

# Effect of Magnetic Field on Particle Emission from the Surface of Neutron Star

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**Abstract:** Studying the effect of magnetic fields on particle emission from the surface of neutron stars is vital for advancing our understanding of neutron star physics and high-energy astrophysical processes. One of the main topics in pulsar magnetospheric physics is the particle emission from neutron stars surface. This study investigates the role of multipolar magnetic fields in neutron star (NS) emission physics by incorporating higher-order field components into the standard dipole model. While past studies have primarily relied on a dipolar field configuration, recent observations suggest the presence of multipole components that significantly influence emission processes. Our findings show that higher-order multipole magnetic fields shape localized particle emission regions near the NS surface, while the dipole field dominates at larger distances. By considering the NS's crust and superfluid core structure, as well as the effects of rapid rotation, we refine the understanding of magnetic field topologies and their impact on radiation mechanisms. This study highlights the necessity of incorporating multipolar magnetic fields for accurate modeling of pulsar and magnetar emissions. By investigating these effects, particularly the role of multipolar magnetic field components, researchers can refine theoretical models of emission mechanisms that go beyond the classical dipole framework. This has significant implications for interpreting observations of pulsars and magnetars, whose emission patterns, spectral features, and temporal variability often demand more complex field geometries. Furthermore, understanding particle emission driven by magnetic fields offers insight into neutron star spin-down evolution, magnetic field decay, and energy loss processes. Future work will involve detailed numerical simulations incorporating general relativistic effects and magnetosphere-plasma interactions.

**Keywords:** Neutron Star, Neutron Star Surface, Multipole Magnetic Fields, Particle Emission

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## 1. Introduction

Neutron stars (NSs) are among the most extreme objects in the universe, formed from the remnants of massive stars undergoing supernova explosions [10]. Their extreme magnetic fields, often reaching  $10^{11}$ - $10^{13}$ G, play a crucial role in their emission physics, influencing charged particle trajectories, surface heating, and radiation anisotropies [14, 23]. NSs provide a stable environment with simultaneous extreme physical conditions, making them ideal for studying matter and plasma properties under extreme limits [1]. A collapsing star initially has a significantly larger surface area compared to the resulting neutron star, leading to a stronger magnetic field due to the conservation of magnetic

flux. However, this straightforward explanation does not fully account for the varying magnetic field strengths observed in neutron stars [23]. Some theories suggest that in certain objects, the magnetic field could be the primary energy source for the radiation observed [27]. Information about the magnetic fields of neutron stars is gathered from factors like the spin down rate of pulsars, observations of cyclotron lines in accreting pulsars, and estimations based on the equilibrium spin period of accreting pulsars [15]. Huge magnetic fields, which reach magnitudes up to  $10^{11}$ - $10^{13}$  G in the majority of NSs, cause the NS atmospheres to have even more unusual properties [19].

It is still unclear how the magnetic field and the surface of

a neutron star interact because it is a complicated physical process [26]. Additionally, by differential rotation, studies of the magnetic fields on the structure of neutron stars have been conducted in recent decades [8, 24]. It has been demonstrated that for stars, the multipole fields are treated by neglecting current density. A distinct theoretical and observational analysis also suggests that the physics of rotation-powered pulsars depends critically on multipolar magnetic fields developing close to neutron stars [4]. However, over the past few decades, studies have been conducted to examine the impact of magnetic multi-poles on the form of neutron stars and their surroundings by ignoring current density [8]. In order to calculate the electromagnetic recoil of a newly formed neutron star, [24] estimated the electromagnetic multi-polar fields. In addition, he corrected the Deutsch solution with some higher order adjustments and computed the off-center magnetic multipoles. It was demonstrated by [12] that limitations might be placed on the magnetic multipole fields' strength for millisecond pulsars. [16] examined the decay of multipolar magnetic components in neutron stars, however they were unable to identify any notable changes that could have altered the radio pulse signature. The distinctive and significant pulse signature of the rotating NS is assumed to be caused by magnetic multipole structure at and around the polar cap [28]. So far it was generally believed that NSs have a dipolar magnetic field. This structure has been assumed mostly for pragmatic reasons, as basic calculations of the electromagnetic and plasma torques acting on dipolar magnetospheres sufficiently describe observations of pulsar spin down or spin up [5]. However, it was explained by [17], that high-order multipolar fields may be intensified close to the atmosphere of a rotating neutron star, And one first reason for introducing multipolar magnetic fields close to the stellar surface lies in the inability for the pure surface dipolar field to provide sufficient pair production, as required for the generation of pulsar radiation emission [4]. It seems that the same reasons are applicable to the direct influence of the small scale component on current losses [29].

While early theoretical studies primarily assumed a dipolar magnetic field Goldreich & Julian, (1969), increasing evidence suggests that NSs possess significant multipolar components [6, 9, 20]. These higher-order fields impact pulsar spin-down, emission localization, and magnetospheric dynamics[4]. Additionally, observational data from X-ray and radio pulsars indicate deviations from a purely dipolar configuration, suggesting the need to account for quadrupole and octupole contributions[7, 22]. Thus, to unlock the physics of neutron star magnetospheres, a thorough examination of the precise topology of the multipolar fields is likely necessary in order to comprehend the many different types of neutron stars. Each multipole moment's relative strength holds the clue [20].

This study builds upon prior literature by examining how multipolar magnetic fields influence emission processes, taking into account the NS's internal structure and rotation. Unlike previous works that neglected current density effects (Gregory, 2011), we incorporate circulating currents and their feasibility in the NS's surface, crust, and core[8]. The rest

of the paper is organized as follows: Section 2 describes theoretical model and derivations of input parameters, Section 3 presents results of the analysis and discussions, and then the final Section 4 come up with summary and conclusion.

## 2. Theoretical Model

The magnetic topology of NSs depends on their internal composition, which includes a solid crust, a superfluid interior, and a highly magnetized plasma atmosphere [13]. The standard dipole approximation is insufficient to explain observed anisotropies in pulsar radiation. We assumed that while the bulk of NS magnetism originates from the core and crust, surface currents, though limited in extent, can modify local field configurations. Hall drift in the crust redistributes magnetic field lines, leading to the emergence of localized multipolar structures [7]. Additionally, electron-positron pair cascades in the magnetosphere can create transient currents influencing field topology [2]. Thus, our assumption of surface currents does not imply that they generate the entire NS field but rather that they contribute to higher-order multipole corrections near the emission regions. The majority of observed NSs exhibit spin periods ranging from milliseconds to seconds, leading to significant relativistic effects. Rotation-induced field distortions impact the magnetosphere's structure and particle acceleration mechanisms [17].

