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Research Article

Modeling and Simulation of Fractional PID Controller for Under-actuated Inverted Pendulum Mechanical System

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Abstract

The stabilization of the non-linear inverted pendulum system requires a robust control strategy, as this system is inherently unstable and sensitive to disturbances. This research utilizes Lagrangian mechanics, a powerful technique in analytical dynamics, to derive the mathematical representation of the system. By applying the principles of Lagrangian dynamics, we can accurately model the energies involved and derive the equations of motion that govern the pendulum's behavior. Following this, state-space feedback is employed to determine the Proportional, Integral, and Derivative (PID) values essential for effective control. This control strategy is particularly useful due to its ability to minimize error over time and ensure stability. To further enhance the control process, a comprehensive mathematical model is developed to establish the transfer function that correlates the pendulum's angle with the displacement of the cart. This relationship is crucial for understanding how changes in the cart's position affect the pendulum's stability. To validate the proposed control law, extensive simulations are conducted, allowing for comparative analysis against an Integer Order Controller. These simulations not only highlight the effectiveness of the PID controller but also provide insights into the dynamic behavior of the system under various conditions. The results demonstrate significant improvements in settling time and overshoot, showcasing enhanced performance metrics for the selected objective functions. This research contributes to the broader field of control systems engineering, suggesting that advanced control strategies can effectively manage complex, non-linear systems.

Keywords

Inverted Pendulum Dynamics, Cart Position Control, Pendulum Angle Stabilization, Fractional PID Control, Integer PID Control

1. Introduction

The inverted pendulum problem represents a fundamental challenge in control theory, serving not only as a classic example of dynamic instability but also as a vital benchmark for the evaluation and development of innovative control algorithms across various applications. In an inverted pendulum system, a pendulum is attached to a cart that can move

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horizontally along a track. The goal is to keep the pendulum upright by moving the cart back and forth, while minimizing the movement of the cart.

This problem has many practical applications, such as in robotics, aerospace, and offshore drilling. By studying and developing control strategies for inverted pendulum systems, researchers can gain insights into how to control other complex systems with unstable dynamics.

Moreover, the inverted pendulum problem is also used as a teaching tool in control theory courses, as it provides a simple and intuitive example to illustrate concepts such as stability, controllability, and observability. By studying this problem, students can gain a deeper understanding of control theory and its practical applications.

The inverted pendulum system is a highly complex and challenging control problem due to its nonlinear dynamics, strong coupling, and instability.

Over the years, researchers have devised a diverse range of advanced control strategies to tackle the inverted pendulum problem, including, but not limited to, traditional double-loop PID control, fuzzy logic systems, state feedback control, sliding mode variable structure control, genetic algorithms, and more sophisticated approaches such as model predictive control, adaptive control, and robust control, each offering unique advantages and insights into the system's dynamics. These methods have their strengths and weaknesses, and the choice of control strategy depends on the specific requirements of the control problem. Modern methods for controlling the inverted pendulum system primarily use traditional PID control, state feedback, fuzzy logic, genetic algorithms, and sliding mode control [1-9].

Fractional order control has been applied to the inverted pendulum control system in recent years, and it has shown promising results. The use of fractional calculus allows for more flexible and adaptable control strategies, which can better handle the nonlinear and uncertain dynamics of the system.

Fractional order control can provide better performance in terms of stability, robustness, and disturbance rejection compared to traditional integer order control methods. It also offers more degrees of freedom in the design of control laws, which can lead to more efficient and effective control strategies.

Overall, the application of fractional order control to the inverted pendulum control system is an exciting development in control theory, and it has the potential to improve the performance of this classic control problem [10-12]. The Integer Order PID (IOPID) controller is generalized to Fractional Order PID (FOPID) by introducing adjustable parameters λ and μ, where $0 < \lambda < 2$ and $0 < \mu < 2$, which modify the orders of the integral and differential operators, resulting in the following mathematical model of the FOPID controller.

The Fractional Order PID (FOPID) controller introduces two adjustable parameters, λ for the integral order and μ for the derivative order, compared to the Integer Order PID (IOPID) controller. This enhanced flexibility enables a broader adjustable range and improves control quality, resulting in superior performance for the control system [7, 13-19].

