

Research Article

# An Analysis of Influencing Factors for Vapour Transfer in Freezing Soils in High-filling Engineering

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## Abstract

Canopy effect has been paid more and more attention in high-filling engineering in cold and arid regions of China. Vapour transfer is considered as the main cause of canopy effect in freezing soil in the literature. However, the influencing factors of vapour transfer in freezing soils have not been systematically analyzed in the literature. Based on the coupled heat and mass transfer model proposed by the authors, the effects of hydraulic parameters and environmental factors on vapour transfer in freezing soils will be analyzed in this paper. The results show that the effects of the hydraulic parameters, such as the fitting parameters of the soil water characteristic curve (SWCC) and the saturated hydraulic conductivity, on the vapour transfer and the total water content are significant, even if the values of these parameters vary within a quite small range. The temperature gradient, the cooling rate, the water flux at the top and the sealing conditions at the bottom can also lead to an increment of the total volumetric water content. Therefore, these hydraulic parameters and environmental factors can all promote vapour transfer under suitable conditions. The effects of the terms related to vapour transfer in the governing equations on the total water volumetric content are also analyzed. In total, the water increment caused by vapour transfer is large, which can then cause frost damage in silt. The research results in this paper are helpful to understand the influence of factors for canopy effect and also have a great significance for guiding the design and maintenance of high-filling engineering.

## Keywords

Vapour Transfer, Water Content, Hydraulic Parameter, Environmental Factor, Canopy Effect, High-filling Engineering

## 1. Introduction

High-filling engineering which is generally defined as fill height greater than 20m, such as airports, highway and railway, have been widely built in the mountainous areas of China. These engineering have high requirements for deformation control, and it is necessary to avoid significant effects caused by frost heave. In the arid and cold areas of northwest China, however, it is found that the water content under an impervious cover of an airport increased significantly in freezing soils, which is called as the canopy effect by Li et al.

[1]. Similar phenomena have also been found in highway embankment [2] and high-speed railway embankment [3]. The concrete pavement of an airport, the pavement of an highway and the track board of a high-speed railway can all be regarded as an impervious cover plate which is an important prerequisite for canopy effect. When the soil temperature is lower than the freezing temperature, the high moisture content will cause significant frost heaving, threatening the safety of high-filling engineering. Vapour transfer is considered as an

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important and main reason for these phenomena [1-3].

Experimental results in the literature show that vapour transfer in unsaturated freezing soils can lead to a significant increase in water content [4-7]. Similar experimental results have been obtained as well [8-10]. The freezing test results show that the vapour transfer can even cause significant frost heave in the coarse soil [11, 12]. However, many theoretical models in the literature ignore or neglect the effect of vapour transfer [13-26]. By establishing the relationship between unfrozen water content, temperature, matrix suction and SWCC, Zhang et al. proposed a three-phase coupling model of liquid water-heat-vapour transfer considering multiple phase transitions [4]. The results obtained by a one-dimensional freezing test of calcareous sand also verify the rationality of this model for vapour transfer [4]. Based on this model, a frost heave model of coarse-grained soil is also improved by Teng et al. [11, 12]. However, these models have only been validated with coarse-grained soils and cannot explain why canopy effect is more common in silt than in clay and sand, which is also true of other models [7, 27, 28]. By improving the calculation of unfrozen water content, the model proposed by the authors can deal with this problem quite well, which is also verified by the analysis of clay, silt and sand [29].

Although theoretical and experimental studies on vapour transfer in freezing soils have been carried out, the research on its influencing factors, including hydraulic parameters and environmental factors, is still insufficient. Therefore, the existing studies on influencing factors of vapour transfer are reviewed and then analyzed in the following paragraphs.

The hydraulic parameters are the main intrinsic factors for vapour transfer, including the SWCC, the saturated hydraulic conductivity, the hydraulic conductivity in the frozen area and the vapour diffusion coefficient [4]. Unquestionably, the vapour diffusion coefficient is a parameter directly related vapour transfer. However, since this parameter is difficult to obtain through test, and its value is greatly affected by subjective factors, so this paper does not carry out research. Although the SWCC and the hydraulic conductivity cannot directly affect vapour transfer, evaporation and condensation will inevitably occur between liquid water and vapour. For example, liquid water will turn into vapour under the effect of the temperature, changing the relative humidity in soils. Vapour will then transfer under this relative humidity gradient. Additionally, the relative humidity is usually considered as a function of the suction in the theory for unsaturated soils. The isothermal and thermal hydraulic conductivities of vapour due to the water pressure head and the temperature are also influenced by the suction in freezing soils [4, 16-19]. Therefore, there is no doubt that these hydraulic parameters all have an impact on vapour transfer. Yao and Wang [30] conducted a study on the influencing factors of canopy effect through a numerical model based on the model proposed by Zhang et al.

[4], including initial water content, freezing period, boundary temperature and isolating layer. It is noted that the shortcomings of this proposed model have been explained in the previous paragraph, so the conclusions obtained are open to question. Therefore, it is necessary to analyze the influence of hydraulic parameters on canopy effect through the model proposed by the authors.

Environmental factors are also quite important to vapour transfer. The ground water table, the freezing period, the temperature, the temperature gradient, the cooling rate, the ground water table, and the sealing condition at the top and bottom of the soil column are the typical factors in practical engineering. It has been known that vapour transfer plays a major role when the initial water content is quite low, especially in freezing soils. And the suction and the temperature are the two fundamental factors to both of liquid water transfer and vapour transfer [4, 29]. The two factors are also chosen as the independent variables in the governing equations for the moisture-heat coupling in freezing soils [17]. These environmental factors can change the temperature or the water content (i.e. suction or water head) distribution and thus have effects on the vapour transfer. Through numerical analysis, He et al. [29] has analyzed the influence of some of these environmental factors on vapour transfer and total water content. Although some meaningful results have been obtained, there are still some factors whose influence has not been studied. In addition, the governing equations of the model proposed by He et al. [29] have strong nonlinearity, where the influence of some terms related to temperature and matric suction is also unclear. Research on this will help to distinguish the magnitude of impacts and thus improve computational and analytical efficiency.