Incorporating rotation, the field structure is modified by frame-dragging effects and rotationally induced currents. The dipole field in a rotating NS experiences an obliquity-dependent modification, leading to enhanced multipolar components [20]. According to the several observational and theoretical evidences [4], the structure of NS is considered as spherical coordinates system  $(r, \theta, \phi)$  where  $r$  is measured from the stellar center,  $\theta$  is the polar angle (in radians) measured from the z-axis, and  $\phi$  is an azimuthal angle (in radians) measured from an arbitrary origin. Z-axis is assumed to be directed along with the dipolar magnetic momentum of the star. Outside the sphere everywhere, consider that the magnetic multipoles of order  $(l, m)$ , are produced by currents moving through the atmosphere. As noted by [21] and a few others, the structure of an NS is thought to be composed of a solid crust, a superfluid interior, and an inner core that is likely likewise solid (neutron solid). A NS may be thought of as a compact object in a plasma state since the majority of the stuff that makes up an NS is composed of neutrons, ions, and electrons in the solid crust and neutrons, proton, and electrons in the outer core. Nonetheless, because electrons are the lightest species in the plasma, any driving force will cause them to react the quickest. For the remainder of this work, we shall treat the plasma as a single component (electron) plasma because of this. we used the addition formula for spherical harmonics that provides a way to express the product of two spherical harmonics of different angles in terms of a sum of spherical harmonics of the same angle. The inverse distance between the two vectors,  $\vec{r}$  and  $\vec{r}'$  can be written as,

$$\frac{1}{|\vec{r} - \vec{r}'|} = \frac{1}{\sqrt{r^2 + r'^2 - 2rr' \cos \gamma}} \quad (1)$$

After using addition theorem for spherical harmonics, we obtained the general expression for the inverse distance between the two vectors as,

$$|\vec{r} - \vec{r}'|^{-1} = \sum_{lm} \frac{r'^l}{r^{l+1}} Y_{lm}^*(\theta', \phi') Y_{lm}(\theta, \phi) \quad (2)$$

## 2.1. Derivation of Multi-pole Magnetic Fields

In the case of a neutron star, the source of current is considered to be the surface charge density, which arises from the presence of ions and electrons on the surface of the star. To derive the magnetic vector potential outside a sphere for a uniform spherical surface charge distribution with radius  $R$  and surface charge density  $\sigma$ , we must first determine the current density  $\vec{J}$ , which is given by;

$$\vec{J} = \sigma \delta(r' - R) \vec{v} \quad (3)$$

But  $\sigma = \frac{Q}{4\pi R^2}$ , and  $\vec{v} = \vec{\omega} \times \vec{r}$ , where  $\vec{\omega}$  is angular frequency,  $\vec{v}$  is velocity and  $\vec{r}$ , is radius. Thus the current density becomes,

$$\vec{J} = \sigma \delta(r' - R) \vec{\omega} \times \vec{r} \quad (4)$$

$$\vec{J} = \sigma \delta(r' - R) \vec{\omega} R \sin(\theta') (-\sin(\phi') \hat{i} + \cos(\phi') \hat{j}) \quad (5)$$

where  $\omega R \sin(\theta') \hat{e}_{\phi'} = \vec{\omega} \times \vec{r}$  which clearly states that  $\vec{J}$  has only  $\hat{\phi}'$  component as;

$$\vec{J} = \sigma \delta(r' - R) \omega R \sin(\theta) \hat{e}_{\phi'} \quad (6)$$

The magnetic vector potential of neutron star at a given field point  $\vec{r}$ , with a net surface charge different from zero,  $\vec{J}$  is the current density, and  $\gamma$  is the angle between  $\vec{r}$  and  $\vec{r}'$  [11] is given by;

$$\vec{A}(r) = \int dv' \frac{\vec{J}(r')}{|\vec{r} - \vec{r}'|}, \quad (7)$$

Since the geometry is spherically symmetric, we choose the observation point in  $XZ$ -plane ( $\phi' = 0$ ). Since the azimuthal integration in Eq.7 is symmetric in  $\phi'$ , the  $i^{th}$  component of the current does not contribute. This leaves only the  $j^{th}$  component, which is  $\vec{A}_{\phi}$ . Then by using Eq.2 in Eq.7, we can express the vector potential as,

$$\vec{A}(r) = \sum_{lm} \frac{4\pi}{2l+1} \int J(r') \frac{r_{<}^l}{r_{>}^{l+1}} Y_{lm}^*(\theta', \phi') Y_{lm}(\theta, \phi) dv' \quad (8)$$

Where  $r_{<} = r < r'$  and  $r_{>} = r > r'$ . Considering the volume element  $d^3r = dv' = r'^2 \sin(\theta) dr' d\theta' d\phi'$  and using the current density given by Eq.5, the vector potential can be written as,

$$\vec{A}^l(r) = \frac{R\omega|Q|}{4\pi R^2} \sum_{lm} \frac{4\pi}{2l+1} Y_{lm}(\theta, \phi) \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} Y_{lm}^*(\theta', \phi') \frac{r_{<}^l}{r_{>}^{l+1}} \delta(r' - R) \sin^2 \theta' \cos \phi' r'^2 dr' d\phi' d\theta' \hat{e}_{\phi'} \quad (9)$$

By neglecting the inner structure as we only consider the atmosphere, the magnetic vector potential everywhere outside the sphere will be obtained by applying  $r_{<} = R$  and  $r_{>} = r$  to Eq.9. Thus we obtain;

$$\vec{A}_{\phi, out}^l = \frac{R\omega|Q|}{4\pi R^2} \sum_{lm} \frac{4\pi}{2l+1} Y_{lm}(\theta, \phi) \times \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} Y_{lm}^*(\theta', \phi') \times \frac{R}{r^{l+1}} \delta(r' - R) \sin^2 \theta' \cos \phi' r'^2 dr' d\phi' d\theta' \hat{e}_{\phi'} \quad (10)$$

