}}{r^{l+2}}\,Y_{20}(\theta,\varphi) \tag{45}$$

$$= 4\pi\,\frac{3}{10}\sqrt{\frac{5}{4\pi}}\frac{Q_{33}}{r^4}\sqrt{\frac{5}{4\pi}}(\frac{3}{2}cos^2\theta - \frac{1}{2})$$

$$= \frac{3}{2}\frac{Q_{33}}{r^4}(\frac{3}{2}cos^2\theta - \frac{1}{2})$$

$$B_r^{(2)} = \frac{3}{4}\frac{Q_{33}}{r^4}(3cos^2\theta - 1) = \frac{3}{4}\frac{\mu_q}{r^4}(3cos^2\theta - 1) \tag{46}$$

where $Q_{33} = Q$ is the quadrupole moment and refer to the axis of symmetry as the "direction" of the quadrupole moment (μ_q). Considering axial multipoles where the space-fixed z-axis of the star (the stellar rotation axis) as the symmetry axis of the multipole being considered then can get the known general multipolar magnetic field equation [29]. Then spherical field components of an axial multipole $(B_\varphi = 0)$ of order l are given by

$$B_r^l = B_*^{l,pole}\left(\frac{R_*}{r}\right)^{l+2} P_l(cos\theta) \tag{47}$$

$$B_\theta^l = \frac{B_*^{l,pole}}{l+1}\left(\frac{R_*}{r}\right)^{l+2} P_{l1}(cos\theta) \tag{48}$$

Where R_* is the star radius,r is a point external to the star, $P_{l1}(cos\theta)$ and $P_l(cos\theta)$ are the m = 1 l^{th} associated Legendre function and the l^{th} Legendre polynomial respectively. Now by applying quadrupole field equations, calculate the quadrupolar magnetic field components of neutron star. In the same way solving for $\vec{B}_\theta$ yields

$$B_\theta^{(2)} = \frac{3Q_{33}}{2r^4}sin\theta cos\theta = \frac{3\mu_q}{2r^4}sin\theta cos\theta \tag{49}$$

where

$$\mu_q = |Q|\Omega R_*^3 \tag{50}$$

3.3. Magnetic Octupolar Field

The octupolar field components are also found from previously determined vector potential of $l = 3$ in chapter one. The result obtained is as follows:

$$B_r^{(3)} \approx \left(\frac{7\Upsilon}{5r^5}\right) cos\theta(5cos^2\theta - 3)\hat{e_r} \tag{51}$$

$$B_\theta^{(3)} \approx \left(\frac{21\Upsilon}{20r^5}\right) sin\theta(5cos^2\theta - 1)\hat{e_\theta} \tag{52}$$

Where $\Upsilon = \Upsilon\hat{\Upsilon} = |Q|\Omega R_*^4$ is the octupole moment. Following the same fashion the octupole magnetic field components are shown as follows. Hence,from the previous relation

$$\Upsilon = B_{*o} R_*^5 \hat{\Upsilon} \tag{53}$$

3.4. Magnetic Dipole-Quadrupole Field Interaction

Considering that magnetic dipole and quadrupole moments are deflected from the rotation axis by same angle then magnetic dipole and quadrupole combination (interaction) of neutron star as:

$$B_r^{d+q} = B_r^d + B_r^q = \frac{2\mu_d}{3r^3}\,cos\theta\,\hat{e_r} + \frac{3}{4}\frac{\mu_q}{r^4}(3cos^2\theta - 1)\hat{e_r} \tag{54}$$

$$B_\theta^{d+q} = B_\theta^d + B_\theta^q = \frac{\mu_d}{3r^3}\,sin\theta\;\hat{e_\theta} + \frac{3\mu_q}{2r^4}sin\theta cos\theta\,\hat{e_\theta} \tag{55}$$