In a word, the research on the influencing factors of canopy effect is insufficient. Based on the model proposed by He et al. [29], this paper will systematically analyze the effects of the hydraulic parameters and the environmental factors on vapour transfer and total water content. And the effects of the terms related to vapour transfer in the governing equations will be analyzed as well. This work will help to deepen the understanding of canopy effect and provide reference for frost heave prevention and deformation control measures of high-filling engineering in cold and arid regions of China.

## 2. A Brief Introduction of the Mathematical Model

In unsaturated freezing soils, the transfer of liquid water and vapour is affected by several effects, such as matrix potential, temperature potential and gravity potential. Liquid water transfer obeys the Darcy's law and vapour transfer obeys the Fick's law. The mass conservation equation can be then expressed as follows [17, 18]

$$\frac{\partial \theta_w}{\partial t} + \frac{\partial \theta_v}{\partial t} + \frac{\rho_i}{\rho_w} \frac{\partial \theta_i}{\partial t} = \frac{\partial}{\partial z} \left[ K'_{wh} \left( \frac{\partial h}{\partial z} + 1 \right) + K_{wT} \frac{\partial T}{\partial z} + K_{vh} \frac{\partial h}{\partial z} + K_{vT} \frac{\partial T}{\partial z} \right] \quad (1)$$

where  $\theta_w$ ,  $\theta_v$  and  $\theta_i$  are the liquid water content, the equivalent vapour content and the pore ice content, respectively.  $\rho_w$  ( $=1000 \text{ kg/m}^3$ ) and  $\rho_i$  ( $=916 \text{ kg/m}^3$ ) are the liquid water density and the ice density, respectively.  $h$  (m) and  $T$  (K) are the water pressure head and the temperature, respectively.  $K'_{wh}$  (m/s) and  $K_{wT}$  ( $\text{m}^2/\text{K/s}$ ) are the isothermal and thermal hydraulic conductivities of liquid water due to the water pressure head and the temperature, respectively.  $K_{vh}$  (m/s) and  $K_{vT}$

( $\text{m}^2/\text{K/s}$ ) are the isothermal and thermal hydraulic conductivities of vapour due to the water pressure head and the temperature, respectively.  $z$  (m) is the spatial coordinate positive upward.  $t$  (s) is the time.

There are three forms of phase change in freezing soils, namely evaporation, condensation and sublimation. The energy conservation equation considering the three forms can be expressed as follows [15-17, 31-32]

$$C_p \frac{\partial T}{\partial t} - L_i \rho_i \frac{\partial \theta_i}{\partial t} + L_v \rho_w \frac{\partial \theta_v}{\partial t} = \frac{\partial}{\partial t} \left[ \lambda \frac{\partial T}{\partial z} \right] - C_w \frac{\partial (q_w T)}{\partial z} - C_v \frac{\partial (q_v T)}{\partial z} - L_v \rho_w \frac{\partial q_v}{\partial z} \quad (2)$$

where  $C_p$  ( $\text{J/m}^3/\text{K}$ ) and  $\lambda'$  ( $\text{W/m/K}$ ) are the equivalent volumetric heat capacity and the thermal conductivity considering soil skeleton, liquid water, vapour and ice, respectively (Wu et al., 2015; Sakai et al., 2009).  $C_w$  ( $=4.18 \times 10^6 \text{ J/m}^3/\text{K}$ ) and  $C_v$  ( $=6.3 \times 10^3 \text{ J/m}^3/\text{K}$ ) are the heat capacities of liquid water and vapour, respectively.  $L_i$  ( $=3.34 \times 10^5 \text{ J/kg}$ ) and  $L_v$  ( $\text{J/kg}$ ) are the latent heats of water freezing and vaporization, respectively.  $q_w$  (m/s) and  $q_v$  (m/s) are the liquid water flux and vapour flux, respectively.

The criterion of pore ice crystal formation can be determined by the following formula [29]

$$\theta_i = \begin{cases} 0 \\ \theta_w - \theta_u (T < T_f \text{ and } \theta_w > \theta_u) \end{cases} \quad (3)$$

where  $\theta_u$  is the maximum unfrozen water content.  $T_f$  (K) is the freezing temperature. Based on the thermodynamic equilibrium theory, the relationship between the maximum matrix potential and temperature in freezing soils is given by [33-37]

$$h_u = L_i \frac{T - T_f}{g T_f} (T < T_f) \quad (4)$$

where  $g$  ( $=9.8 \text{ m/s}^2$ ) is the gravitational acceleration. And  $h_u$  (m) is the maximum water pressure head which is consistent with the maximum unfrozen water content.

These equations constitute the main framework for the model of liquid water-vapour-heat transfer in unsaturated freezing soils. In this model, the method of pore ice content and maximum unfrozen water content are established respectively, which can only be expressed as functions of two independent variables, i.e. maximum matric potential and temperature. The detailed derivation, validation of this model and effects of the soil type can refer to the author's previous research results [29]. Additional details of this model are

provided in the Appendix. This model can then be used to analyze the effects of hydraulic parameters, environmental factors and other factors on vapour transfer and the total water content.