Using integration over the delta function yields  $\int_0^{\infty} \delta(r' - R) r'^2 dr' = R^2$ , as the result the vector potential outside the sphere can be written as;

$$\vec{A}_{\phi, out}^l = \frac{R\omega|Q|}{4\pi R^2} R^2 \sum_{lm} \frac{4\pi}{2l+1} Y_{lm}(\theta, \phi) \times \int_0^{2\pi} \int_0^{\pi} Y_{lm}^*(\theta', \phi') \times \frac{R}{r^{l+1}} \sin^2(\theta') \cos \phi' d\phi' d\theta' \quad (11)$$

This expression shows that the magnetic vector potential outside the sphere is proportional to the distance from the center of the sphere and depends on the angles  $\phi$  and  $\theta$ . Where,  $R$  is the star radius, and  $r$  is a point external to the star. Therefore, in order to derive the multi-polar magnetic field components, we used to consider different values of  $l$  to calculate corresponding vector potentials first. i.e.  $l = 1, 2, 3, 4$  for the multi-pole fields such as dipole, quadrupole, hexapole, and octupole, respectively.

For dipole field let  $l = 1$ , the corresponding vector potential is calculated as;

$$\vec{A}_{\phi, out}^1 = \frac{R\omega |Q|}{4\pi R^2} R^2 \sum_{lm} \frac{4\pi}{3} Y_{11}(\theta, \phi) \times \int_0^{2\pi} \int_0^\pi Y_{11}^*(\theta', \phi') \times \frac{R}{r^2} \sin^2(\theta') \cos(\phi') d\phi' d\theta', \quad (12)$$

From the normalization and orthogonality condition [11],

$$\int_0^{2\pi} \int_0^\pi Y_{lm}^*(\theta', \phi') Y_{lm}(\theta, \phi) \sin(\theta') d\phi' d\theta' = \delta_{l1} \delta_{m1} \quad (13)$$

So that,

$$\delta_{l1} \delta_{m1} = \begin{cases} 1, & \text{if } l = 1 \text{ and } m = 1 \\ 0, & \text{if } l \neq 1 \text{ or } m \neq 1 \end{cases} \quad (14)$$

Then taking the spherical harmonic value and its conjugate, we obtain;

$$\vec{A}_{\phi, out}^1 = \frac{R\omega |Q|}{4\pi R^2} \frac{R}{r^2} \frac{R^2 4\pi}{3} \sqrt{\frac{3}{8\pi}} \sin(\theta') \hat{e}_{\phi'} \left( -\sqrt{\frac{8\pi}{3}} \right) \times \int_0^{2\pi} \int_0^\pi Y_{11}(\theta, \phi) Y_{11}^*(\theta', \phi') \sin(\theta') d\phi' d\theta' \quad (15)$$

After proceeding some steps and applying normalization and orthogonality condition; we get the dipole vector potential,

$$\vec{A}_{\phi, out}^1 = -\frac{\omega |Q|}{3} \frac{R^2}{r^2} \sin(\theta) \hat{e}_{\phi'} \quad (16)$$

For quadrupole field, we employ the spherical harmonic  $Y_{21} = -\sqrt{\frac{15}{8\pi}} \sin(\theta) \cos(\theta) \hat{e}_{\phi'}$ , and its conjugate  $Y_{21}^*$  for the vector potential, and running integration of each pole for  $\theta$  from 0 to  $\pi/2$ , the quadrupole vector potential will be;

$$\vec{A}_{\phi, out}^2 = -\frac{\omega |Q|}{4} \frac{3R^3}{r^3} \sin(\theta') \cos(\theta) \hat{e}_{\phi'} \quad (17)$$

When the value of  $Y_{31} = -\frac{1}{8} \sqrt{\frac{15}{\pi}} \sin(\theta) (5 \cos^2(\theta) - 1) \hat{e}_{\phi'}$ , is substituted into the same approach to determine the integrals, these integral returns zero in terms of  $\theta$ . Thus, in order to take integrals, we ultimately apply the geometry of those poles and multiply the value by the quantity of poles. The vector potential can be calculated as;

$$\vec{A}_{\phi, out}^3 = \frac{R\omega |Q|}{4\pi R^2} \sum_{l,m} \frac{R^2 4\pi}{7} \times \int_0^{2\pi} \int_0^\pi Y_{31}(\theta, \phi) Y_{31}^*(\theta', \phi') \times \frac{R^3}{r^4} \sin^2 \theta' \cos \phi' d\phi' d\theta' \quad (18)$$

Assuming that the integration of  $\theta$  goes from 0 to  $\pi/3$  for each unique pole, we obtained the octupole vector potential as ;

$$\vec{A}_{\phi, out}^3 = -\frac{\omega |Q|}{25} \frac{4R^4}{r^4} \sin(\theta) (\sin^2(\theta) - 1) \hat{e}_{\phi'} \quad (19)$$

For octupole vector potential, we have used the same procedures and the value of  $Y_{41} = -\frac{3}{8} \sqrt{\frac{5}{\pi}} \sin(\theta) (7 \cos^3(\theta) - 3 \cos(\theta)) \hat{e}_{\phi'}$  to find the integrals as we did for the quadrupole and hexapole fields. We have the octupole vector potential after the integration runs for individual poles as  $\theta$  run from 0 to  $\pi/4$  that can be expressed as;

$$\vec{A}_{\phi, out}^4 = -\frac{\omega |Q|}{64} \frac{15R^5}{r^5} \sin(\theta) \cos(\theta) (7 \cos^2(\theta) - 3) \hat{e}_{\phi'} \quad (20)$$

Multipolar fields are important not only for neutron stars but also for main-sequence stars, or any other star possessing a non-negligible magnetic field. To study quantitatively the influence of those components, it is illuminating to get simple closed analytical expressions for those fields. Thus in this section, we give the solutions for the dipole, quadrupole, hexapole and octupole fields. The neutron star's overall magnetic field can be obtained from the sum of individual multipolar magnetic field components. i.e.