2. Inverted Pendulum System Modeling

The physical representation of a first order inverted pendulum system is shown in Figure 1. The inverted pendulum on a cart system is a classic example of a control problem that is commonly used to illustrate concepts in control theory. The system consists of a cart that moves along a horizontal track and a pendulum that is mounted on the cart and is free to rotate in the vertical plane. To model this system, we can use the principles of Newtonian mechanics. Let's assume that the cart has mass M and the pendulum has mass m. The cart is free to move in the horizontal direction and is subject to a force F. The pendulum is subject to the force of gravity and a torque τ that acts on it due to its angular acceleration. We can define the position of the cart along the track as x, and the angle of the pendulum with respect to the vertical as θ. Using these variables, we can write the equations of motion for the system as follows:

$$M\ddot{x} = F - ml\sin(\theta)\dot{\theta}^{2} \tag{1}$$

$$ml^{2}\ddot{\theta} + mlg\sin(\theta) = u \tag{2}$$

Where $\ddot{x}$ and $\ddot{\theta}$ represent the second derivative of x and θ with respect to time, respectively.

These equations can be used to design a control system for the inverted pendulum on a cart. One approach is to use a feedback control strategy, where the position and velocity of the cart are measured, and the torque u is adjusted to keep the pendulum upright. This can be done using a proportional-integral-derivative (PID) controller or other control algorithms.

Another approach is to use a state-space representation of the system, which allows for more advanced control strategies such as optimal control or model predictive control. In the state-space representation, the system is described by a set of differential equations that relate the state variables $(x, \theta, \dot{x}, \theta)$ to the input (F) and output (θ). The equations can be written as follows:

$$\dot{x} = \dot{x} \tag{3}$$

$$\dot{\theta} = \dot{\theta} \tag{4}$$

$$M\ddot{x} = F - ml\sin(\theta)\dot{\theta}^{2} \tag{5}$$

$$ml^{2}\ddot{\theta} + mlg\sin(\theta) = u \tag{6}$$

This representation can be used to design a control law that stabilizes the system by controlling the state variables. The control law can be designed using techniques such as pole placement, LQR control, or Kalman filtering. In summary, the modeling of an inverted pendulum on a cart system can be done using Newtonian mechanics, and a control system can be designed using feedback or state-space control strategies. The system is a classic example of a control problem and is commonly used to illustrate concepts in control theory.

The pendulum angle θ(s) and displacement x(s) are represented by the following transfer function:

$$P(s) = \frac{\theta(s)}{X(s)} = \frac{m l s^2}{(1+ml^2)s^2 + m g l} \tag{7}$$

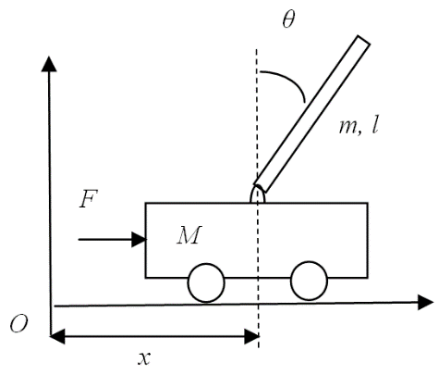

Figure 1. First-order inverted pendulum physical model.

3. Designing PID & Fractional Order PID Controllers

Based on the context, Figure 2 presents the block diagram of the entire controlled system, referred to as the Pendulum-on-a-Cart (POAC) system. The controller is designed based on this overall configuration. In order to simulate the system, you consider the following parameters:

Here,

Cart mass (M) = 0.5 kg

Pendulum mass (m) = 0.2 kg

Acceleration due to gravity (g) = 9.8 m/s^2

Pendulum length (h) = 1 meter

The open-loop characteristics of the inverted pendulum system, P(s), are marked by unstable poles in the right half-plane and open-loop zeros, indicating inherent instability [20-26]. Following stabilization, the transfer function of the generalized controlled object for the inverted pendulum system is expressed as follows:

$$P_1(s) = \frac{K}{(T_1 s+1)\,(T_2 s+1)} \tag{8}$$

The system parameters are specified as follows: sampling time T = 0.005 s, cart mass M = 0.5 kg, pendulum mass m = 0.2 kg, damping coefficient b = 0.1, pendulum length l = 0.3 m, moment of inertia I = 0.006 kg.m ², time constants T_1 = 0.1784 and T_2 = 7.008, gain K = 4.5455, cut-off frequency ωc = 1 rad/s, and phase margin θm = 70°.