### 3. An Analysis of the Hydraulic Parameters

Canopy effect is usually observed in silt [1], so a kind of silt which is widely used in high-filling engineering in north-western China is chosen for analysis in this paper. The basic physical properties of silt obtained through test by Zhang et al [4, 38] are shown in Table 1.  $\theta_s$  and  $\theta_r$  are the saturated and residual water content.  $\alpha$  (1/m),  $n$  and  $m$  ( $=1-1/n$ ) are the fitting parameters of the SWCC.  $K_s$  (m/s) is the saturated hydraulic conductivity.  $l$  ( $=0.5$ ) is the fitting parameter in Mualem model [23]. The initial conditions and the boundary conditions are shown in Table 2, which represent the typical hydrogeology and climatic conditions where canopy effect occurs. The values of initial temperature and upper and lower boundary temperature of soil column are also adopted in other literature [4-5, 11-12, 29]. These basic physical properties and conditions also correspond to that measured, obtained and then adopted by Zhang et al. [4]. The analysis condition is shown in Table 3. Soil-water characteristic curve is closely related to moisture transfer and is the focus in this section. It should be noted that the hydraulic conductivity in the frozen area is hardly to measure and its theoretical expression has been not widely recognized [17]. And the vapour diffusion coefficient in a soil is also hardly to be obtained by test although its effect on vapour transfer is obviously direct [18]. Therefore, the two factors will be not considered in this section. In addition to hydraulic parameters, there may be other internal factors in Table 1 affecting water vapor transfer, but they are not within the scope in this analysis due to their unknown influence mechanism.

**Table 1.** Basic physical properties of silt (unchanged).

$\theta_s$	$\theta_r$	$m$	$l$	$\alpha(1/m)$	$n$	$K_s(\times 10^{-6} \text{ m/s})$	$\rho_d(\text{g/cm}^3)$	Liquid limit (%)	Plastic limit (%)
0.49	0.065	1-1/n	0.5	0.546	2.32	2.55	1.61	24.8	15.5

**Table 2.** Initial conditions and boundary conditions.

Initial water content (%)	Initial temperature (°C)	Temperature at the top(°C)	Temperature at the bottom(°C)
16	15	-10	15
Freezing period(d)	Height(m)	The ground water table(m)	Water flux at the top(m/s)
90	20	0	0

**Table 3.** Analysis condition.

Case	$\alpha(1/m)$	$n$	$K_s(\times 10^{-6} \text{ m/s})$
1	0.4	2.32	2.55
2	0.546	2.32	2.55
3	0.8	2.32	2.55
4	0.546	2	2.55
5	0.546	2.7	2.55
6	0.546	2.32	1
7	0.546	2.32	4.1

### 3.1. The Effect of Parameter $\alpha$

Parameter  $\alpha$  is one of the fitting parameters of soil-water characteristic curve. The slope of the SWCC curve is controlled by parameter  $\alpha$  and a larger value of this parameter can generate a flatter curve, which can change the water holding characteristics of soils and then influence the moisture transfer. The test results of parameter  $\alpha$  and its simulated results by van Genuchten (VG) model [39] are shown in Figure 1. The values of parameter  $\alpha$  are 0.4, 0.546 and 0.8, corresponding to case 1, 2 and 3 in Table 3, respectively. The other fitting parameters for the SWCC all remain unchanged. It can be seen that the simulated results all agree well with the test results. It should be noted that the change of parameter  $\alpha$  must ensure that the difference between the simulated result and the test result lies within a reasonable range because a great difference means that the type of the soil changes. For example, the SWCC will remarkably shift to the right or the left if parameter  $\alpha$  dramatically decreases or increases, which means that to achieve the same level of the water content, the suction will be quite larger or smaller. And the other fitting parameters

will change as well. This is obviously unreasonable for the same type of soil.

The effect of parameter  $\alpha$  on the total water content is shown in Figure 2. The total water content is defined as the sum of the liquid water content, the vapour content and the ice content. And no vapour means that vapour transfer is not considered at all in the governing equations. This paper mainly focuses on the total water content at the top, corresponding to the main characteristic of canopy effect, so the curves are shown only within the top several meters. It can be seen that a larger value of parameter  $\alpha$  can lead to a greater total water content with considering vapour transfer. A similar trend is also obtained without considering vapour transfer. Additionally, the increased amount of the total water content due to vapour transfer in frozen area also increases with the increase of parameter  $\alpha$ , so does the total increased amount. This increased amount is defined as that the difference between the total water content with and without considering the vapour transfer at a height. And the total increased amount is defined as the sum of the increased amount in the whole frozen area. This difference may be caused by human factors, instrument factors and environmental factors respectively or together, which is inevitable and easy to be ignored. However, because the errors are within a reasonable range, the fitting of the model is still acceptable.

The test for the SWCC and its fitting result predicted by VG model all have certain errors. For example, with considering vapour transfer, the maximum total water contents when the values of parameter  $\alpha$  are 0.4 and 0.8 are 0.21 and 0.24, respectively. This difference reaches 0.03 which is almost equal to that due to vapour transfer with the same value of parameter  $\alpha$ . Therefore, Figure 2 suggests that even though this error is small, a large change of the total water content still occurs. This means that a more reasonable fitting parameter should be selected based on the test result, which requires that more attentions should be paid to in future.

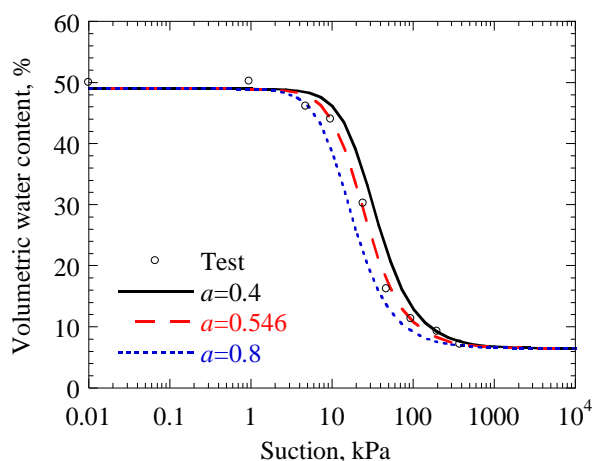


Figure 1. SWCC with parameter  $\alpha$  changing.