$$B(r, \theta, \phi) = \sum_{l=0}^{\infty} B^{l,m}(r, \theta, \phi) \quad (21)$$

But as implied by Maxwell's equations, the multi-polar magnetic field components can be derived from vector potential as  $B = \nabla \times \vec{A}$ , which can be written as;

$$\nabla \times \vec{A} = \hat{e}_r \frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (\sin \theta \vec{A}_\phi) - \frac{\partial \vec{A}_\theta}{\partial \phi} \right] + \hat{e}_\theta \left[ \frac{1}{r \sin \theta} \frac{\partial \vec{A}_r}{\partial \phi} - \frac{1}{r} \frac{\partial (r \vec{A}_\phi)}{\partial r} \right] + \hat{e}_\phi \frac{1}{r} \left[ \frac{\partial}{\partial r} (r \vec{A}_\theta) - \frac{\partial \vec{A}_r}{\partial \theta} \right] \quad (22)$$

But due to the reason that we stated before, we have only  $\hat{\phi}$  component of the vector potential, then the magnetic field will be;

$$\nabla \times \vec{A} = \frac{1}{r \sin(\theta)} \left[ \frac{\partial}{\partial \theta} (\sin(\theta) \vec{A}_\phi) \hat{e}_r + \frac{-1}{r} \left( \frac{\partial \vec{A}_\phi}{\partial r} \right) \hat{e}_\theta \right], \quad (23)$$

Thus, each magnetic multipole with indices  $l$  and  $m$  can be associated with a vector potential  $\vec{A}^{lm}$ . Each multipolar component of the magnetic field vector  $B^{l0}(r, \theta)$ , in the symmetric and uniform case with arbitrary  $l$  and with  $m = 0$ , logically comes from a potential vector,  $\vec{A}^{l0}(r, \theta)$ . The potential vector only contains an azimuthal component  $\vec{A}_\phi(r, \theta)$ , which can be given for each specific magnetic multipole having indices  $l > 1$  and  $m = 0$ , assuming that the magnetic field is produced by an alignment of charged particles in plasma. Therefore, the multipolar magnetic fields outside the sphere i.e. on the atmosphere of NS, can be calculated

by setting different values of  $l$ , where  $l = 0, 1, 2, 3, 4, \dots$  and  $m = -l, -l + 1, \dots, l - 1, l$  in the spherical harmonics (In our magnetic case there are no real monopoles, and the series will start with the dipole  $l = 1$ , terms). Despite the condition stated by Eq.23, only  $B_r^{l,m}(r, \theta)$  and  $B_\theta^{l,m}(r, \theta)$  would survive, and according to Eq.21, for the case with  $m = 0$ , the components of the total magnetic field close to the atmosphere of the star can be expressed in a general form as a sum of multi-polar magnetic field components, which can be expressed as;

$$B_r^{l,m}(r, \theta) = \frac{e_r}{r \sin \theta} \frac{d}{d\theta} (A_\phi^{lm}(r, \theta) \sin \theta) B_\theta^{l,m}(r, \theta) = -\frac{e_\theta}{r} \frac{d}{dr} (r A_\phi^{lm}(r, \theta)) B_\phi^{l,m}(r, \theta) = 0 \quad (24)$$

But in the context of the general multipolar magnetic field equation of a neutron star, the symbols  $\hat{e}_{r'}$  and  $\hat{e}_{\theta'}$  typically represent unit vectors in a spherical coordinate system. Then applying Eq.24 in accordance with Eq.16-Eq.20, the radial and polar components of multipole magnetic fields can be calculated.

The dipole magnetic field with radial and polar components can be written as,

$$\vec{B}_{dip} = \frac{2Q\omega R^2}{3r^3} \cos(\theta) \hat{e}_{r'} + \frac{Q\omega R^2}{3r^3} \sin(\theta) \hat{e}_{\theta'} \quad (25)$$

For  $l = 2$ , the radial and polar components of quadrupole magnetic field is ;

$$\vec{B}_{qu} = \frac{3Q\omega R^3}{4r^4} (\cos^2(\theta) - 1) \hat{e}_{r'} + \frac{3Q\omega R^3}{2r^4} \sin(\theta) \cos(\theta) \hat{e}_{\theta'} \quad (26)$$

The same step was performed for hexapolar magnetic field where we used  $l = 3$  and obtain,

$$\vec{B}_{hex} = \frac{16Q\omega R^4}{25r^5} \cos(\theta) (5 \cos^2(\theta) - 3) \hat{e}_{r'} + \frac{12Q\omega R^4}{25r^5} \sin(\theta) (5 \cos^2(\theta) - 1) \hat{e}_{\theta'} \quad (27)$$

Finally for  $l = 4$ , the radial and polar octupole magnetic field components for the axisymmetric mode  $m = 0$  are given in an orthonormal basis by;

$$\vec{B}_{oct} = \frac{15Q\omega R^5}{64r^6} (35 \cos^4(\theta) - 30 \cos^2(\theta) + 3) \hat{e}_{r'} + \frac{15Q\omega R^5}{16r^6} \sin(\theta) \cos(\theta) (7 \cos^2(\theta) - 3) \hat{e}_{\theta'} \quad (28)$$

Where, the terms in left hand side and right hand side of Eq.25- Eq.28, represents radial and polar components of the field respectively.

## 2.2. Method

This study is grounded in a theoretical framework centered on a highly magnetized isolated neutron star (NS). We take into account both the solid crust and the superfluid interior of the

neutron star. The crust sustains a robust magnetic field that is anchored by currents located deep within the star, rather than being limited to the thin atmospheric layer. The methodology incorporates the following key aspects. The study derives multipole magnetic fields  $l, m$  for various values of  $l = 1$ ,

2, 3, and 4 corresponding to dipole, quadrupole, hexapole, and octupole fields. The current density distribution are used to determine the magnetic vector potential outside the NS atmosphere. The multipole expansion of the vector potential is performed using spherical harmonics and the addition theorem. Maxwell's equations are employed to derive the total magnetic field from the computed vector potential, considering contributions from different multipole components.

### 3. Results

In this section, we present the analytical result of the multipole magnetic field. Most likely, the magnetic field of neutron stars is the result of magnetic flux compression carried over from the progenitor star. According to this theory, the spinning separated charged particles that are produced by the plasma diffusion process are the sources of the NSs surface magnetic field [25].