This interaction become minimum at the point where the first derivative of the equation with respect to r or θ is zero, this happen at:

$$-\frac{3\mu_q}{2r\mu_d} = \frac{cos\theta}{(3cos^2\theta - 1)} \qquad (for\ the\ radial\ component)$$

$$|r| = \frac{3\mu_q}{2\mu_d cos\theta}(3cos^2\theta - 1) \tag{56}$$

For instance consider $\theta = 0$ the value of r at which the interaction is minimum will be $\mid r \mid = 3\frac{\mu_q}{\mu_d}$ and so on for other angles. After some calculation for polar component then get

$$-\frac{9\mu_q}{2r\mu_d} = \frac{cos\theta}{(cos^2\theta - sin^2\theta)} \qquad (for\ polar\ angle\ \theta) \tag{57}$$

$$cos\theta = \frac{-9\mu_q}{2r\mu_d}(1 - 2sin^2\theta) \tag{58}$$

Note, that r is measured from the center of the star. Figure 1 shows the field is very strong for dipole plus quadrupole. Both are reducing at the point far from the star. However, the dipolar field is strongest of all very far from the star because the far field potential is dominated by the first non-zero moment $(l = 1)$. According to our assumption on this graph if $r = 1$ it is on the surface of the star.

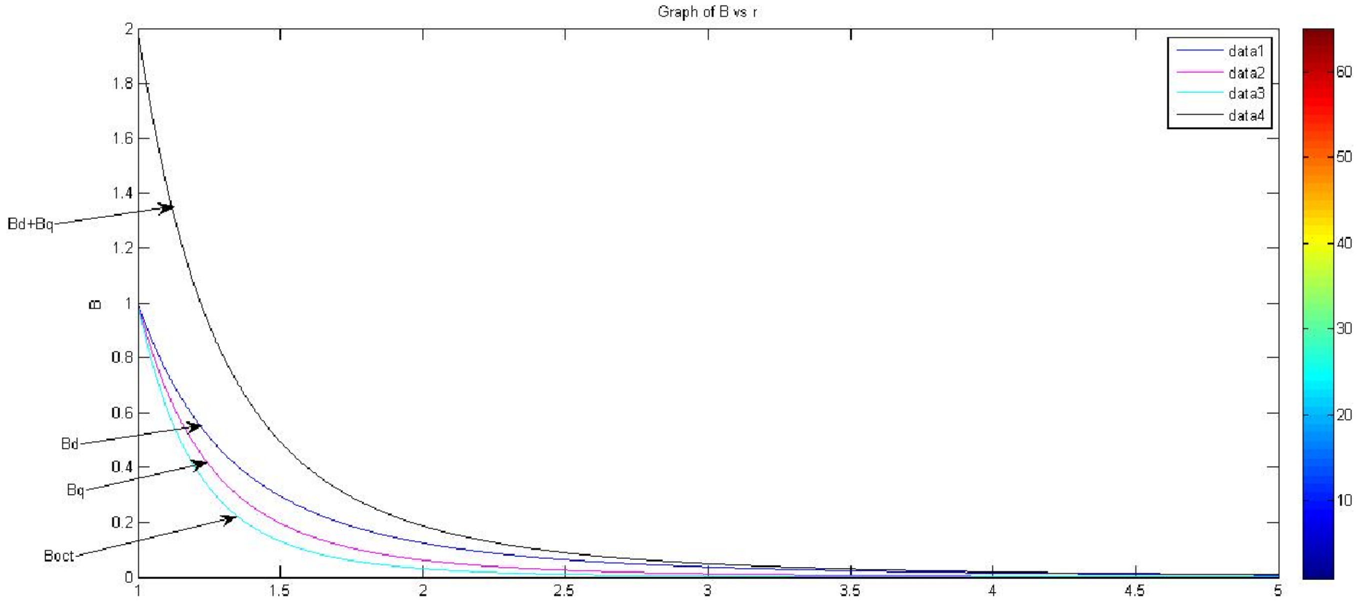

Figure 1. Graphical comparison of neutron star magnetic multipole fields and the dipole-quadrupole interaction field vs r(distance from the center of the star). As seen from the graph octupolar field is fast decaying field as r increases where as the dipole-quadrupole interaction field is strongest at the surface of the star.

3.5. Magnetic Multipole Field Line Equations of Neutron Star

Qualitatively, a field line for any vector field $\vec{V}$ is a curve that is tangential to $\vec{V}$ at every point along the line. The concept of a field line may be given a mathematical description by writing down the equation for the field line. A convenient starting point is the parametric equations for the field line in cartesian coordinates:

$$\frac{dx}{B_x} = \frac{dy}{B_y} = \frac{dz}{B_z} \tag{59}$$

The above parametric equations (may be written in terms of any other orthogonal coordinate system. Accordingly, the magnetic field line equations for different poles and model their geometry can be derived.