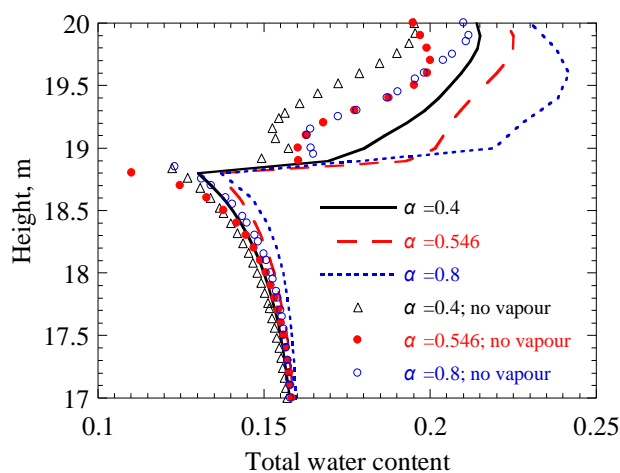


Figure 2. Effects of parameter  $\alpha$  on the total water content.

### 3.2. The Effect of Parameter $n$

Parameter  $n$  is the other of the fitting parameters of soil-water characteristic curve. This parameter controls the shape of SWCC curve and influences the moisture transfer as well. The test results of parameter  $n$  and its simulated results by VG model [39] are shown in Figure 3. The values of parameter  $n$  are 2, 2.32 and 2.7, corresponding to case 2, 4 and 5 in Table 3, respectively. The other conditions also remain unchanged. Similarly, the change of parameter  $n$  must also ensure that the difference between the test result and the simulated result is within a reasonable range. If this change is quite great, it may change the other soil properties as well.

The effect of parameter  $n$  on the total water content is shown in Figure 4. It can be seen that a larger value of parameter  $n$  will lead to a larger total water content no matter whether vapour transfer is considered. It is also obvious that the total water content is greater with considering vapour transfer than that without considering vapour transfer. And the increased amount of the total water content due to vapour

transfer in frozen area also increases with the increase of parameter  $n$ , so does the total increased amount. Figure 4 also suggests that even though the error of parameter  $n$  between the test result and the simulated result is small, a large change of the total water content still occurs. Similar to parameter  $\alpha$ , the fitting of parameter  $n$  should also be in a good agreement with the test result. A good fitting result is quite helpful to avoid non-negligible errors in the analysis of vapour transfer.

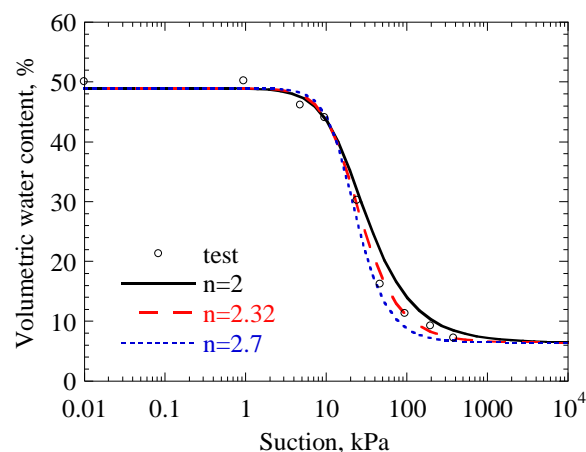


Figure 3. SWCC with parameter  $n$  changing.

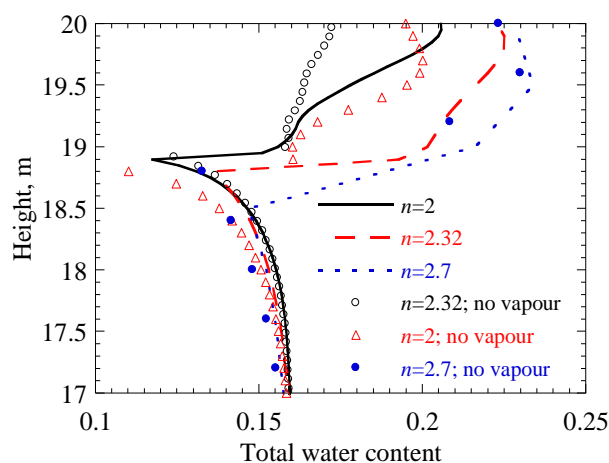


Figure 4. Effects of parameter  $n$  on the total water content.

### 3.3. The Effect of the Saturated Hydraulic Conductivity

The saturated hydraulic conductivity is also an important parameter for moisture transfer. This parameter is related to soil property and pore ratio. The higher the value of this parameter, the more channels that allow moisture to migrate. The effect of the saturated hydraulic conductivity on the total water content is shown in Figure 5. The values of the saturated hydraulic conductivity are  $1 \times 10^{-6}$  m/s,  $2.55 \times 10^{-6}$  m/s and  $4.1 \times 10^{-6}$  m/s, corresponding to case 2, 6 and 7 in



Table 3, respectively. The orders are all the same and also correspond to that in silt. Otherwise, a quite larger or smaller order will be more related to sand or clay. Figure 5 shows that the total water content will increase with the saturated hydraulic conductivity increasing no matter whether vapour transfer is considered. And the total increased amount has a similar trend as well, which means that the saturated hydraulic conductivity can also increase the water content due to vapour transfer. The difference of the value of permeability coefficient analyzed in this paper is quite small and this difference can even be caused by the test error. The results, however, suggest that a small change of the hydraulic conductivity will also have an obvious effect on the total water content, the increased amount and the total increased amount. This kind of influence was easily ignored in the past and should be paid enough attention in the future.