#### 3.1. Multi-pole Magnetic Field Effect on Particle Acceleration

We start our discussion with the well-known magnetic dipole field of the NS and how its component varies with polar angle  $\theta$ . Because axial multipoles are rotationally symmetric about the z-axis and reflectionally symmetric in the x-y plane, we only considered values of polar angle,  $\theta$  between zero and  $\pi$ . The following plots illustrates the relationship between the multi-pole magnetic field strength and the polar angle  $\theta$  for various values of  $h$ , likely representing different radial distances from the neutron star. In figure 1-a, the magnetic field strength is found to vary sinusoidally with the polar angle  $\theta$  for all values of  $h$ . It reaches its maximum at  $\theta=0$  and  $\theta=\pi$  (the poles), but it reaches its minimum at  $\theta=\pi/2$ , which indicates that the charged particles within the neutron star's atmosphere are not as accelerated. The sinusoidal dependence of the magnetic field strength on the polar angle confirms the dipole nature of the field, with stronger fields at the poles and weaker fields at the equator. As the radial distance increases, the magnetic field strength decreases significantly, and the variation with polar angle becomes less pronounced, indicating the weakening influence of the dipole field at greater distances.

Again in figure 1-b, quadrupole fields are more complex than dipole fields, exhibiting multiple peaks in the angular distribution of field strength. The presence of peaks at the poles and equator suggests that the quadrupole field has a more intricate angular dependence, with significant variations even at intermediate angles. At small radial distances (low  $h$  values), the quadrupole field strength is relatively strong and shows clear angular variation. From figure 1-c, as the distance increases, the field strength diminishes rapidly, and the angular dependence becomes less pronounced, indicating the rapid decay of hexapole fields with distance compared to lower-order multipole fields like dipole and quadrupole fields.

The field strength can be positive (indicating outward direction) or negative (indicating inward direction) depending

on the polar angle. Charged particles emitted from the star's surface follow trajectories along the field lines. The pronounced peaks at the poles and the rapid decrease in field strength with distance reflect the complex nature and rapid decay of higher-order multi-pole fields. Hexapole field is a significant factor in shaping the structure of the neutron star atmosphere. It introduces perturbations in the atmospheric density and temperature profiles.

In addition, graph 1-d demonstrates how the strength of octupole magnetic field changes with respect to the polar angle around the neutron star. It may show regions where the field is stronger or weaker, indicating areas of magnetic dominance and potential interaction with the plasma especially near the atmosphere for small radial distance. Variations in field strength and orientation at different polar angles may alter the emission spectra, polarization, and intensity of radiation emitted from the plasma-filled regions. Again as the radial distance increases, the octupole field strength decays rapidly, and the angular dependence becomes less pronounced, indicating that octupole fields decay faster with distance compared to dipole field.

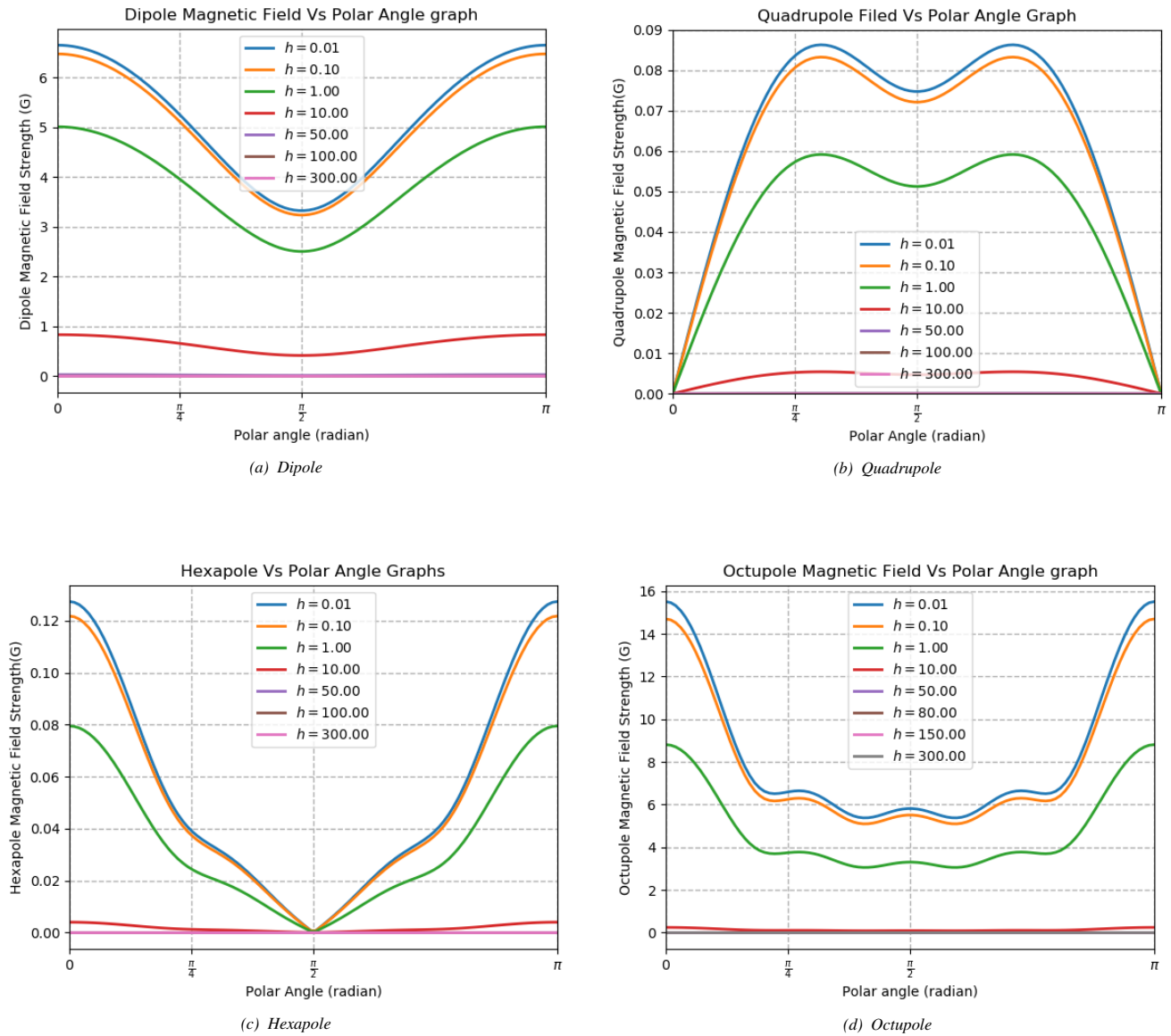
In terms of visualization, the octupole magnetic field around the neutron star atmosphere may exhibit a complex and asymmetric pattern, characterized by multiple regions of magnetic influence, field lines with varying orientations, and potential interactions with the surrounding plasma. The field configuration would reflect the intricate interplay between the stellar magnetic field and the plasma environment, manifesting in diverse spatial distributions and gradients of magnetic strength with respect to the polar angle. We observed that at higher radial distance, dipole field is relatively strong, playing role in particle acceleration and emissions of radiation, and our findings corroborate the result made by [6] that neutron star magnetic fields are roughly dipolar at radio emission altitudes where high-energy emission is generated.

