

Research Article

Suborbital Graphs of Direct Product of the Symmetric Group Acting on the Cartesian Product of Three Sets

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Abstract

The study of suborbital graphs is a key area in group theory for it provides a graphical representation of a group action on a set. Moreover, it helps in understanding the combinatorial structures of the action of a group on a set. In this paper, we construct suborbital graphs based on the group action of the direct product of the symmetric group on Cartesian product of three sets through computation of the ranks and subdegrees of the group action. Suborbital graphs are constructed by the use of Sims theorem. The properties of the suborbital graphs are analyzed. In the study it is proven that the rank of the group action of direct product of the symmetric group acting on the Cartesian product of three sets is 8 for all $n \geq 2$ and the suborbits are length 1, $(n-1)$, $(n-1)$, $(n-1)$, $(n-1)^2$, $(n-1)^2$, $(n-1)^2$, $(n-1)^3$. We show that the suborbits of the group action are self-paired. Furthermore, it is demonstrated that each graph has a girth of 3 for all $n > 2$ and suborbital graphs of the group action are undirected. It is shown that graphs Γ_2 and Γ_3 are regular of degree $n-1$, graphs Γ_4 , Γ_5 and Γ_6 of degree $(n-1)^2$ and graph Γ_7 is regular of degree $(n-1)^3$. The suborbital graphs $\Gamma_i (i=1, 2, \dots, 6)$ are disconnected, with the number of connected components equal to n^2 while suborbital graph Γ_7 is connected for all $n > 2$.

Keywords

Rank, Subdegrees, Suborbit, Suborbital Graphs

1. Introduction

The group action of the direct product of the symmetric group on Cartesian product of three sets i.e. $S_n \times S_n \times S_n$ on $X \times Y \times Z$ is defined as $(g_1, g_2, g_3)(x, y, z) = (g_1x, g_2y, g_3z) \forall g_1, g_2, g_3 \in S_n$, $x \in X, y \in Y, z \in Z$ [1]. This paper investigates properties of the suborbital graphs that arise from this group action.

The *degree* of a vertex in any graph is the number of vertices adjacent to that vertex. A vertex is isolated if it has degree 0 [6].

The length of the shortest cycle in graph is called its girth.

A *connected* graph is one where every pair of vertices is connected by a path; otherwise, it is disconnected. A maximal connected subgraph of a graph is a connected component of that graph. In this case every vertex and edge of the graph belongs to exactly one connected component of the graph [2].

If a group G acts transitively on a set X and if Δ is an orbit of the stabilizer of G on X , then $\Delta^* = \{gx | g \in G, x \in \Delta\}$. In this case Δ^* is also an orbit of the stabilizer of G and is called the G -suborbit paired with Δ [7]. Clearly $|\Delta| = |\Delta^*|$ and $\Delta^{**} = \Delta$. If $\Delta^* = \Delta$, then Δ is

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said to be self-paired [8].

Theorem 1.1(Sims Theorem) If Γ_i^* are the suborbital graphs corresponding to the suborbital α_i^* . If the suborbits $\Delta_i (i=0,1,\dots,r-1)$ correspond to the suborbital α_i . Then Γ_i is undirected if Δ_i is self-paired and Γ_i is directed if Δ_i is not self-paired [7].

2. Main Results

2.1. Ranks and Subdegrees

Theorem 2.1 The rank of $G = S_n \times S_n \times S_n$ acting on $X \times Y \times Z$ is 8, for all $n \geq 2$

Proof. Let $X = \{1, 2, \dots, n\}$, $Y = \{n+1, n+2, \dots, 2n\}$ and $Z = \{2n+1, 2n+2, \dots, 3n\}$. The number orbits of $G = S_n \times S_n \times S_n$ acting on $X \times Y \times Z$ are given as follows;

Let $B = \{1, n+1, 2n+1\}$ then

Table 1. The rank of $S_n \times S_n \times S_n$ acting on $X \times Y \times Z$.