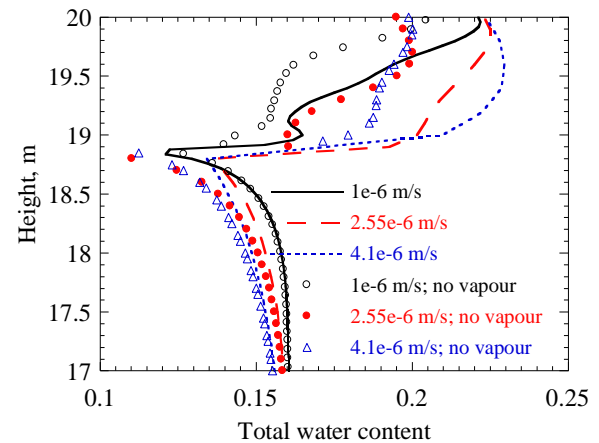


Figure 5. Effects of the saturated hydraulic conductivity on the total water content.

## 4. An Analysis of the Environmental Factors

Table 4. Environmental conditions.

Case	Temperature gradient ( °C/m)	Cooling rate ( °C/h)	Vapour flux at the top (m/s)	Sealing condition at the bottom
1	0.8	$\infty$	0	open
2	1.25	$\infty$	0	open
3	0.8	0.1	0	open
4	0.8	$\infty$	$1 \times 10^{-8}$	open
5	0.8	$\infty$	$1 \times 10^{-10}$	open
6	0.8	$\infty$	$1 \times 10^{-12}$	open
7	0.8	$\infty$	0	closed

Environmental factors are important extrinsic factors that affect the moisture transfer in freezing soils, including the temperature gradient, the cooling rate, the ground water table, the freezing period, the vapour flux at the top and the sealing condition at the bottom. The effects of the ground water table and the freezing period on the total water content and the vapour transfer have been analyzed by He et al. [29], so only the other four environmental factors are analyzed here.

The environmental conditions are shown in Table 4.  $\infty$  represents the value of the cooling rate is infinite, which means that the temperature is imposed instantaneously. The vapour flux at the top boundary is the transfer rate of vapour. And a positive or a negative value of the vapour flux indicates that the transfer direction is inward and outward, respectively. The outward vapour flux will reduce soil moisture and then weaken the canopy effect, which is not analyzed in this paper.

In the sealing condition, open indicates that the ground water table locates here and close means that there is no water supply at all. The other conditions are the same as that of case 2 in Table 3.

### 4.1. The Effect of the Temperature Gradient

The fluctuation of the temperature in Winter will change the surface temperature and then cause different temperature gradients in soils. Temperature gradient is one of the main factors to vapour transfer and is then needed to analyze. The effect of the temperature on an enough deep soil is negligible, so it can be assumed that the temperature at the bottom of the simulated soil column remains constant. The temperature gradient can then be obtained by setting a temperature at the top. Although the temperature gradient of shallow soil is generally large and the temperature gradient of deep soil is

quite small, the influence depth of temperature is limited and the difference of the influence depth is small for the same soil. Therefore, the average temperature gradient is reasonable and has also practical engineering significance. The temperature gradients are  $0.8\text{ }^{\circ}\text{C/m}$  and  $1.25\text{ }^{\circ}\text{C/m}$ , respectively. The other conditions are the same as that of case 1 and 2 in Table 4.

The effect of the temperature gradient on the total water content is shown in Figure 6. It can be seen that the maximum total water content when the temperature gradient is  $0.8\text{ }^{\circ}\text{C/m}$  is about 0.19, smaller than that when the temperature gradient is  $1.25\text{ }^{\circ}\text{C/m}$ . A smaller temperature gradient will lead to a lower freezing front and a smaller ice content as well. And vapour transfer has few effects on the total water content when the temperature gradient is quite low. Additionally, if the vapour transfer is not considered at all, a similar trend is also observed. And it can be seen that the total increased amount increases with the temperature gradient increasing as well.

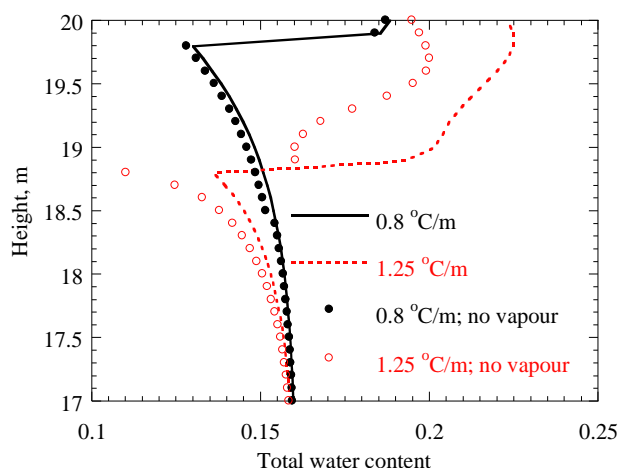


Figure 6. Effects of the temperature gradient on the total water content.

## 4.2. The Effect of the Cooling Rate

The fluctuation of the temperature in winter will cause different cooling rates in soils as well. During test, the temperature is not applied instantaneously. The initial temperature in soils is the same as the environmental temperature. The cooling rate can be then obtained by the circulation of the cooling liquid. Similarly, the effect of the temperature on a deep soil can be also negligible, so it can be then assumed that the temperature at the bottom of the simulated soil column remains constant as well. The cooling rates at the top are  $0.1\text{ }^{\circ}\text{C/h}$  and  $\infty\text{ }^{\circ}\text{C/h}$ , respectively. The other conditions correspond to that of case 1 and 3 in Table 4, respectively. The effect of the cooling rate on the total water content is shown in Figure 7. It can be seen that no matter whether vapour transfer

is considered, the larger the cooling rate is, the larger the total water content is. However, the effect of vapour transfer slightly increases with the cooling rate increasing, which means that the total increased amount does not obviously increase.

