

Research Article

Explicit Expressions for Calculating the Turbulent Friction Coefficient: A Review

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Abstract

Research on friction in pipes remains complex. This complexity is due to the numerous parameters that must be considered when calculating the coefficient of friction f . Due to this complexity, many researchers have proposed multiple formulas for determining the friction coefficient. Today, there are a large number of explicit expressions for calculating the friction coefficient for each flow regime, whether for smooth or rough pipes. In this review, we set out to study the evolution of work on the friction coefficient and to present it. For this purpose, a large number of reviews have been used. Once this has been done, we identified more than 120 expressions, most of which were explicit. We have thus noted that many of these expressions are approximations of the implicit Colebrook-White equation. Similarly, a good number are derived either from direct developments using computer tools, or from mathematical methods that are sometimes very long. However, in general, we have found that all these expressions only take into account the three parameters present in the Colebrook-White formula. These parameters are: Reynolds number Re , the absolute roughness ϵ and the diameter D . We would like to point out that these parameters were deemed insufficient to minimise the difference between the calculated values and the values observed on site with regard to the friction coefficient.

Keywords

Friction Coefficient, Colebrook-White, Explicit Approximation, Explicit Expression, Implicit Expression

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Received: 30 May 2026; **Accepted:** 22 June 2026; **Published:** 28 July 2026



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1. Introduction

To determine the flow rate of a fluid in a pipe or pipe network, it is important to first calculate the head losses caused by friction on the walls. To achieve this, it is first necessary to calculate the coefficient of friction (f). This coefficient is linked to both head losses and the viscous effects of the fluid [1]. Traditionally, the coefficient of friction is estimated using the Colebrook-White equation or the Moody diagram.

In general, the determination of the value of the coefficient of friction depends on the fluid's flow regime in the pipe and the pipe's physical characteristics. Consequently, there are several possible scenarios.

For smooth pipes, the expressions for calculating f are similar regardless of the flow regime. The literature suggests that there are no major difficulties in this case. Once the flow regime becomes turbulent, as it is most often the case in hydraulic structures, we observe a wide variety of expressions for calculating friction coefficient f . This variety is even greater when the pipe is rough. Some of these expressions have been obtained by processing experimental data [2].

The Moody diagram which has been used by engineers since 1945 is semi-empirical and the conditions it describes (fully developed flow, isothermal, incompressible, dissipative, quasi-stationary) are rarely sufficiently achieved in a real-world scenario [3].

The literature review conducted as part of this work shows that there are two main approaches to calculating friction coefficient. There is an implicit approach fully represented by the Colebrook-White formula, and an explicit approach with a wide variety of expressions in the literature. It should be noted that a large number of explicit approaches are based on the Colebrook-White equation.

The Colebrook and White formula is one of the most frequently used expressions for calculating the coefficient of friction in the turbulent regime [4]. It is given by Eq. (1) below.

$$f = -2 \log \left(\frac{\varepsilon}{3.7D} + \frac{2.51}{Re\sqrt{f}} \right) \quad (1)$$

With $\frac{\varepsilon}{D}$ the relative pipe roughness and Re the Reynold's number.

This formula is not practical, as the expression is implicit. Iterations are required to perform the calculation. To find the value of f defined in this equation, you need to use numerical algorithms, which is not easy. This is why its use is not recommended in practical calculations by engineers or for students in courses related to fluid mechanics [5]. The difficulties in determining the coefficient of friction using Eq. (1) have led many authors to develop explicit equations that can be used directly.

In this review, we present a comprehensive set of formulas for calculating the coefficient of friction in pipes.

2. Current Situation

Table 1 below shows the platforms used for publishing new coefficient of friction formulas; scientific articles published by various journals with a high percentage for a high percentage (around 85%) of the total sources consulted.

Table 1. Bibliographical sources consulted.

Consulted	Number
Newspaper articles	118
Books	10
Reports	5
Lectures	2
Book chapters	1
Unpublished	3

Table 2 below shows the authors who published at least two expressions for calculating friction.

Table 2. Authors with the highest number of publications.