Suborbit	Formula	Number of suborbits
Orbit comprising no element from B	3C_0	1
Orbits comprising a single element from B	3C_1	3
Orbits comprising two elements from B	3C_2	3
Orbit comprising three elements from B	3C_3	1

Hence the rank of the rank of $G = S_n \times S_n \times S_n$ acting on $X \times Y \times Z$ is $1 + 3 + 3 + 1 = 8$ [3].

The 8 orbits of $G_{(1,n+1,2n+1)}$ on $X \times Y \times Z$ are:

a) Suborbit whose every element comprises 3 elements from B

$$\Delta_0 = Orb_{G_{(1,n+1,2n+1)}}(1, n+1, 2n+1) = \{(1, n+1, 2n+1)\} \text{-the trivial orbit.}$$

b) Suborbits whose every element contains 2 elements from B

$$\Delta_1 = Orb_{G_{(1,n+1,2n+1)}}(1, n+1, 2n+2) = \{(1, n+1, 2n+2), (1, n+1, 2n+3), \dots, (1, n+1, 3n)\}, n \geq 2.$$

$$\Delta_2 = Orb_{G_{(1,n+1,2n+1)}}(1, n+2, 2n+1) = \{(1, n+2, 2n+1), (1, n+3, 2n+1), \dots, (1, 2n, 2n+1)\}, n \geq 2.$$

$$\Delta_3 = Orb_{G_{(1,n+1,2n+1)}}(2, n+1, 2n+1) = \{(2, n+1, 2n+1), (3, n+1, 2n+1), \dots, (n, n+1, 2n+1)\}, n \geq 2.$$

c) Suborbits whose every element contains three elements from B

$$\Delta_4 = Orb_{G_{(1,n+1,2n+1)}}(1, n+2, 2n+2) = \{(1, n+2, 2n+2), (1, n+3, 2n+3), \dots, (1, 2n, 3n)\}, n \geq 2.$$

$$\Delta_5 = Orb_{G_{(1,n+1,2n+1)}}(2, n+1, 2n+2) = \{(2, n+1, 2n+2), (3, n+1, 2n+3), \dots, (n, n+1, 3n)\}, n \geq 2.$$

$$\Delta_6 = Orb_{G_{(1,n+1,2n+1)}}(2, n+2, 2n+1) = \{(2, n+2, 2n+1), (3, n+2, 2n+1), \dots, (n, 2n, 2n+1)\}, n \geq 2.$$

d) Suborbit containing no element from B

$$\Delta_7 = Orb_{G_{(1,n+1,2n+1)}}(2, n+2, 2n+2) = \{(2, n+2, 2n+2), (3, n+2, 2n+2), \dots, (n, 2n, 3n)\}, n \geq 2.$$

Table 2. The subdegrees of $S_n \times S_n \times S_n$ acting on $X \times Y \times Z$.

Suborbit length	1	$n - 1$	$(n - 1)^2$	$(n - 1)^3$
number of suborbits	1	3	3	1

2.2. Suborbital Graphs of $S_2 \times S_2 \times S_2$ Acting on $X \times Y \times Z$ and Their Properties

Theorem 2.2 All Suborbits of $S_2 \times S_2 \times S_2$ acting on $X \times Y \times Z$ are self-paired.

Proof. It was proven that the number of orbits of $G_{(1,3,5)}$ acting on $X \times Y \times Z$ is 8 orbits. The suborbital graph corresponding to Δ_0 is a null graph. The remaining suborbits $\Delta_1, \Delta_2, \dots, \Delta_7$ are self-paired. Consider for the example

$$\Delta_2 = \{(1, 4, 5)\} \text{ for } (g_1, g_2, g_3) \in G.$$

$$\Rightarrow (g_1, g_2, g_3)(1, 4, 5) = (1, 3, 5)$$

$$\Rightarrow g_1(1) = 1, g_2(4) = 3 \text{ and } g_3(5) = 5, \text{ therefore this group action can be given by } (g_1, g_2, g_3) = (1, (4\ 3), (5)).$$

$$\Rightarrow (g_1, g_2, g_3)(1, 3, 5) = (1, 4, 5) \in \Delta_2 \Rightarrow \Delta_2^* = \Delta_2$$

Therefore, self-paired.