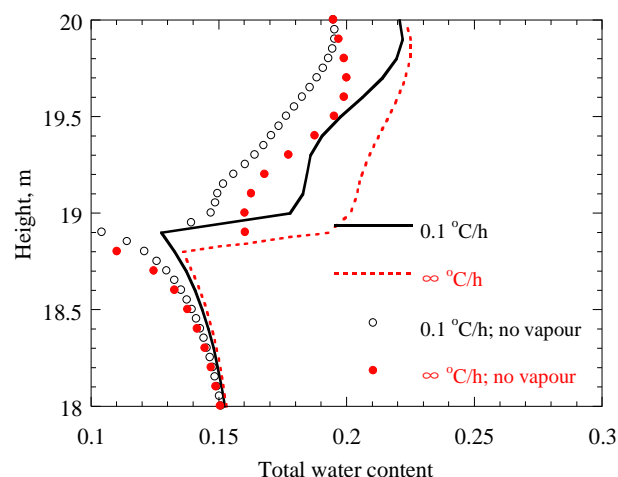
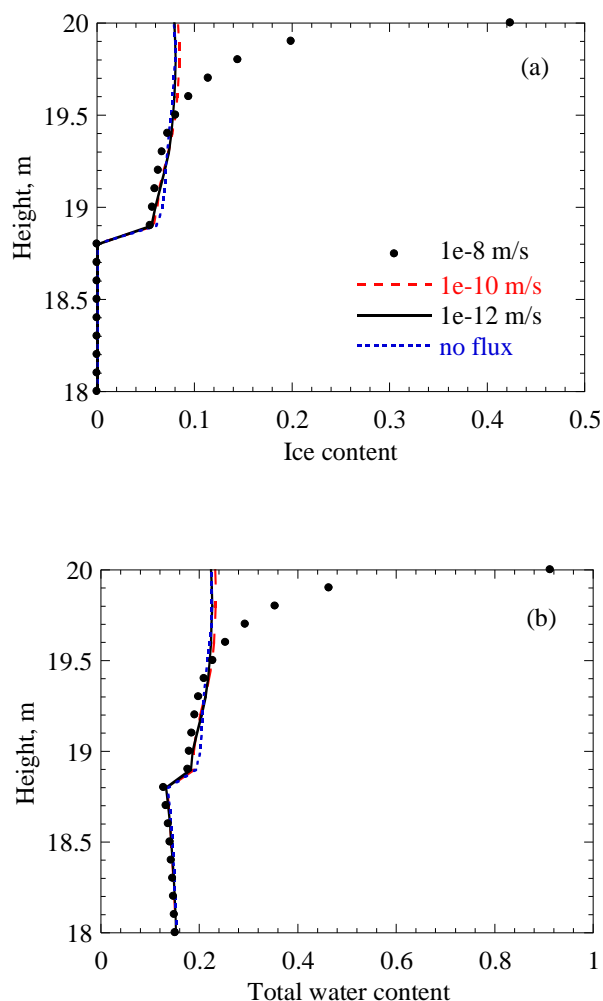


Figure 7. Effects of the cooling rate on the total water content.

## 4.3. The Effect of the Vapour Flux at the Top

During freezing test, vapour in the environment can enter into the specimen through the gap between the cooling plate and the wall of the sample cylinder, then changing the total water content in soils. In practical engineering, vapour in the atmosphere can also enter through the cracks in the cover. Snow and rainfall can infiltrate downwards along these cracks, thereby causing a series of frost damage. The gap during test and the cracks in practical engineering have been all observed. It is then needed to analyze the effect of the flux at the top on the total water content.

According to the research results by He et al. [29], the order of vapour flux in this silty soil is about  $10^{-10}$ . Therefore, the values of the vapour flux at the top are set as  $1 \times 10^{-8}\text{ m}^2/\text{s}$ ,  $1 \times 10^{-10}\text{ m}^2/\text{s}$ ,  $1 \times 10^{-12}\text{ m}^2/\text{s}$  and no flux, corresponding to case 4, 5, 6 and 7 in Table 4, respectively. The other conditions remain constant as well. The simulated results are shown in Figure 8. It can be seen that the ice content and the total water content all hardly increase with the vapour flux at the top increasing. However, when the flux at the top is  $1 \times 10^{-8}\text{ m}^2/\text{s}$ , the ice content and the total water content reach 0.42 and 0.9, respectively. Obviously, ice lens have generated in this case. It should be note that this order of the flux at the top is a kind of the liquid water flux instead of the vapour flux. Therefore, it can be also concluded that the gap and the cracks must be all perfect dealt with for avoiding frost damage.



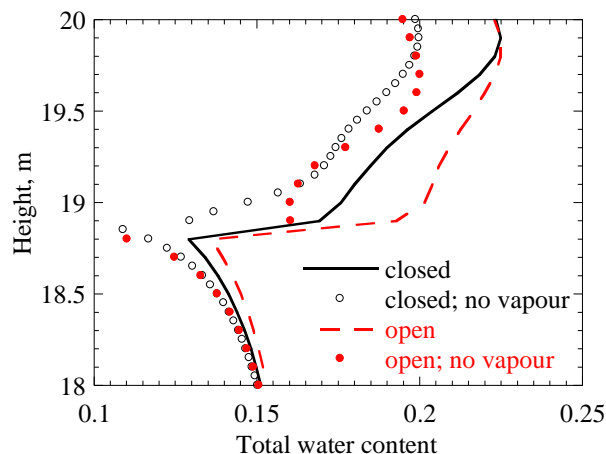
**Figure 8.** Effects of the vapour flux at the top on (a) the ice content and (b) the total water content.

#### 4.4. The Effect of the Sealing Condition at the Bottom

Although the depth of the ground water table is quite large in the cold and arid areas in northwestern China, some drainage facilities should still be installed, such as the drainage ditch, the water-resisting layer and the clay layer with a very low permeability because a sudden rainstorm may still occurs [1, 4]. Otherwise, moisture can transfer upwards into the soil column under the capillary effect. These drainage facilities cause that moisture hardly transfer upwards, which is considered as a sealing condition.