4. Magnetic Dipole Field Line

Here by "equation of the field lines" it means an expression of the form r = $r(\theta)$ which describes the path (shape) of the field lines in a spherical coordinate system. An expression for the field lines of an axisymmetric (axial) multipole of arbitrary degree is derived. In this context the order of a magnetic multipole for($l = 1$ and $l = 2$) can be thought of as the number of polarity changes in the surface field between the north and south pole of the star. In this paper the attention given to axial multipoles, which generate planar field lines. For example, an axial dipole ($l = 1$),quadrupole $l = 2$, and octupole ($l = 3$). Throughout our work r and θ are standard spherical polar coordinates, with θ measured from the rotation pole of the star; $\theta = 0$ corresponds to the stellar rotation pole, $\theta = \frac{\pi}{2}$ corresponds to the equatorial plane.The origin of the coordinate system is at the center of the star, and thus the stellar surface corresponds to $r = R_*$. Only azimuthally symmetric multipoles, known as axial multipoles are considered, i.e, $B_\varphi = 0$. The differential equation describing the path of the field lines in spherical coordinate is

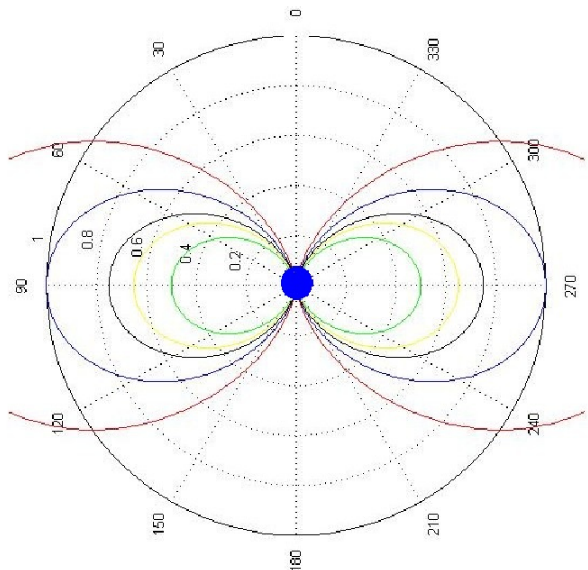

Figure 2. Pure magnetic dipole field line geometry for $K_d > 1$.

$$dr = dr\hat{r} + rd\theta\hat{\theta} + rsin\theta d\varphi\hat{\varphi} \tag{60}$$

In two dimension (2-D) magnetic field can be written as

$$\vec{B} = B_r\hat{r} + B_\theta\hat{\theta} \tag{61}$$

Here $l = 1$ dividing these equation one by the other then get

$$\frac{B_r^{(1)}\hat{r}}{dr\hat{r}} = \frac{B_\theta^{(1)}\hat{\theta}}{rd\theta\hat{\theta}} \Longrightarrow \frac{B_r^{(1)}}{dr} = \frac{B_\theta^{(1)}}{rd\theta} \tag{62}$$

$$\Longrightarrow \frac{dr}{r} = \frac{B_r^{(1)}}{B_\theta^{(1)}}d\theta = K_d$$

Where k_d is a constant related to the field curvature which, upon substituting for B_r and B_θ from above and integrating, yields the well known result.

$$\frac{dr}{rd\theta} = \frac{2cos\theta}{sin\theta}$$

$$\Rightarrow \frac{dr}{r} = 2cot\theta d\theta$$

$$\Rightarrow \frac{dr}{r} = \int 2cot\theta d\theta$$

$$\Rightarrow lnr = 2ln|sin\theta| + lnk_d = ln|sin\theta|^2 + lnk_d$$

$$| r(\theta) |= k_d sin^2\theta \tag{63}$$

This is the general field line equation for magnetic dipole field. Figure 2 implies if a particular closed field line loop of the dipole reaches a maximum radial extent of r_{max} in the stellar equatorial plane, where $\theta = \pi/2$, then the integration constant in the above equation (63) is equal to r_{max} [29]. Thus for a dipole the equation of the particular closed field line will be