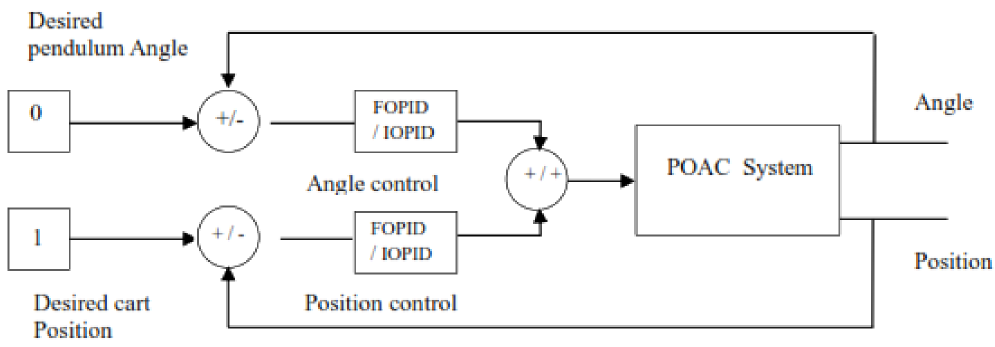

Figure 2. POAC system's block diagram.

3.1. Integer PID Controller: Design & Simulation

The transfer function of an Integer Order PID (IOPID) controller can be represented as:

$$C_1(s) = K_p + \frac{K_i}{s} + K_d s \tag{9}$$

Based on the parameter tuning rules outlined in Section 2 of this paper, the following values can be obtained by solving the constraint equations of the controller K_p = 1.2, K_i = 0.4 and K_d = 0.01.

Therefore, IOPID controller's transfer function $C_1(s)$ is:

$$C_1(s) = 1.2 + \frac{0.4}{s} + 0.01s \tag{10}$$

Figure 3 shows the integer order PID controller's step response for the inverted pendulum system, indicating that PID control is effective and meets design specifications.

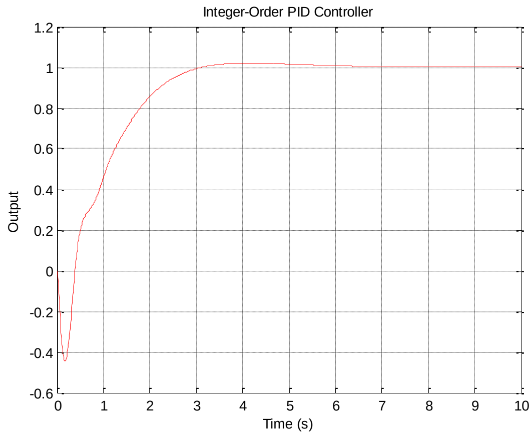

***Figure 3**. Integer Order PID Control System Simulation Results.*

3.2. Fractional FOPID Controller: Design & Simulation

Fractional Order PID controller's transfer function is expressed as:

$$C_2(s) = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu} \tag{11}$$

where, $0 < \lambda < 2$ and $0 < \mu < 2$. Based on the parameter tuning rules from Section 2 of the paper, the values can be obtained as $K_p = 1$, $K_i = 0.3$, $K_d = 0.01$, $\lambda = 0.9$ and $\mu = 0.9$. The transfer function of the FOPID controller is expressed as:

$$C_2(s) = 1 + \frac{0.3}{s^{0.9}} + 0.01 s^{0.9} \tag{12}$$

Figure 4 displays the step response curve of the FOPID controller in the inverted pendulum system.

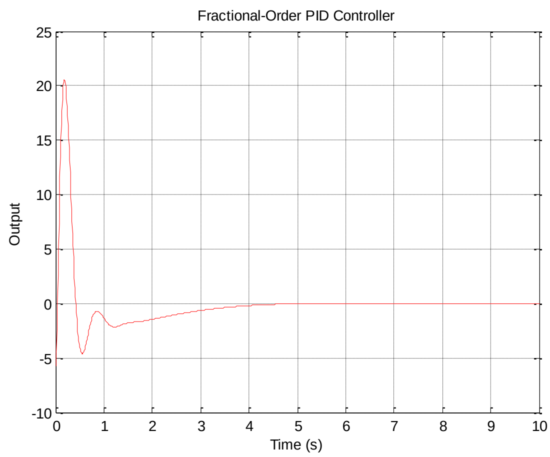

***Figure 4**. Fractional Order PID Control System Simulation Results.*

4. Conclusion

We successfully stabilized the POAC system using a Fractional Order (FO) PID controller to control both the pendulum angle and cart position. Additionally, comparing the FOPID and Integer Order PID controllers revealed that the FOPID controller is more effective for stabilizing the inverted pendulum on a cart system. Fractional Order PID controllers offer some distinct advantages over their integer order counterparts, including improved robustness and increased flexibility in the control design. However, they also require more complex mathematical models and can be more difficult to implement in practice. Overall, this work is an important contribution to the field of control engineering, and it will be interesting to see how these findings can be applied in other systems and applications.