From derivations in the method section-2, the multi-pole magnetic field of a NS atmosphere is inversely proportional to the radial distance from its center. From figure 2, it was shown that the strength of magnetic field decreases with increasing radial distance from the star's center, but the rate of decrease depends on order of the multipole magnetic moment. In a dipole magnetic field, the magnetic field strength at a point in space depends on the radial distance from the dipole center and the angle  $\theta$  from the dipole axis. The magnetic field strength in the plane of the dipole (perpendicular to the dipole axis) decreases with the cube of the distance from the dipole as the analytical result shows in Eq.25. Therefore, for all angles  $\theta$ , the magnetic field strength decreases as the radial distance from the dipole increases. This is expected as the magnetic field strength for a dipole field falls off with distance (typically as  $1/r^3$ ) as shown in figure 2-a. Unlike a dipole field, a quadrupole field has a more complex spatial dependence and decays faster with distance, especially by  $1/r^4$  and as shown in figure 2-b, for all  $\theta$ , the quadrupole magnetic field strength decreases rapidly as the radial distance from the quadrupole center increases. Again in figure 2-c, the rapid decay with distance and significant variation with angle highlight the

complex nature of hexapole fields compared to dipole and quadrupole fields.

From figure 2-d, we have octupole magnetic field that decreases with increasing radial distance. The octupole magnetic field configuration can have significant effects on the behavior and dynamics of the neutron star, influencing processes such as its rotation, emission of radiation, and interactions with its environment. The graphs provides a clear depiction of how the multi-pole magnetic field strength varies with radial distance and orientation. The rapid decay with distance and significant variation with angle highlight the complex nature of non-dipole fields compared to dipole fields.

Near the atmosphere of a neutron star, the dominant magnetic field component is typically the dipole component. The dipole magnetic field is the simplest and most significant component of the magnetic field near the surface of a neutron star. It is responsible for shaping the overall structure of the magnetic field and has a strong influence on the behavior and emission properties of the neutron star. While higher-order multipole components such as the quadrupole, hexapole, and octupole fields also exist and contribute to the overall complexity of the magnetic field, their influence near the surface is generally overshadowed by the dominant dipole component.



**Figure 1.** Particle emission via multipole magnetic field against polar angle.

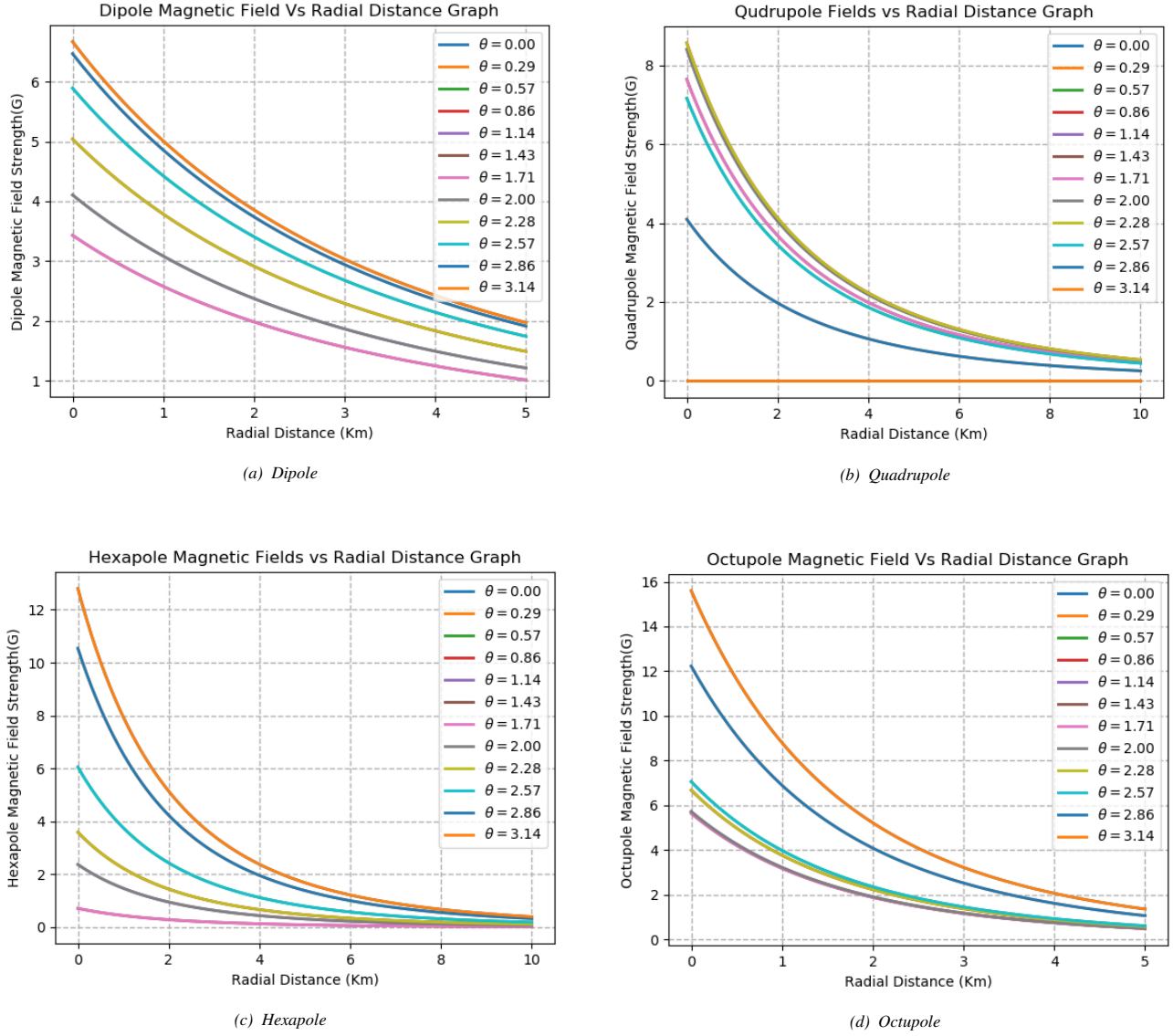


Figure 2. Multipole magnetic field strength versus radial distance.