$$r(\theta) = r_{max}sin^2\theta \tag{64}$$

Different values of r_{max} correspond to different field lines. There are two lobes in a spam of 360 degrees or in the range $(0, 2\pi)$ for a specific k_d, where k_d is a constant related to the dipolar field curvature. For $r < r_m$ the magnetic energy-density dominates and the field lines are closed. The surfaces where magnetic stress energy balanced with matter stress $(P + \rho v^2 = \frac{B^2}{8\pi})$ i.e, the surface $\beta = 1$ separates the regions of magnetically dominated and matter dominated plasma $\left(\beta = (P + \rho v^2)/\frac{B^2}{8\pi}\right)$.

5. Magnetic Quadrupole Field Line

Here, again applying the same method as dipole field line

$$dr = dr\hat{r} + rd\theta\hat{\theta} + rsin\theta d\varphi\hat{\varphi} \tag{65}$$

As usual in 2D magnetic field can be written as

$$B = B_r\hat{r} + B_\theta\hat{\theta} \tag{66}$$

$$\frac{B_r^{(2)}}{dr} = \frac{B_\theta^{(2)}}{rd\theta} \Longrightarrow \frac{dr}{r} = \frac{B_r^{(2)}}{B_\theta^{(2)}} = k_q \tag{67}$$

Substituting the quadrupolar field $B_r^{(2)}$ and $B_\theta^{(2)}$ from chapter 2 and have

$$\frac{dr}{r} = \frac{\frac{3\mu_q}{4r^4}(3cos^2\theta - 1)}{\frac{3\mu_q}{2r^4}sin\theta cos\theta} \tag{68}$$

$$\Rightarrow \int \frac{dr}{r} = \int \frac{1}{2}\frac{(3cos^2\theta - 1)}{sin\theta cos\theta}$$

$$lnr = \frac{1}{2}\int \frac{(3cos^2\theta - 1)}{sin\theta cos\theta}d\theta = \frac{1}{2}\int \frac{3(1 - sin^2\theta) - 1}{sin\theta cos\theta}d\theta$$
$$= \frac{1}{2}\int \frac{3 - 3sin^2\theta - 1}{sin\theta cos\theta}d\theta = \frac{1}{2}\int \frac{2 - 3sin^2\theta}{sin\theta cos\theta}d\theta$$

$$= \int \frac{1}{sin\theta cos\theta}d\theta - \frac{3}{2}\int \frac{sin^2\theta}{sin\theta cos\theta}d\theta$$
$$= \int \frac{1}{sin\theta cos\theta}d\theta - \frac{3}{2}\int \frac{sin\theta}{cos\theta}d\theta$$
$$= \int \frac{1}{sin\theta cos\theta}d\theta - \frac{3}{2}\int tan\theta d\theta$$
$$= \int csc\theta sec\theta d\theta - \frac{3}{2}(-ln)|cos\theta| + lnk_q$$
$$= ln|tan\theta| + \frac{3}{2}\, ln\, |cos\theta| + lnk_q$$
$$r(\theta) = ln|tan\theta| + ln\, |cos\theta|^{3/2} + lnk_q$$
$$| r(\theta) |= k_q\, tan\theta\, cos^{\frac{3}{2}}\theta \tag{69}$$