Abbreviations

POAC	Pendulum-on-a-Cart
IOPID	Integer Order Proportional, Integral, and Derivative
FOPID	Fractional Order Proportional, Integral, and Derivative

Author Contributions

Djamel Boucherma: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review & editing

Mohamed Lotfi Cherrad: Conceptualization, Data curation, Formal Analysis, Resources, Visualization, Writing – review & editing

Khaled Chettah: Conceptualization, Formal Analysis, Visualization, Writing – review & editing

Toufik Achour: Data curation, Formal Analysis, Methodology, Resources, Software, Visualization, Writing – review & editing

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Billel Hamadi: Conceptualization, Formal Analysis, Methodology, Resources, Visualization, Writing – review & editing

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] Q. Wang and Z. Shen, "Control comparison of inverted pendulum system based on PID," in *IOP Conference Series: Materials Science and Engineering*, 2018, vol. 394, no. 5, p. 052065: IOP Publishing. https://doi.org/10.1088/1757-899X/394/5/052065

[2] Q. Qian, L. Ting, L. Feng, T. Yong-chuan, and W. Shu-li, "Parameter optimization of the fuzzy nine-point controller for inverted pendulum system based on genetic algorithm [J]," *Journal of Southwestern University (Natural Science Edition)*, vol. 36, no. 11, pp. 191-199, 2014.

[3] Y. Haotian, "Analysis and Design of Fractional Sliding Mode Control for Nonlinear Systems [J]," *Journal of Naval Aeronautical and Astronautical University*, vol. 31, no. 4, pp. 407-414, 2016.

[4] C. Wang, M. Li, S. Jiang, and Y. Wang, "Design of fractional order control system," *National Defense Industry Press, Beijing*, 2014.

[5] S. Jiang, M. Li, and C. Wang, "Design and simulation of fractional order PID controller for an inverted pendulum system," in *2017 IEEE International Conference on Manipulation, Manufacturing and Measurement on the Nanoscale (3M-NANO)*, 2017, pp. 349-352: IEEE.

[6] M. Mahmoodabadi and T. Soleymani, "Optimum fuzzy combination of robust decoupled sliding mode and adaptive feedback linearization controllers for uncertain under-actuated nonlinear systems," *Chinese Journal of Physics*, vol. 64, pp. 241-250, 2020. https://doi.org/10.1016/j.cjph.2019.11.015

[7] K. Orman, A. Basci, and A. Derdiyok, "Speed and direction angle control of four wheel drive skid-steered mobile robot by using fractional order PI controller," *Elektronika ir Elektrotechnika*, vol. 22, no. 5, pp. 14-19, 2016. https://doi.org/10.5755/j01.eie.22.5.16337

[8] H. O. Erkol, "Optimal PI $^{{\lambda}} $ D $^{{\mu}} $ controller design for two wheeled inverted pendulum," *IEEE Access*, vol. 6, pp. 75709-75717, 2018. https://doi.org/10.1109/ACCESS.2018.2883504

[9] R. El-Khazali, "Fractional-order PIλDμ controller design," *Computers & Mathematics with Applications*, vol. 66, no. 5, pp. 639-646, 2013. https://doi.org/10.1016/j.camwa.2013.02.015

[10] L. Youxin, "New Fractional PID Controller and Its Simulation Research [J]," *Journal of Harbin Institute of Technology*, vol. 5, pp. 215-217, 2009.

[11] K. Bingi, R. Ibrahim, M. N. Karsiti, S. M. Hassan, and V. R. Harindran, "A comparative study of 2DOF PID and 2DOF fractional order PID controllers on a class of unstable systems," *Archives of Control Sciences*, pp. 635-682-635-682, 2018.

[12] S. K. Pradhan and B. Subudhi, "Position control of a flexible manipulator using a new nonlinear self-tuning PID controller," *IEEE/CAA Journal of Automatica Sinica*, vol. 7, no. 1, pp. 136-149, 2018. https://doi.org/10.1109/JAS.2017.7510871

[13] N. Esmaeili, A. Alfi, and H. Khosravi, "Balancing and trajectory tracking of two-wheeled mobile robot using backstepping sliding mode control: design and experiments," *Journal of Intelligent & Robotic Systems*, vol. 87, pp. 601-613, 2017. https://doi.org/10.1007/s10846-0170486-9

[14] A. Nikranjbar, M. Haidari, and A. A. Atai, "Adaptive sliding mode tracking control of mobile robot in dynamic environment using artificial potential fields," *Journal of Computer & Robotics*, vol. 11, no. 1, pp. 1-14, 2018.