Author	Number of publications
Brkic D.	9
Vatankhah A.	4
Swamee P. K.	3
Achour B.	2
Avci A	2
Bedjaoui A.	2
Chen J.	2
Chernikin V.	2
Churchill S.	2
Genic S.	2
Giustolisi O.	2
Goudar C.	2
Jain A. K.	2
Karagoz I.	2
Kouchakzadeh S.	2
Mileikovskiy V.	2
Niazkar M.	2
Petresin E.	2
Praks P.	3

Author	Number of publications
Sonnad J.	2
Tkachenko T.	2
Zigrang D.	2

Similarly, we aimed to place the development of research on this parameter over time. Figure 1 below illustrates the evolution of research into the coefficient of friction in pipes, showing a high level of interest after the year 2000.

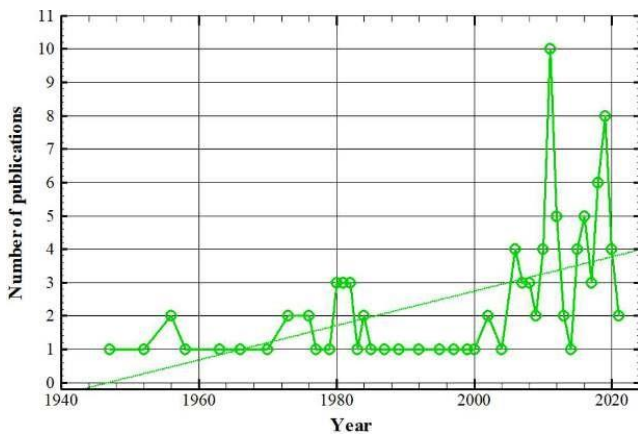


Figure 1. Publication of friction coefficient formulas by year.

3. Approaches to Solving the Colebrook-White Equation

In general, there are six approaches available for solving the Colebrook-White equation [6]:

- 1) Graphical solutions, such as those using Moody or Rouse diagrams.
- 2) The iterative schemes, for example, the Newton-Raphson method [7].
- 3) Explicit approximate relations [8, 9].
- 4) Lambert's W function [6, 10].
- 5) The trial-and-error method [7, 11].
- 6) Artificial intelligence models [12-14].

Of all these approaches, the second is technically considered not only the most accurate, but also the reference solution for calculating friction coefficient. However, it requires more computational effort, which is its only drawback for proper implementation.

Authors have used various techniques and methods to propose new explicit expressions for calculating the turbulent friction coefficient. Computerized methods are increasingly being used to determine friction in pipes. Below, we list some authors along with the methods used to calculate the coefficient of friction:

- 1) Artificial Intelligence (AI) models, including Artificial Neural Networks (ANNs) [12, 15-19];
- 2) The adaptive neurofuzzy technique [13];
- 3) Gene expression programming [14, 20-22],
- 4) The adaptive neuro-fuzzy inference system [23].

4. Classification of Explicit Relations

In this section, we listed almost all the explicit formulas for calculating the turbulent friction coefficient available in the literature. They are presented chronologically and separately for both rough and smooth pipes. We note, however, that many of them are approximations of the Colebrook and White equation.

4.1. Expressions for Rough Pipes

We examine the explicit formulations developed to calculate the turbulent friction coefficient in rough pipes. Over the decades, a vast number of approximations of the Colebrook-White implicit equation have been proposed. Given the large number of these equations, it is impossible to present them all in the main body of the text. Therefore, Table 3 presents a rigorous selection of these explicit relationships. This subset highlights pioneering historical formulas, the most widely used approximations, high-precision mathematical models, as well as recent advances based on artificial intelligence or Lambert's W-function. For a comprehensive overview, an exhaustive compilation of all explicit relationships identified in this study for rough pipes is provided in the Supplementary Material.

Table 3. Selected explicit approximations for rough pipes.

Author and Year	Formula	Method / Main characteristic	Range of application
Moody [24]	$f = 0.001375 \left(1 + \left(2 \cdot 10^4 \frac{\varepsilon}{D} + \frac{10^6}{Re} \right)^{\frac{1}{3}} \right)$	The first explicit form proposed for the Colebrook-White equation.	$4 \times 10^3 \leq Re \leq 10^8$ and $0 \leq (\varepsilon/D) \leq 10^{-2}$
Wood [25]	$f = 0.094 \left(\frac{\varepsilon}{D} \right)^{0.225} + 0.53 \left(\frac{\varepsilon}{D} \right) + 88 \left(\frac{\varepsilon}{D} \right)^{0.44} \cdot Re^{-v}$	An attempt to provide a simplified formula.	$Re > 4000$; and $0 \leq (\varepsilon/D) \leq 5 \times 10^{-2}$