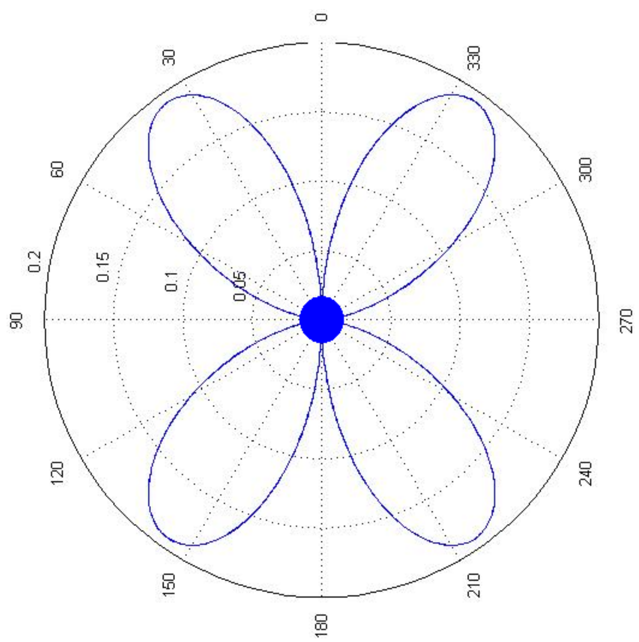

Figure 3. *Pure quadrupole field line geometry for the solution derived above.*

Figure 3 indicates the quadrupole field has four lobes in the range $(0, 2\pi)$ or in a spam of 360 degrees.which have one solution for a single degree in one quadrant for a specific k_q.

6. Results

6.1. Equation of Magnetic Dipole-quadrupole Interaction Field Line and Its Geometry

Summing up our previous dipole and quadrupole filed equations as:

$$B_r^{(d+q)} = B_r^{(1)} + B_r^{(2)} \quad , \quad B_\theta^{(d+q)} = B_\theta^{(1)} + B_\theta^{(2)} \tag{70}$$

Where the indices 1 and 2 indicate the dipole ($l = 1$) and quadrupole ($l = 2$) respectively.

$$B_r^{d+q} = \frac{2\mu_d}{3r^3}cos\theta + \frac{3Q}{4r^4}(3cos^2\theta - 1) \tag{71}$$

$$B_\theta^{d+q} = \frac{\mu_d}{3r^3}sin\theta + \frac{3Q}{2r^4}sin\theta cos\theta \tag{72}$$

$$\frac{dr}{rd\theta} = \frac{B_r}{B_\theta} \tag{73}$$

$$\Longrightarrow \int \frac{dr}{r} = \int \frac{B_r}{B_\theta}d\theta$$

$$\Longrightarrow lnr = \int \frac{B_r}{B_\theta}d\theta \tag{74}$$

Inserting the values of B_r and B_θ from above, and find

$$lnr(\theta) = \int \frac{\frac{2\mu_d}{3r^3}cos\theta + \frac{3\mu_q}{4r^4}(3cos^2\theta - 1)}{\frac{\mu_d}{3r^3}sin\theta + \frac{3\mu_q}{2r^4}sin\theta cos\theta}d\theta \tag{75}$$

$$= \int \frac{\frac{2\mu_d}{3r^3}cos\theta}{\frac{\mu_d}{3r^3}sin\theta + \frac{3\mu_q}{2r^4}sin\theta cos\theta}d\theta + \int \frac{\frac{3\mu_q}{4r^4}(3cos^2\theta - 1)}{\frac{\mu_d}{3r^3}sin\theta + \frac{3\mu_q}{2r^4}sin\theta cos\theta}d\theta$$

the integration yields

$$lnr(\theta) = \frac{9logsin(\theta/2)/cos(\theta/2)}{11} - \frac{log(4+18cos\theta)}{cos\theta + 1}/2$$
$$-\frac{162log(4+18cos(\theta)}{cos(\theta)+1)}/(8-162) - 3log\frac{1/cos(\theta/2)^2}{2} + 4log\frac{sin(\theta/2)}{cos(\theta/2)}/11 + lnk \tag{76}$$

Simplifying this one can arrive at the solution keeping magnetic moments and r constant.

$$| r(\theta) | = k\left[\frac{(tan(\theta/2))^{\frac{13}{11}}}{\left[\left(\frac{4+18cos\theta}{cos\theta+1}\right)^{\frac{162}{8-162}-1/2}\left(\frac{1}{1/2cos\theta+1/2}\right)\right]^{3/2}}\right] \tag{77}$$