The effect of the sealing condition on the total water content is shown in Figure 9. The simulated conditions correspond to case 2 and 8 in Table 4, respectively. The other conditions are all the same. It can be seen that when the bottom is closed, the total water content is larger no matter whether vapour transfer is considered. And vapour transfer can also increase the total water content no matter whether the bottom is closed. This is logical because the soil column is not completely dry so the liquid water still exists and can then turn

into vapour through evaporation under the effect of temperature. This simulated example shows that the sealing condition at the bottom also slightly works, especially for vapour transfer.



**Figure 9.** Effects of the sealing conditions at the bottom on the total water content.

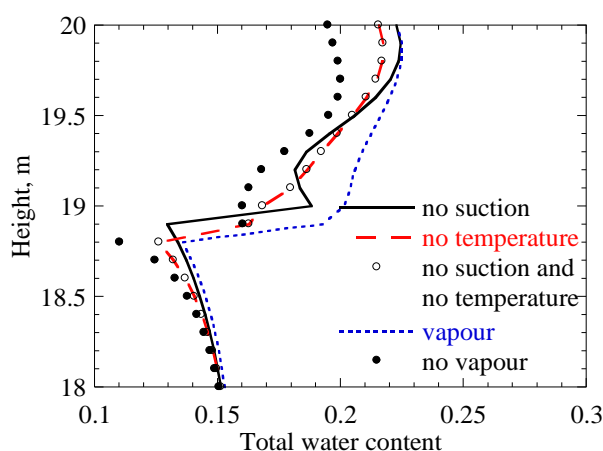
### 5. An Analysis of Other Factors

The governing equations (1) and (2) are highly non-linear because the coefficients or terms in the two equations will vary with time and they can affect each other as well. The suction and the temperature are the major two factors to vapour transfer [18]. To determine which factor dominates, we will analyze the total water content at the top by simplifying some terms related to vapour transfer in the governing equations (1) and (2) in this section. The simulated conditions are the same as that of case 2 in Table 3. The simulated results are shown in Figures 10 and 11. In this section,  $K_{vh}\partial h/\partial z$ ,  $K_{vT}\partial T/\partial z$ ,  $-C_v\partial(q_v T)/\partial z$  and  $-L_w\rho_w\partial q_v/\partial z$  are subjectively, simply and also not reasonably called the suction, the temperature, the heat convection and the heat latent, respectively. In Figure 10, no suction and no temperature represents that  $K_{vh}\partial h/\partial z$  and  $K_{vT}\partial T/\partial z$  are all not considered in the governing equations. In Figure 11, no heat convection and no heat latent represents that  $-C_v\partial(q_v T)/\partial z$  and  $-L_w\rho_w\partial q_v/\partial z$  are all not considered in the governing equations. Vapour and no vapour in both Figures 10 and 11 represent that vapour transfer is considered and is not considered at all, respectively.

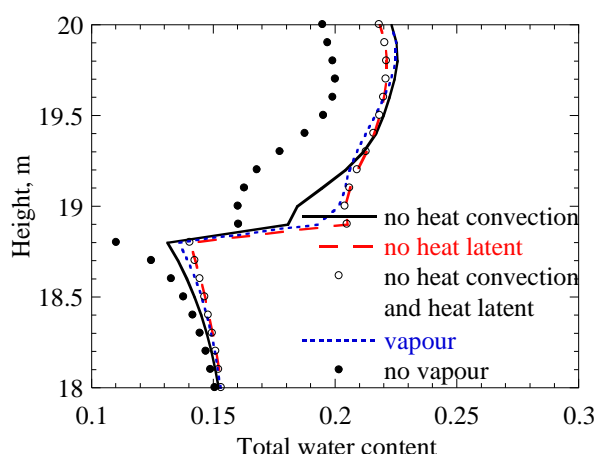
When  $K_{vh}\partial h/\partial z$  or  $K_{vT}\partial T/\partial z$  is not considered, the total water content at the top decreases to some extent. And when  $-C_v\partial(q_v T)/\partial z$  or  $-L_w\rho_w\partial q_v/\partial z$  is not considered, the total water content at the top just slightly decreases. It can be then concluded that the effects of  $K_{vh}\partial h/\partial z$  and  $K_{vT}\partial T/\partial z$  on the total water content at the top are slightly greater than that of  $-C_v\partial(q_v T)/\partial z$  and  $-L_w\rho_w\partial q_v/\partial z$ . Therefore, the effects of the suction on the total water content are slightly larger than that of the temperature as well. Although the effect of each of the



four terms is relatively small, vapour transfer or all these terms has a remarkable effect on the total water content. Figures 10 and 11 also indicate that some terms related to vapour transfer can be ignored for the quite small difference of the total water content without considering these terms. These simulated results also suggest that ignoring some terms is reasonable and acceptable if the required simulated precision is not quite high, which can decrease the nonlinearity and complexity of the calculation and increase the convergence as well. However, this is not inconsistent with the analysis in the previous sections, because these terms are only a part of the effects of the previous parameters, but not all of them.