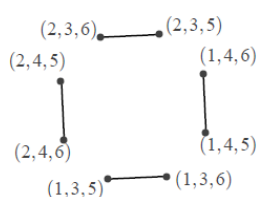
By Theorem 1.1, their corresponding suborbital graphs $\Gamma_1, \Gamma_2, \dots, \Gamma_7$ are undirected due to the fact that all suborbits are self-paired.

The suborbital graph corresponding to Δ_0 is a null graph and the remaining seven suborbital graphs are given as follows;

Assume that A and B are two distinct points from $X \times Y \times Z$. Then the corresponding graphs are constructed as follows;

The suborbital $\Delta_1 = \{(g_1, g_2, g_3)(1, 3, 5)(g_1, g_2, g_3)(1, 3, 6) | (g_1, g_2, g_3) \in G\}$.

Consequently, in graph Γ_1 , the suborbital graph corresponding to Δ_1 , there is an edge from A to B iff the first two coordinates of A are identical to the first two coordinates of B and the third coordinate of A is not identical to the third coordinate of B [5].

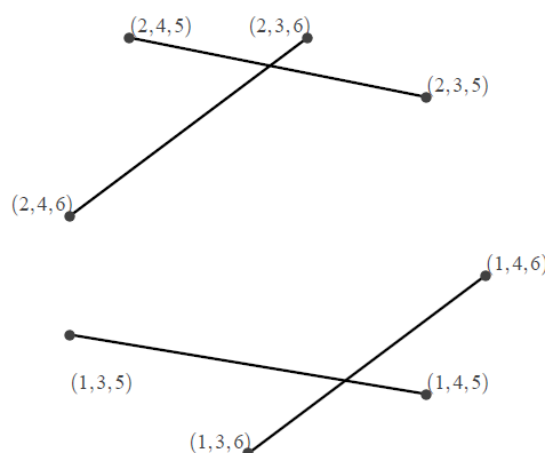
**Figure 1.** The suborbital graph Γ_1 .

Properties of the graph Γ_1

1. The graph has no cycle and hence a girth of zero.
2. The graph is disconnected.
3. The graph is a regular of degree 1.

The suborbital $\Delta_2 = \{(g_1, g_2, g_3)(1, 3, 5)(g_1, g_2, g_3)(1, 4, 5) | (g_1, g_2, g_3) \in G\}$.

Consequently, in graph Γ_2 , the suborbital graph corresponding to Δ_2 , there is an edge from A to B iff the first and third coordinates of A are identical to the first and third coordinates of B and the second coordinate of A is not identical to the second coordinate of B .

**Figure 2.** The suborbital graph Γ_2 .

Properties of graph Γ_2

1. The graph has no cycle and hence has a girth zero.
2. The graph is disconnected.
3. The graph is regular of degree 1.

The suborbital $\Delta_3 = \{(g_1, g_2, g_3)(1, 3, 5)(g_1, g_2, g_3)(2, 3, 5) | (g_1, g_2, g_3) \in G\}$.

Therefore, in graph Γ_3 , the suborbital graph corresponding to Δ_3 , there is an edge from A to B iff the last two coordinates of A are identical to the last two coordinates of B and the first coordinate of A is not identical to the first coordinate of B .

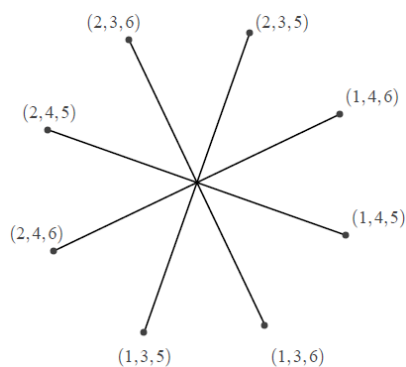


Figure 3. The suborbital graph Γ_3 .

Properties of graph Γ_3

- i. The graph has a girth zero.
- ii. The graph is disconnected.
- iii. The graph is regular of degree 1.