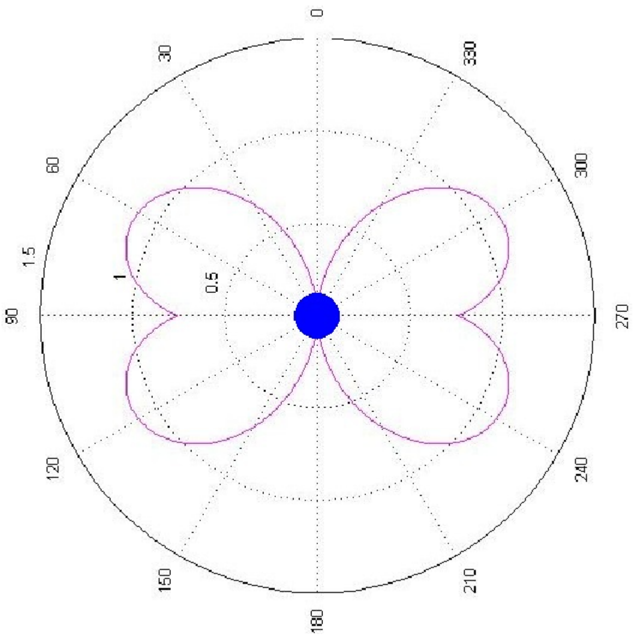

Figure 4. *Dipole-Quadrupole interaction field line geometry for specific k.*

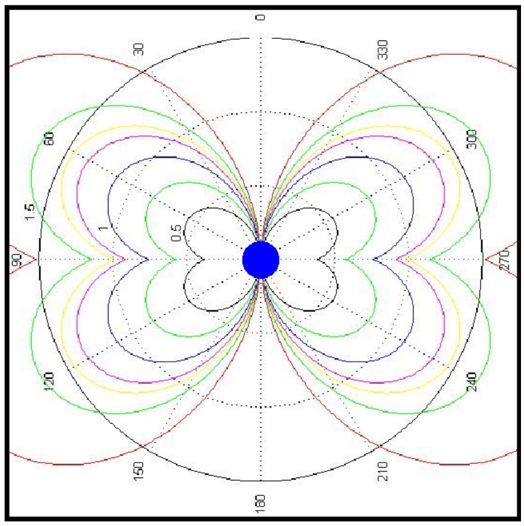

Figure 5. *Dipole-Quadrupole interaction field lines geometry for $k = n$, as number of k increases large space needed to see full line here.*