[15] Z. Bingul and O. Karahan, "Comparison of PID and FOPID controllers tuned by PSO and ABC algorithms for unstable and integrating systems with time delay," *Optimal Control Applications and Methods*, vol. 39, no. 4, pp. 1431-1450, 2018. https://doi.org/10.1002/oca.2419

[16] N. D. Bernardes, F. A. Castro, M. A. Cuadros, P. F. Salarolli, G. M. Almeida, and C. J. Munaro, "Fuzzy logic in auto-tuning of fractional PID and backstepping tracking control of a differential mobile robot," *Journal of Intelligent & Fuzzy Systems*, vol. 37, no. 4, pp. 4951-4964, 2019. https://doi.org/10.3233/JIFS-181431

[17] R. Cajo, T. T. Mac, D. Plaza, C. Copot, R. De Keyser, and C. Ionescu, "A survey on fractional order control techniques for unmanned aerial and ground vehicles," *IEEE Access*, vol. 7, pp. 66864-66878, 2019. https://doi.org/10.1109/ACCESS.2019.2918578

[18] R. Rajesh, "Optimal tuning of FOPID controller based on PSO algorithm with reference model for a single conical tank system," *SN Applied Sciences*, vol. 1, no. 7, p. 758, 2019. https://doi.org/10.1007/s42452-019-0754-3

[19] J. Bhookya and R. K. Jatoth, "Optimal FOPID/PID controller parameters tuning for the AVR system based on sine–cosine-algorithm," *Evolutionary Intelligence*, vol. 12, pp. 725-733, 2019. https://doi.org/10.1007/s12065-019 00290-x

[20] W. Cu, J. Fang, Q. Yuan, J. Wei, G. Yang, and Z. Zhou, "Optimization and design of fractional-order PID controller based on an improved sparrow search algorithm," 2022. https://doi.org/10.21203/rs.3.rs-1897616/v1

[21] M. Bongulwar and B. Patre, "Design of FOPID controller for fractional-order plants with experimental verification," *International Journal of Dynamics and Control*, vol. 6, no. 1, pp. 213-223, 2018. https://doi.org/10.1007/s40435-017-0305-y

[22] P. T. Garran and G. Garcia, "Design of an optimal PID controller for a coupled tanks system employing ADRC," *IEEE Latin America Transactions*, vol. 15, no. 2, pp. 189-196, 2017. https://doi.org/10.1109/TLA.2017.7854611

[23] J. Viola, L. Angel, and J. M. Sebastian, "Design and robust performance evaluation of a fractional order PID controller applied to a DC motor," *IEEE/CAA Journal of Automatica Sinica*, vol. 4, no. 2, pp. 304-314, 2017. https://doi.org/10.1109/JAS.2017.7510535

[24] L. Zhang, L. Liu, and S. Zhang, "Design, Implementation, and Validation of Robust Fractional-Order PD Controller for Wheeled Mobile Robot Trajectory Tracking," *Complexity*, vol. 2020, no. 1, p. 9523549, 2020. https://doi.org/10.1155/2020/9523549

[25] A. Al-Mayyahi, W. Wang, and P. Birch, "Design of fractional-order controller for trajectory tracking control of a non-holonomic autonomous ground vehicle," *Journal of Control, Automation and Electrical Systems*, vol. 27, pp. 29-42, 2016. https://doi.org/10.1007/s40313-015-0214-2

[26] Z. A. Karam and O. A. Awad, "Design of active fractional PID controller based on whale's optimization algorithm for stabilizing a quarter vehicle suspension system," *Periodica Polytechnica Electrical Engineering and Computer Science*, vol. 64, no. 3, pp. 247-263, 2020. https://doi.org/10.3311/PPee.14904

Research Fields

Djamel Boucherma: Fractional order calculus, Fractional order control, Trajectory tracking of manipulator robot, Fractional differential equations, Control of mechanical systems

Mohamed Lotfi Cherrad: Vibrations and acoustics, Random vibrations, Linear vibration theory, Acoustic and vibration frequency analysis

Khaled Chettah: Materials physics, Physics, astronomy, technology, engineering, Applications to specific physical systems, Biophysics and nanoparticles

Toufik Achour: Applied mechanics, Computational mechanics, Dynamical systems in statistical mechanics, Structured surfaces and interfaces, rupture and damage

Mohamed Chaour: Heat and mass transfer, Differential equations with applications, Applied mechanics, Mechanical application in engineering

Sofiane Boulkroune: Heat and mass transfer, Differential equations with applications, Applied mechanics, Mechanical application in engineering

Billel Hamadi: Applied mechanics, Mechanical Engineering, Manufacturing engineering and optimization