The

suborbital

$$o_4 = \{(g_1, g_2, g_3)(1,3,5)(g_1, g_2, g_3)(1,4,6) | (g_1, g_2, g_3) \in G\}$$

Therefore, in graph Γ_4 , the suborbital graph corresponding to o_4 , there is an edge from A to B iff the first coordinate of A is identical to the first coordinate of B and the last two coordinates of A are not identical to the last two coordinates of B .

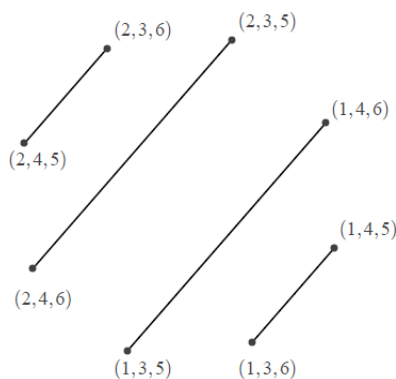


Figure 4. The suborbital graph Γ_4 .

Properties of graph suborbital Γ_4

1. The graph has a girth zero.
2. The graph is disconnected.
3. The graph is regular of degree 1.

The

suborbital

$$o_5 = \{(g_1, g_2, g_3)(1,3,5)(g_1, g_2, g_3)(2,3,6) | (g_1, g_2, g_3) \in G\}$$

Consequently, in graph Γ_5 , the suborbital graph corresponding to o_5 , there is an edge from A to B iff the second coordinates of A are identical to the second coordinate of B and the first and third coordinates of A are not identical to the first and third coordinates of B .

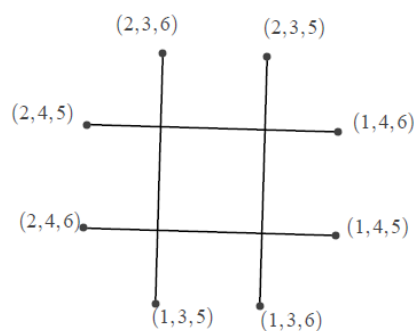


Figure 5. The suborbital graph Γ_5 .

Properties of graph Γ_5

1. The graph has a girth zero.
2. The graph is disconnected.
3. The graph is regular of degree 1.

The

suborbital

$$o_6 = \{(g_1, g_2, g_3)(1,3,5)(g_1, g_2, g_3)(2,4,5) | (g_1, g_2, g_3) \in G\}$$

Thus, in graph Γ_6 , the suborbital graph corresponding to o_6 , there is an edge from A to B iff the last coordinate of A is identical to the last coordinate of B and the first two coordinates of A are not identical to the first two coordinates of B .

The properties of the graph Γ_6 are; the graph has no cycle and hence has a girth zero. It is disconnected, regular of degree 1.

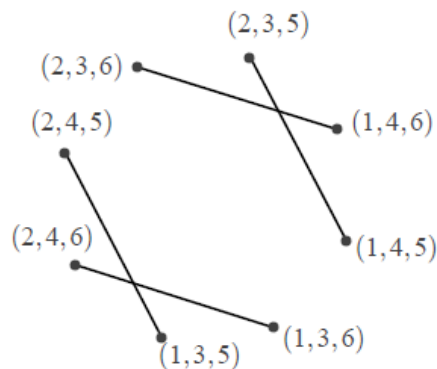


Figure 6. The suborbital graph Γ_6 .

The

tal $o_7 = \{(g_1, g_2, g_3)(1,3,5)(g_1, g_2, g_3)(1,4,6) | (g_1, g_2, g_3) \in G\}$ Thus in graph Γ_7 , the suborbital graph corresponding to o_7 , there is an edge from A to B iff there is no coordinate of A is identical to any coordinate of B [4].

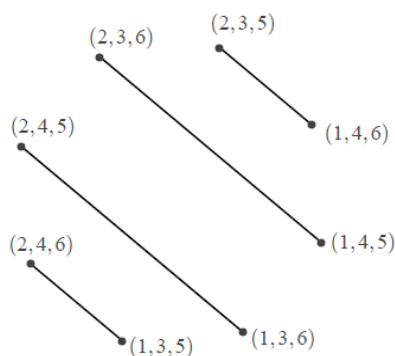


Figure 7. The suborbital graph Γ_7 .