### 3.2. Particle Emission Rates Due to Multipole Fields

Higher-order components are important for, small-scale anisotropic surface emissions of particles or thermal radiation, modifying the large-scale geometry of particle acceleration regions, and explaining hotspot localization observed in pulsars and magnetars. The following results are shown as four polar contour plots of emission rates. Dipole Field (Top Left), quadrupole Field (Top Right), hexapole Field (Bottom Left), and octupole Field (Bottom Right). Near the Surface, polar emission dominance occurs due to particle confinement along dipolar field lines, which converge near the poles. Emission is concentrated near the magnetic polar regions ( $\theta = 0$ ).

This aligns with theoretical models of pulsar emission where polar caps serve as regions of intense particle acceleration and thermal emission [9]. Thus, dipole field dominates at large distances and determines large-scale emission geometry (e.g., polar caps in pulsars). The dipole-dominated large-scale structure aligns with radio and X-ray observations of pulsar magnetospheres [3]. Quadrupole Field (Top Right),

emission is still polar-focused but splits into two distinct lobes symmetrically about the poles. The quadrupole field decays as  $r^{-7/4}$ , faster than the dipole. Near the Surface, quadrupole fields introduce angular variations in the field geometry, leading to localized hotspots and anisotropic emission. This behavior matches theoretical models that incorporate multipole fields to explain surface temperature anisotropies in neutron stars [7]. It introduces additional angular structure near the surface, contributing to non-uniform polar emission. Far from the surface; the contribution of the quadrupole field diminishes quickly, leaving the dipole component dominant. Hexapole Field (Bottom Left), particle emission becomes more localized and structured with multiple peaks near the poles. Near the Surface, the hexapole structure produces intricate, small-scale emission regions due to the rapid angular variations in the magnetic field. This aligns with models where localized multipolar fields cause patchy surface heating and complex magnetic hotspots, observed in magnetar X-ray spectra [22, 30].



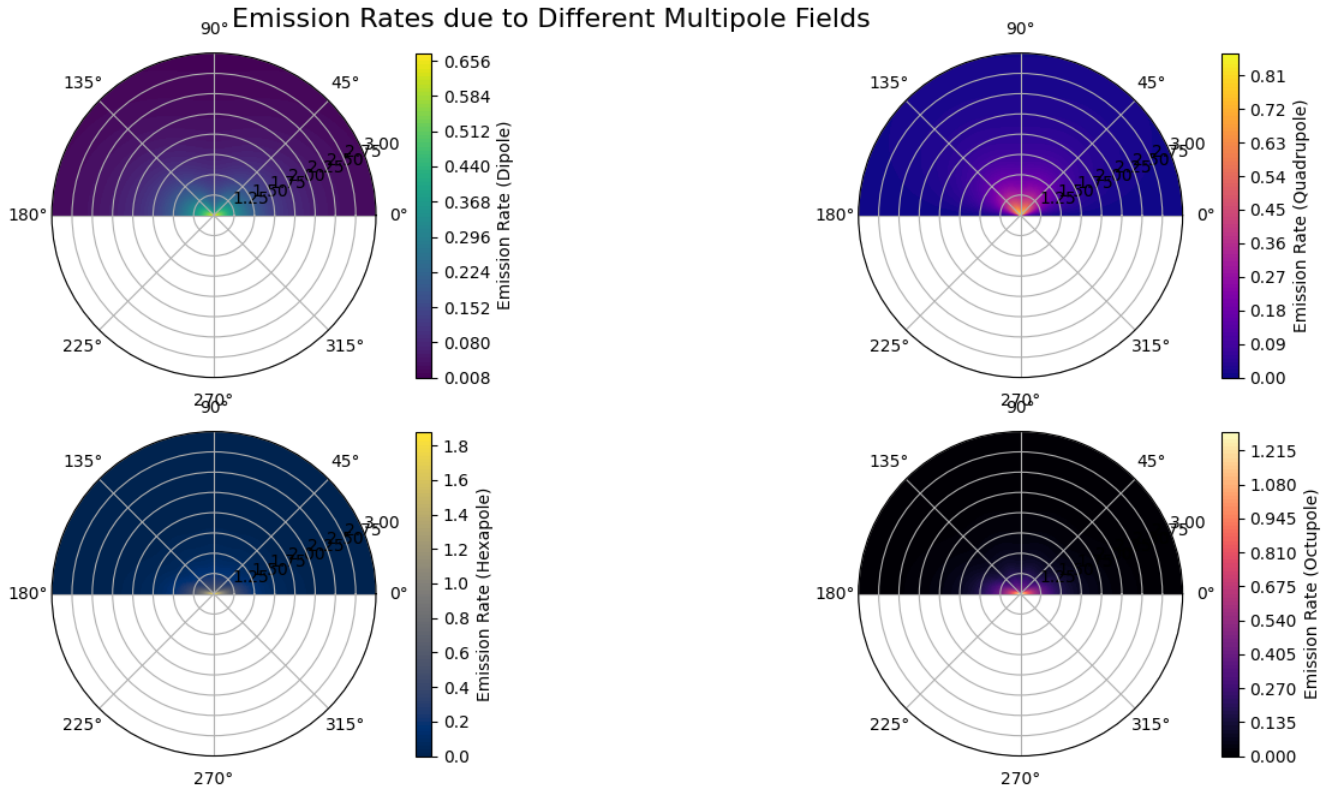


Figure 3. Emission rates due to multipole fields.

But, far from the Surface, hexapole field decays much faster than the dipole and quadrupole, becoming negligible at larger distances. Octupole Field (Bottom Right), the particle emission shows even more pronounced localization with higher-order angular structures, reflecting the octupole symmetry. The field decays as  $r^5$ , intermediate between quadrupole and hexapole fields. Near the Surface, octupole fields produce additional emission hotspots and fragmented regions, influencing surface thermal and particle emissions. Far from the Surface, the octupole field's contribution diminishes more rapidly than the dipole but slower than the hexapole. This aligns with results suggesting that octupole fields contribute to small-scale surface temperature variations in neutron stars [7].