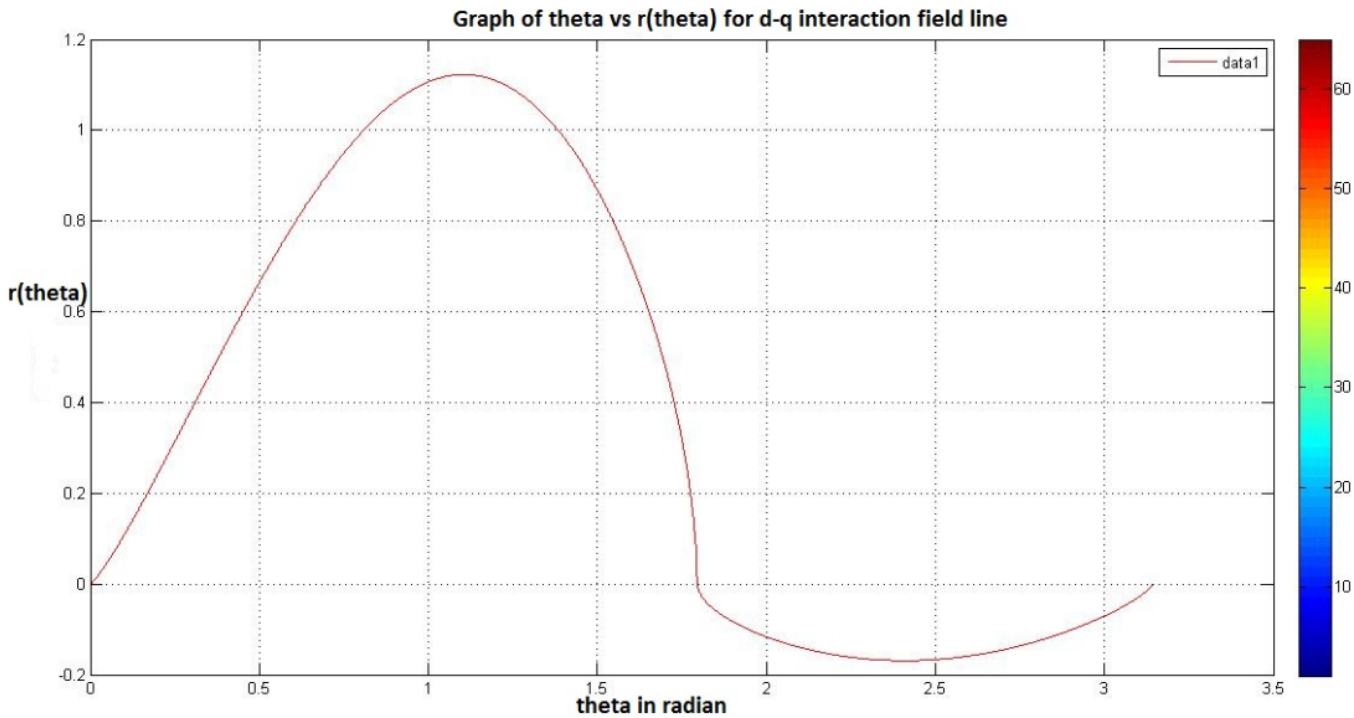

Figure 6. *Graphical representation of field lines for neutron star magnetic dipole-quadrupole interaction,where θ is in radians.*

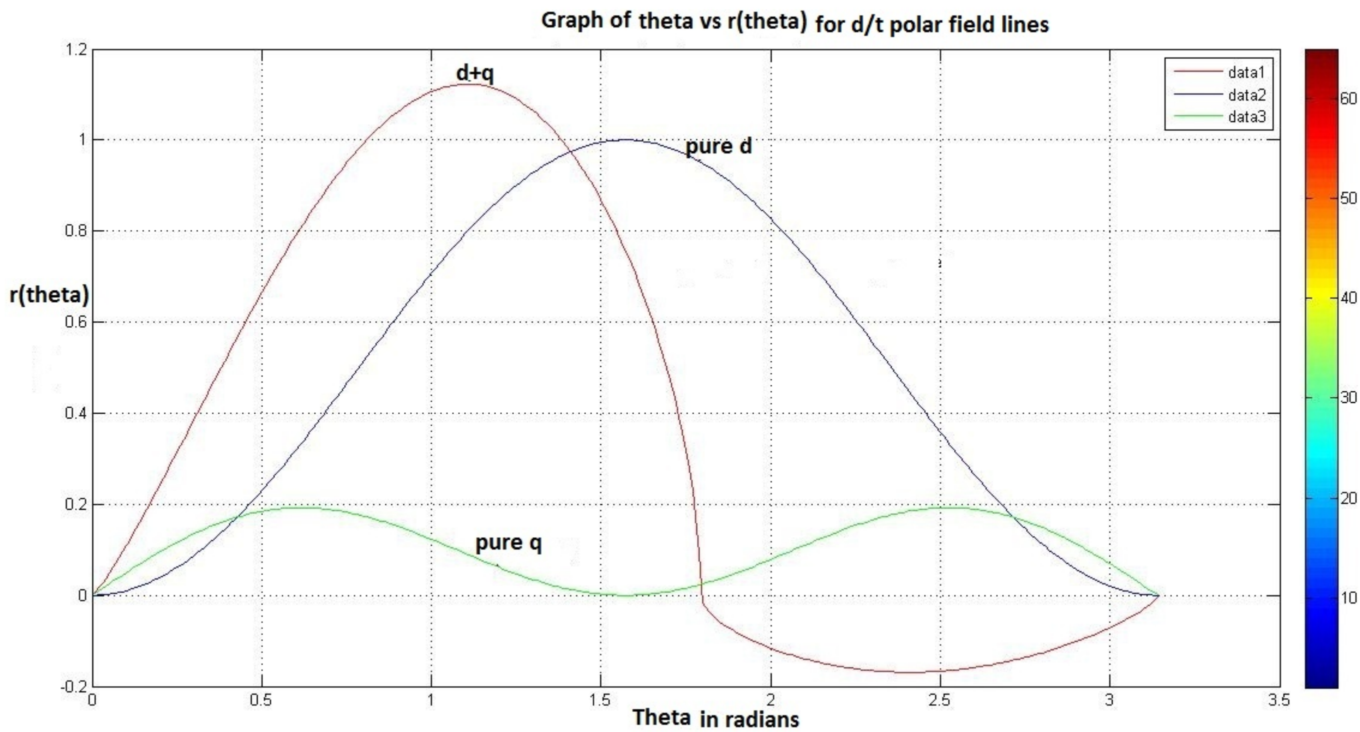

Figure 7. *Graph of magnetic pure dipole and quadrupole field lines and their interaction.*