Properties of The graph Γ_7

- The graph has no cycle and hence has a girth zero.
- It is disconnected.
- It is regular of degree 1.

2.3. Suborbital Graphs of $S_3 \times S_3 \times S_3$ Acting on $X \times Y \times Z$ and their Properties

As seen in the previous section, the suborbital graph corresponding to Δ_0 is a null graph and the remaining seven suborbital graphs of $S_3 \times S_3 \times S_3$ Acting on $X \times Y \times Z$ are given as follows;

Assume that A and B are two distinct points from $X \times Y \times Z$. Then the graphs are constructed as follows;

The suborbital $\sigma_1 = \{(g_1, g_2, g_3)(1, 4, 7)(g_1, g_2, g_3)(1, 4, 8) | (g_1, g_2, g_3) \in G\}$.

Therefore, in graph Γ_1 , the suborbital graph corresponding to σ_1 , there is an edge from A to B iff the first two coordinates of A are identical to the first two coordinates of B and the third coordinate of A is not identical to the third coordinate of B .

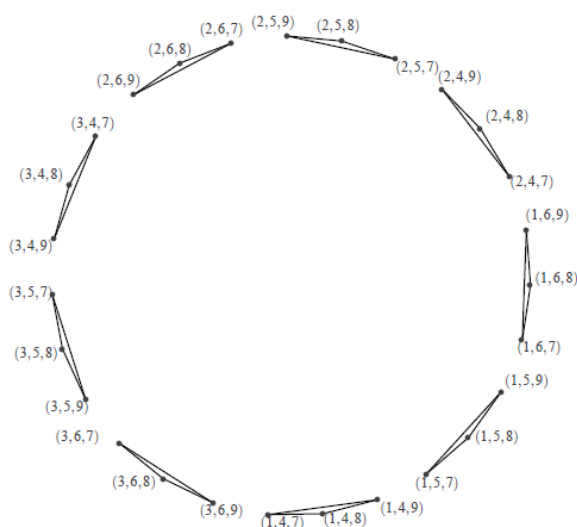


Figure 8. The suborbital graph Γ_1 .

Properties of the graph Γ_1

- The graph is disconnected graph with 9 connected components.
- It is regular of degree 2
- It has a girth three.

The suborbital $\sigma_2 = \{(g_1, g_2, g_3)(1, 4, 7)(g_1, g_2, g_3)(1, 5, 7) | (g_1, g_2, g_3) \in G\}$.

Therefore, in graph Γ_2 , the suborbital graph corresponding to σ_2 , there is an edge from A to B iff the first and third coordinates of A is identical to the first and third coordinates of B and the second coordinate of A is not identical to the second coordinate of B .

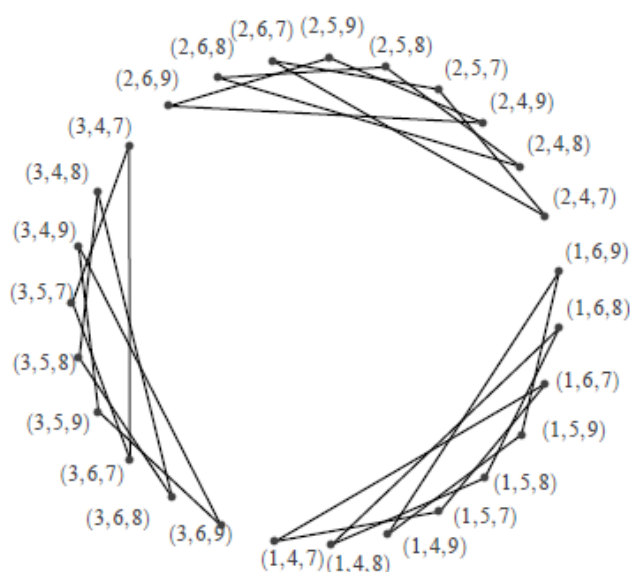


Figure 9. The suborbital graph Γ_2 .