Author and Year	Formula	Method / Main characteristic	Range of application
	where ν is given by: $\nu = 1.62 \left(\frac{\varepsilon}{D}\right)^{0.134}$		
Churchill [26]	$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\varepsilon}{3.71D} + \left(\frac{7}{Re}\right)^{0.9} \right)$	An empirical expression that replaces the Colebrook-White equation.	$4000 < Re < 10^8$ and $10^{-6} < \varepsilon/D < 5 \times 10^{-2}$
Swamee-Jain [8]	$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\varepsilon}{3.71D} + \frac{5.74}{Re^{0.9}} \right)$ $f = 8 \left(\left(\frac{8}{Re}\right)^{12} + \frac{1}{(A+B)^{3/2}} \right)^{1/12}$ Where:	Widely used and considered the best explicit approximation.	$5 \times 10^3 < Re < 10^8$ and $10^{-6} < \varepsilon/D < 10^{-2}$
Churchill [27]	$A = \left(2.457 \ln \left(\frac{1}{\left(\frac{7}{Re}\right)^{0.9} + \frac{0.27\varepsilon}{D}} \right) \right)^{16}$ $B = \left(\frac{37530}{Re} \right)^{16}$	This formula applies to all fluid flow regimes.	$4 \times 10^3 < Re < 10^8$ and $10^{-6} < \varepsilon/D < 0.05$
Chen [28]	$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\varepsilon}{3.7065D} - \frac{5.0452}{Re} \log \left(\frac{1}{2.8257} \left(\frac{\varepsilon}{D}\right)^{1.1098} + \frac{5.8506}{Re^{0.8981}} \right) \right)$	Detailed solution.	$4 \times 10^3 \leq Re \leq 4 \times 10^8$ and $10^{-7} \leq (\varepsilon/D) \leq 5 \times 10^{-2}$
Round [29]	$\frac{1}{\sqrt{f}} = -1.8 \log \left(\frac{Re}{0.135Re \left(\frac{\varepsilon}{D}\right) + 6.5} \right)$	A relatively simple explicit approximation.	$4 \times 10^3 \leq Re \leq 10^8$ and $0 \leq (\varepsilon/D) \leq 10^{-2}$
Barr [30]	$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\varepsilon}{3.7D} + \frac{4.518 \log \left(\frac{Re}{7} \right)}{Re \left(1 + \frac{Re^{0.52}}{29} \left(\frac{\varepsilon}{D}\right)^{0.7} \right)} \right)$	Does not require any internal iterative calculations.	$2300 \leq Re \leq 10^8$ and $0 \leq (\varepsilon/D) \leq 5 \times 10^{-2}$
Zigrang and Sylvester [31]	$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\varepsilon/D}{3.7} - \frac{5.02}{Re} \log \left(\frac{\varepsilon/D}{3.7} - \frac{13}{Re} \right) \right)$ $\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\varepsilon/D}{3.7} - \frac{5.02}{Re} \log \left(\frac{\varepsilon/D}{3.7} - \frac{5.02}{Re} \log \left(\frac{\varepsilon/D}{3.7} + \frac{13}{Re} \right) \right) \right)$	It does not use an internal iterative procedure to achieve high accuracy.	$4 \times 10^3 \leq Re \leq 10^8$ and $10^{-6} \leq (\varepsilon/D) \leq 5 \times 10^{-2}$
Haaland [32]	$\frac{1}{\sqrt{f}} = -\frac{1.8}{n} \log \left(\left(\frac{\varepsilon}{3.75D}\right)^{1.1n} + \left(\frac{6.9}{Re}\right)^n \right)$ $\frac{1}{\sqrt{f}} = A - \frac{(B-A)^2}{C-2B+A}$ With:	A simple and clear formula, a straightforward solution.	$3 \times 10^3 \leq Re \leq 1.5 \times 10^8$ and $0 \leq (\varepsilon/D) \leq 5 \times 10^{-2}$
Serghides [33]	$A = -2 \log \left(\frac{\varepsilon}{3.7D} + \frac{12}{Re} \right)$ $B = -2 \log \left(\frac{\varepsilon}{3.7D} + \frac{2.51A}{Re} \right)$ $C = -2 \log \left(\frac{\varepsilon}{3.7D} + \frac{2.51B}{Re} \right)$	An approximation of the implicit form that is valid for all ranges.	$2300 \leq Re \leq 10^8$ and $10^{-6} \leq (\varepsilon/D) \leq 5 \times 10^{-2}$
Romeo et al., [34]	$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\varepsilon/D}{3.7065} - \left(\frac{5.0272}{Re}\right) \log \left(\frac{\varepsilon/D}{3.827} - \left(\frac{4.567}{Re}\right) \log \left(\left(\frac{\varepsilon/D}{7.7918}\right)^{0.9924} + \left(\frac{5.3326}{208.815+Re}\right)^{0.9345} \right) \right) \right)$	Obtained using nonlinear multivariate regression.	$3 \times 10^3 \leq Re \leq 1.5 \times 10^8$ and $0 \leq (\varepsilon/D) \leq 5 \times 10^{-2}$
Sonnad and Goudar [35]	$\frac{1}{\sqrt{f}} = 0.8686 \ln \left(\frac{0.4587 Re}{S^{S/(S+1)}} \right)$ With: $S = 0.124 Re \frac{\varepsilon}{D} + \ln(0.4587 Re)$	A mathematical approximation used as a basis by many authors.	$4 \times 10^4 \leq Re \leq 10^8$ and $10^{-6} \leq (\varepsilon/D) \leq 5 \times 10^{-2}$
More [36]	$f = \frac{1}{c \left(W_0 \left(\frac{\exp(\frac{a}{bc})}{bc} \right) - \frac{a}{b} \right)^2}$ W_0 is the main branch of Lambert's W function	Use d'Alembert's method and the principal branch of Lambert's W-function.	Not specified.