This is the solution obtained. Figures 4 and 5 are geometries of this filed line equation for a single k and several k's respectively. There are two lobes in a spam of 360 degrees or in the range $(0, 2\pi)$ for a specific k, where k is a constant related to the interaction field curvature. The geometry is some what different in shape from pure dipole field line geometry as it is to be.This field line geometry indicates that there is only one solution for a particular degree in a quadrant. It also shows that the field geometry become open at a given distance from the star this means, if the value of k is increased one can obtain the point at which field lies become open (see figures 4 and 5). Figure 6 shows the maximum field lines at the for the angle of deflection of magnetic moments from axis of rotation.

The geometry for single k has only one loop in a given quadrant and for many k there are many loops. Since, k is related to the interaction field curvature, as the value of this increases the field line open at a given distance from the surface of the star. This point has some physical meaning or importance to understand the flow of matter to a compact star. The magnetic stresses thus increase much more steeply with decreasing radius than the material stresses do. Therefore, generically one expects that far from the star, material stresses must dominate. Close to the star, magnetic stresses dominate.

This is the graph of our field line solution for neutron star dipole-quadrupole magnetic field interaction. It is plotted for the angle of deflection of magnetic moments from axis of rotation versus $r(\theta)$ between zero degree and π or 3.14 rad and help us to indicate the point of neutral lines. Here the graph indicates there are three neutral lines as expected. The three neutral lines for this interaction part are the two axes, $\theta = 0$ and $\theta = \pi$ and a circle in the plane (r, θ) on which r-solutions correspond to θ-angles in the domain $[\frac{\pi}{2}, \frac{3\pi}{2}]$ radians.

7. Discussion

There are three neutral lines for the interaction part as expected: the two axes, $\theta = 0$ and $\theta = \pi$ and a circle in the plane (r, θ) on which r-solutions correspond to θ-angles in the domain $[\frac{\pi}{2}, \frac{3\pi}{2}]$ degrees. The three field lines have the same solution at some degrees as both have the same at π. Values of the magnetic field on the lines $\theta = 0$, or $\theta = \pi$ obtained from the equations of magnetic field components, it is clear that on the lines $\theta = 0$ and $\theta = \pi$, $B_\varphi(r, \theta) = 0$, as it is zero every where, $B_\theta(r, \theta) = 0$, since $\sin\theta$ is in factor, but in particular, on the line $\theta = \pi$, there is a point at which $B_r(r, \theta) = 0$. This point show that it may be a neutral point for particular configurations of the magnetic field in the star magnetosphere. These neutral points may or may not have physical meaning.

8. Conclusion

In this paper the multipolar magnetic fields of neutron star have been calculated and derived from a vector potential produced by spinning surface charges. This paper investigated magnetic dipole-quadrupole interaction field of neutron star as well as simulated its geometry of field lines in 2D for a single field curvature ($k = 1$) and for different field curvatures ($k = n$). Simulations showed that magnetic field lines are closed if magnetic field is strong enough or (near the surface of the star) and open far from the star this can happen at a maximum radial extent of r_{max} in the stellar equatorial plane (a given distance from the surface of a star). Geometry of these field lines can affect the size of matter flow to the compact star. As seen from geometry of field line the size of accretion disc could be thick.

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Author Contributions

Gemechu Muleta Kumssa: Conceptualization, Formal Analysis, Funding acquisition, Investigation, Methodology, Project administration, Validation, Visualization, Writing - original draft, Writing - review & editing

Legesse Wetro Kebede: Investigation, Project administration, Supervision

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Conflicts of Interest

The authors disclose that this article is derived from Gemechu Muleta Kumssa's MSc Thesis in 2013, which was supervised by the late astrophysicist Dr.legesse Wetro Kebede. The authors assert that they do not have any identifiable conflicting financial interests or personal relationships that could have been perceived to impact the research presented in this paper.

The authors declare no conflicts of interest.

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