Properties of the graph Γ_2

- The graph is disconnected graph with 9 connected components
- It is regular of degree 2
- It has a girth three.

The suborbital $\sigma_3 = \{(g_1, g_2, g_3)(1, 4, 7)(g_1, g_2, g_3)(2, 4, 7) | (g_1, g_2, g_3) \in G\}$.

Therefore, in graph Γ_3 , the suborbital graph corresponding to σ_3 , there is an edge from A to B iff the last two coordinates of A are identical to the last two coordinates of B and the first coordinate of A is not identical to the first coordinate of B .

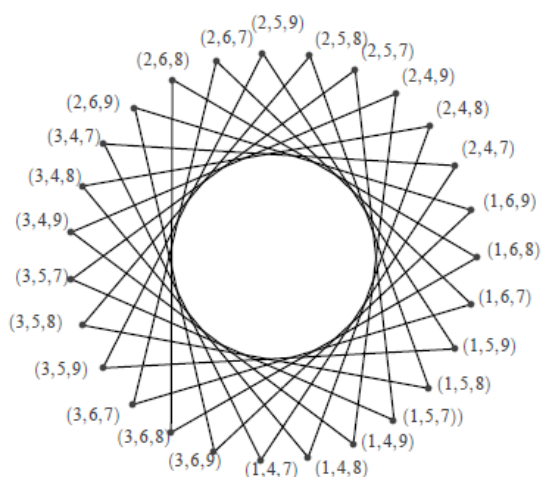


Figure 10. The suborbital graph Γ_3 .

Properties of the graph Γ_3

1. The graph is disconnected graph with 9 connected components.
2. The graph is regular of degree 2.
3. The graph has a girth three.

The

suborbital

$$o_4 = \{(g_1, g_2, g_3)(1, 4, 7)(g_1, g_2, g_3)(1, 5, 8) | (g_1, g_2, g_3) \in G\}$$

Therefore, in graph Γ_4 , the suborbital graph corresponding to o_4 , there is an edge from A to B iff the first coordinate of A is identical to the first coordinate of B and the last two coordinates of A are not identical to the last two coordinates of B.

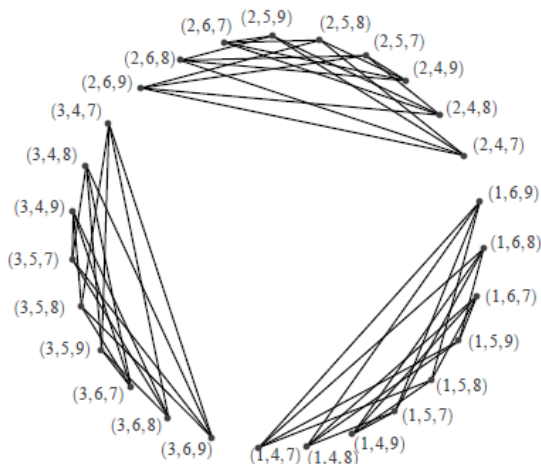


Figure 11. The suborbital graph Γ_4 .

Properties of the graph Γ_4

1. The graph is disconnected graph with 9 connected components
2. It is regular of degree 4.
3. It has a girth three.

The

suborbital

$$o_5 = \{(g_1, g_2, g_3)(1, 4, 7)(g_1, g_2, g_3)(2, 4, 8) | (g_1, g_2, g_3) \in G\}$$

$G\}$

Thus in Γ_5 , the suborbital graph corresponding to o_5 , there is an edge from A to B iff the second coordinates of A are identical to the second coordinate of B and the first and third coordinates of A are not identical to the first and third coordinates of B.

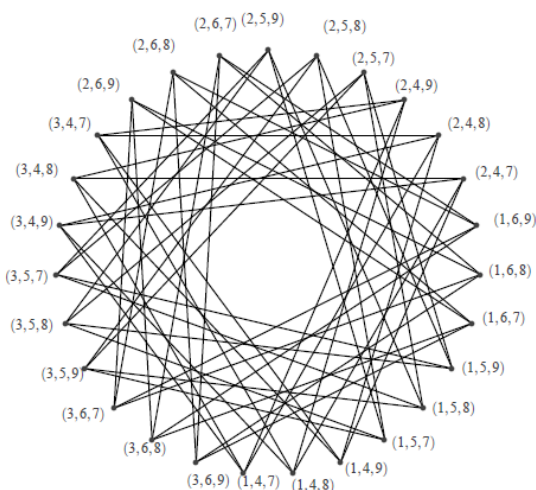


Figure 12. The suborbital graph Γ_5 .