**Figure 10.** Effects of the suction and the temperature on the the total water content.



**Figure 11.** Effects of the heat convection and the heat latent on the total water content.

## 6. Conclusion

Based on a new model for coupled liquid water-vapour-heat transfer in unsaturated freezing soils proposed

by the authors, this paper systematically analyzes the effects of the hydraulic parameters, environmental factors and other factors on vapour transfer and the total water content. This paper will help to deepen the deep understanding of these parameters, which has a certain practical reference significance for high-filling engineering. Although the interaction of these factors has not been studied due to a quite difficulty of analysis, some important conclusions can still be then obtained as follow:

- 1) The effects of the hydraulic parameters such as the fitting parameters (i.e.  $\alpha$  and  $n$ ) of the SWCC and the saturated hydraulic conductivity on the vapour transfer and the total water content are significant, even if the values of these parameters vary within a quite small range. It is inevitable that there are small differences between the test results and the fitting results of these parameters, but the changes should be within a reasonable range to ensure that those match the types and properties of the soils. These small differences and their effects on vapour transfer and total water content have been easily overlooked in the past and should be given sufficient attentions in the future.
- 2) Environmental factors also have remarkable effects on the vapour transfer and the total water content, including the temperature gradient, the cooling rate, the vapour flux at the top and the sealing condition at the bottom. These factors are quite common and typical in practical engineering, representing different actual working conditions. A larger value of each factor can lead to a larger total water content. And vapour transfer can also increase the total water content to some extent for every environmental factor. These research results on the effects of environmental factors are helpful to provide some important guidances for the design and maintenance of high-filling engineering.
- 3)  $K_{vh}\partial h/\partial z$ ,  $K_{vT}\partial T/\partial z$ ,  $-C_v\partial(q_v T)/\partial z$  and  $-L_{wp}\partial q_v/\partial z$  can all slightly decrease the total water content. Therefore, ignoring some terms is reasonable and acceptable if the required simulated precision is not quite high, which can decrease the nonlinearity and complexity of the calculation and increase the convergence as well.

## Abbreviations

SWCC	Soil Water Characteristic Curve
VG	Van Genuchten

## Conflicts of Interest

The authors declare no conflicts of interest.

## Appendix

### Appendix 1: Hydraulic Parameters

The SWCC is written as follows [39]

$$S_e = [1 + (-\alpha h)^n]^{-m} \quad (A-1)$$

where,  $S_e = (\theta_w - \theta_r) / (\theta_s - \theta_r)$  is the effective degree of saturation.

The isothermal hydraulic conductivities of liquid water due to the water pressure head  $K'_{wh}$  (m/s) in freezing soils is expressed as [13, 23]

$$K'_{wh} = 10^{-\Omega \theta_i} K_{wh} = 10^{-\Omega \theta_i} K'_s S_e^l \left[ 1 - (1 - S_e^{1/m})^m \right]^2 \quad (A-2)$$

where,  $K_{wh}$  (m/s) is the isothermal hydraulic conductivities of liquid water due to the water pressure head in unfrozen soils.  $\Omega$  is an empirical parameter.

The thermal hydraulic conductivities of liquid water due to the temperature  $K_{wT}$  (m<sup>2</sup>/K/s) is expressed as [18, 40-42]

$$K_{wT} = K_{wh} h G_{wT} \frac{1}{\gamma_0} \frac{d\gamma}{dT} \quad (A-3)$$

where,  $G_{wT}$  is the gain factor.  $\gamma$  ( $=75.6 - 0.1425(T - 273.15) - 2.38 \times 10^{-4}(T - 273.15)^2$  g/s<sup>2</sup>) is the surface tension of soil water.  $\gamma_0$  ( $=71.89$  g/s<sup>2</sup>) is the surface tension at 25 °C.

The isothermal hydraulic conductivities of vapour due to the water pressure head  $K_{vh}$  (m/s) is given as [18, 43, 44]

$$K_{vh} = \frac{D}{\rho_w} \rho_{vs} \frac{Mg}{RT} H_r \quad (A-4)$$

where,  $D$  ( $=\tau \eta_a n_a D_0$  m<sup>2</sup>/s) is the vapour diffusion coefficient in soils.  $\tau$  is the tortuosity factor.  $\eta_a$  is a strengthening factor.  $n_a$  is the fraction of air.  $D_0$  (m<sup>2</sup>/s) is the vapour diffusion coefficient in air.  $M$  ( $=0.018$  kg/mol) is the molecular weight of liquid water.  $g$  (m/s<sup>2</sup>) is the gravitational acceleration.  $R$  ( $=8.341$  J/mol/K) is the universal gas constant.  $H_r$  ( $=\exp(hMg/RT)$ ) is the relative humidity. The fraction of the vapour  $\theta_v$  is equal to  $\rho_{vs} H_r (\theta_s - \theta_r) / \rho_w$  and the saturated vapour density is given as

$$\rho_{vs} = \exp \left( 31.37 - \frac{6014.79}{T} - 7.92 \times 10^{-3} T \right) \times \frac{10^{-3}}{T} \quad (A-5)$$

The thermal hydraulic conductivities of vapour due to the temperature  $K_{vT}$  (m<sup>2</sup>/K/s) is given as [18]

$$K_{vT} = \frac{D}{\rho_w} \eta H_r \frac{d\rho_{vs}}{dT} \quad (A-6)$$

where,  $\eta$  is an enhancement factor.

### Appendix 2: Thermal Parameters

The effective heat capacity  $C_p$  (J/m<sup>3</sup>/K) is written as [31, 32]

$$C_p = C_n \theta_n + C_w \theta_w + C_v \theta_v + C_i \theta_i \quad (A-7)$$

where,  $\theta_n$  is the fraction of the soil skeleton.  $C_x$  (J/m<sup>3</sup>/K) is the heat capacity of each phase and  $x=n, w, v$  and  $i$ .

The effective thermal conductivity  $\lambda'$  (W/m/K) is expressed as [45]

$$\lambda' = (\lambda_n)^{\theta_n} (\lambda_w)^{\theta_w} (\lambda_v)^{\theta_v} (\lambda_i)^{\theta_i} \quad (A-8)$$

where,  $\lambda_x$  (W/m/K) is the thermal conductivity of each phase and  $x=n, w, v$  and  $i$ .

The latent heats of water vaporization  $L_v$  (J/kg) can be expressed as [46]

$$L_v = 2.501 \times 10^6 - 2369.2 \times T \quad (A-9)$$

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