Until recent years, it was believed that, the neutron star magnetic field is dipolar, and different works discuss about pulsars based on this kind of field [2]. But as we mentioned before, there are different evidences that shows the presence of this field [5, 16, 18], which is different from the dipolar component, our result also shows this. In this paper, we have observed that magnetic multipole fields can have significant effects on the atmosphere of a neutron star. Neutron stars possess extremely strong magnetic fields, which can be highly complex and vary in structure from simple dipoles to higher-order multipoles (quadrupole, hexapole, octupole, etc...).

Magnetic multipole fields can cause non-uniform temperature distributions on the surface of the neutron star. They can also create hot and cold spots on the star's surface, affecting the thermal emission observed from the star. Areas

with stronger magnetic field components, especially near the poles of the star, can channel accreting material or energy more effectively, leading to localized heating and the formation of hot spots. They can influence the regions where high-energy emission originates, leading to variability and anisotropy in X-ray and gamma-ray emissions.

The distribution of plasma in the magnetosphere can be heavily influenced by the multipole field structure, affecting the star's emission properties and interactions with the interstellar medium. Changes in the magnetic field structure, especially if higher-order multipoles are involved, can lead to starquakes and reconfiguration events. These events can emit gravitational waves detectable by sensitive instruments. Moreover, multipole components can alter particle trajectories, magnetic reconnection regions, and the overall surface field distribution, influencing how and where particles are accelerated and emitted. This in turn affects observable phenomena like X-ray and gamma-ray emissions, pulsar radio signals, and could lead to more intricate emission patterns and spectral variations than dipole-dominated models predict.

The axisymmetric multipole magnetic field effect on the surface of stars was described by Gregory [8]. His treatment considers rotation, azimuthally symmetric multipoles of large scale magnetic field of the star and ignores other important parameters like current density, even if it provides a useful insight into the magnetic field and plays a crucial role on particle acceleration. Gregory model assumed magnetostatic field, and rotational symmetry to derive multipole magnetic field of star by an expansion of the magnetostatic scalar

potential. Unlike Gregory, we used an expansion of the magnetostatic vector potential. However, Gregory excludes the NS's current density, whereas this study considers and examines the effect, ultimately arriving at the same conclusion.

## 4. Conclusion

In this investigation, we explored the analytical results for multipole field through the use of classical electrodynamics equations of magnetic vector potential. To do this, rotation is incorporated, and the field structure is modified by frame-dragging effects and rotationally induced currents. The dipole field in a rotating NS experiences an obliquity-dependent modification, leading to enhanced multipolar components. Since spherical coordinate is the most appropriate to employ when describing stars, planets, and their immediate surroundings, we have focused on deriving the equation of the field for a magnetic multipole in this coordinate. The vector potential for the multipole expansion, particularly for dipole, quadrupole, hexapole, and octupole fields, has also been mentioned. We obtained the radial and angular magnetic field components of the neutron star's atmosphere from this vector potential using spherical harmonics, however the azimuthal magnetic field is zero. The findings confirm that while dipole fields dominate at emission altitudes, near-surface regions require a multipolar treatment to explain thermal and non-thermal radiation features. The magnetic field plays a crucial role in shaping the surface of a neutron star. Overall, this study explores the influence of complex, multipolar magnetic field configurations on particle emissions from the surface of neutron stars (NSs). Departing from traditional models that primarily assume a dipolar magnetic topology, this work incorporates higher-order multipole components to better reflect observed anisotropies in emission profiles. We demonstrate that while the global magnetic field retains a dipolar character at larger distances, localized regions near the stellar surface are significantly shaped by multipolar structures. These arise due to internal factors such as crustal Hall drift and magnetospheric processes like pair production cascades.

By integrating the effects of NS internal structure-including a solid crust, superfluid core, and magnetized atmosphere-alongside relativistic frame-dragging due to rapid rotation, the model provides an improved description of the magnetic topology and its consequences for particle acceleration and radiation mechanisms. The results underscore the importance of accounting for surface currents and higher-order field terms in modeling pulsar and magnetar emissions. This refined approach paves the way for future numerical simulations incorporating general relativistic corrections to further unravel the complexities of neutron star magnetospheres. We briefly summarized our conclusions as follows;

1. Multipolar field has been ignored by several authors throughout the years, and they have approximated the NS magnetic field as dipolar. However, as we already indicated, there are numerous indications of the

existence of this field, which is distinct from the dipolar component, and our findings support these indications.

2. Compared to dipole magnetic field, the non-dipole fields, i.e multipole field of the NS surface decreases enormously with the radial distance  $r$ . This shows that dipole field is more dominant over neutron star's atmosphere.
3. Higher-order multipoles influence crustal magnetic fields, leading to patchy heating and temperature gradients, as observed in thermal spectra of magnetars.
4. While the dipole magnetic field dominates at large distances, higher-order multipole fields (e.g., quadrupole, hexapole, and octupole fields) become prominent near the surface of the star due to their rapid spatial decay.
5. It also has been shown that near the surface of NS, the magnetic field can bend the trajectories of charged particles, which can create strong currents in the atmosphere. These currents can heat up the atmosphere and produce non-thermal radiation.
6. Further more, it is shown that the acceleration of charged particles, production of high-energetic particles, emission of radiation and local explosion of events from NS atmosphere depends on the intensity of magnetic field.
7. Unlike dipole fields that lead to relatively symmetric emission patterns, multipole fields produce directional, anisotropic emissions.

## 5. Recommendation

Accurate modeling of neutron star magnetic fields requires incorporating both dipole and higher pole components to fully capture the complexity of the field, especially at different distances and angles. Future work will involve detailed numerical simulations incorporating general relativistic effects and magnetosphere-plasma interactions.

## Abbreviations

NS	Neutron Star
NSs	Neutron Stars

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## Conflicts of Interest

The authors state that they have no financial interests, affiliations, or personal relationships that could be perceived as influencing the research presented in this paper. All authors have disclosed any relevant connections, and no conflicts of interest are known to exist that might impact the integrity of this work.

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