Properties of the graph Γ_5

1. The graph is disconnected.
2. It is regular of degree 4.
3. It has a girth three.

The

suborbital

$$o_6 = \{(g_1, g_2, g_3)(1, 4, 7)(g_1, g_2, g_3)(2, 5, 7) | (g_1, g_2, g_3) \in G\}$$

Thus in Γ_6 , the suborbital graph corresponding to o_6 , there is an edge from A to B iff the last coordinate of A is identical to the last coordinate of B and the first two coordinates of A is not identical to the first two coordinates of B.

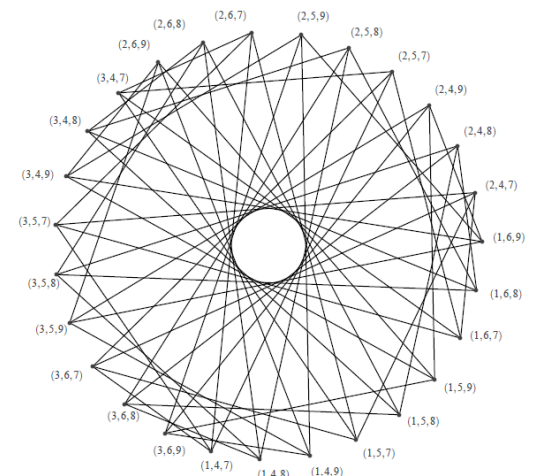


Figure 13. The suborbital graph Γ_6 corresponding to the suborbit Δ_6 of $S_3 \times S_3 \times S_3$ acting on $X \times Y \times Z$.

Properties of the graph Γ_6

1. The graph is disconnected.
2. It is regular of degree 4.
3. It has a girth three.

The

suborbital

$$o_7 = \{(g_1, g_2, g_3)(1, 4, 7)(g_1, g_2, g_3)(2, 5, 8) | (g_1, g_2, g_3) \in G\}$$

Thus in Γ_7 , the suborbital graph corresponding to o_7 , there is an edge from A to B iff there is no coordinate of A is identical to any coordinate of B.

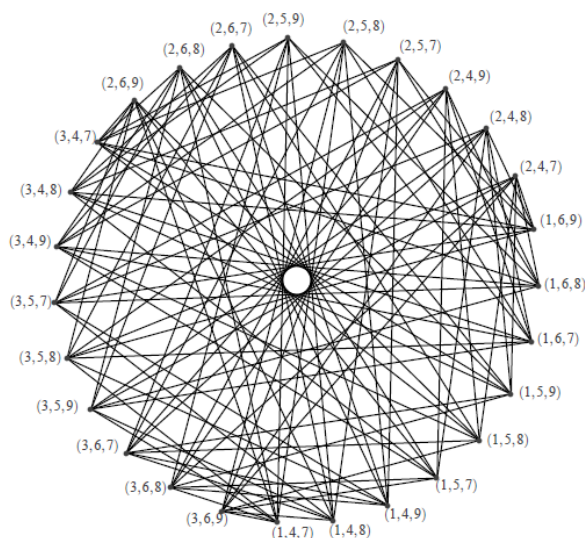


Figure 14. The suborbital graph Γ_7 .

Properties of the graph Γ_7

1. the graph is connected.
2. It is regular of degree.
3. It has a girth three.

2.4. Suborbital Graphs of $S_n \times S_n \times S_n$ Acting on $X \times Y \times Z$ and Their Properties

Theorem 2.4.1 Suborbits $\Delta_1, \Delta_2, \dots, \Delta_7$ of $S_n \times S_n \times S_n$ acting on $X \times Y \times Z$ are self-paired.

Proof. The proof of Δ_0 is trivial.

Abbreviations

Δ	The Suborbit of G on X
Δ^*	The G – suborbit paired with Δ
Γ	The Suborbital Graph Corresponding to the Suborbit Δ
$X \times Y \times Z$	The Cartesian product of sets X, Y and Z
$S_n \times S_n \times S_n$	External direct product of S_n

Consider $\Delta_1 = \{(1, n+1, 2n+2), \dots, (1, n+1, 3n)\}$

$\forall n \geq 2$ is self-paired for if $(g_1, g_2, g_3)\{(1, n+1, 2n+2), \dots, (1, n+1, 3n)\}$. Consider $g_3\{2n+2, 2n+3, \dots, 3n\}$ since the rest are constant. Thus $g_3\{2n+2, 2n+3, \dots, 3n\} = \{1, 2, \dots, n-1\}$, then take g_3 to be $(2n+21)(2n+32), \dots, (3nn-1)$ and $g_3\{1, 2, \dots, n-1\} = \{2n+2, 2n+3, \dots, 3n\} \in \Delta_1$. This shows that Δ_1 is self-paired.

Similarly, consider $\Delta_2 = \{(1, n+2, 2n+1), \dots, (1, 2n, 2n+1)\} \forall n \geq 2$. If

$(g_1, g_2, g_3)\{(1, n+2, 2n+1), \dots, (1, 2n, 2n+1)\}$ Consider $g_2\{2n+2, 2n+3, \dots, 3n\}$ since the rest are constant. Thus $g_2\{2n+2, 2n+3, \dots, 3n\} = \{1, 2, \dots, n-1\}$, then take g_2 to be $(2n+21)(2n+32), \dots, (3nn-1)$ and $g_2\{1, 2, \dots, n-1\} = \{2n+2, 2n+3, \dots, 3n\} \in \Delta_2$.

This show that Δ_2 is also self-paired. With similar arguments $\Delta_3, \Delta_4, \Delta_5$ and Δ_6 are self-paired.

Finally, consider

$\Delta_7 = \{(2, n+2, 2n+2), \dots, (n, 2n, 3n)\} \forall n \geq 2$. If $(g_1, g_2, g_3)\{(2, n+2, 2n+2), \dots, (n, 2n, 3n)\}$ consider $g_1\{2, 3, \dots, n\}$ since for the cases of g_2 and g_3 are already shown that are self-paired. Thus $g_1\{2, 3, \dots, n\} = \{1, 2, \dots, n-1\}$, then take g_1 to be $(21), (32), \dots, (nn-1)$ and $g_1\{1, 2, \dots, n-1\} = \{2, 3, \dots, n\} \in \Delta_7$. Showing that Δ_7 is a self-paired.

Theorem 2.4.2 All the suborbitals graphs of G are undirected.

Proof. From Theorem 1.1 and Theorem 2.4.1 the following results are straight forward due to the fact that all suborbits are self-paired.

Corollary 2.4.3 Let $G = S_n \times S_n \times S_n$ act on $X \times Y \times Z$ then the suborbital graph $\Gamma_i (i = 1, 2, \dots, 6)$ is disconnected with the number of connected components equal to n^2 and suborbital graph Γ_7 is connected for all $n > 2$.

Corollary 2.4.4 Let $G = S_n \times S_n \times S_n$ act on $X \times Y \times Z$ then the suborbital graph $\Gamma_i (i = 1, 2, \dots, 7)$ has a girth of 3 if $n > 2$ and a girth is zero if $n=2$.

Corollary 2.4.5 Let $G = S_n \times S_n \times S_n$ act on $X \times Y \times Z$ graphs Γ_1, Γ_2 and Γ_3 are regular of degree $n-1$, graphs Γ_4, Γ_5 and Γ_6 are regular of degree $(n-1)^2$ and graph Γ_7 is regular of degree $(n-1)^3$. This agrees with subdegrees of $S_n \times S_n \times S_n$ acting on $X \times Y \times Z$.

Conflicts of Interest

The authors declare no conflicts of interest.

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