Author and Year	Formula	Method / Main characteristic	Range of application
	(Chapeau-Blondeau and Monir, 2002). With: $a = \frac{1}{3.7} \frac{\varepsilon}{D}; b = \frac{1.257}{Re}; c = 1.7372$		
Brkic [37]	$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{2.18}{Re} \left(\frac{Re}{1.816 \ln \left(\frac{1.1 Re}{1+1.1 Re} \right)} \right) + \frac{\varepsilon}{3.71 D} \right)$	Use different solutions of Lambert's W-function.	$2300 \leq Re \leq 10^8$ and $0 \leq (\varepsilon/D) \leq 5 \times 10^{-2}$
Samadianfard [14]	$f = \left(\frac{Re^{\varepsilon/D} - 0.6315093}{Re^{1/3} + Re(\frac{\varepsilon}{D})} \right) + 0.0275308 \left(\frac{6.929841}{Re} + \left(\frac{\varepsilon}{D} \right)^{\frac{1}{9}} + \left(\frac{10^{\varepsilon/D}}{\varepsilon + 4.781616} \right) \left(\sqrt{\frac{\varepsilon}{D}} + \frac{9.997001}{Re} \right) \right)$	Explicit calculation based on genetic programming.	$4 \times 10^3 \leq Re \leq 10^8$ and $0 \leq (\varepsilon/D) \leq 5 \times 10^{-2}$
Vatankhah [38]	$f = \left(\frac{\frac{2.51}{Re} + 1.1513 \delta}{\delta - \frac{\varepsilon/D}{3.71} - 2.3026 \delta \log(\delta)} \right)^2$ Where: $\delta = \frac{6.0173}{Re(0.07(\frac{\varepsilon}{D}) + Re^{-0.885})^{0.109}} + \frac{\varepsilon/D}{3.71}$	Optimization of relative error as an objective function.	$4 \times 10^3 \leq Re \leq 10^8$ and $10^{-6} \leq (\varepsilon/D) \leq 5 \times 10^{-2}$
Offor and Alabi [16]	$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\varepsilon}{3.71 D} - \frac{1.975}{Re} \ln \left(\left(\frac{\varepsilon}{3.93 D} \right)^{1.092} + \frac{7.627}{Re + 395.9} \right) \right)$	Use of artificial neural networks (ANNs).	$4 \times 10^3 \leq Re \leq 10^8$ and $10^{-6} \leq (\varepsilon/D) \leq 5 \times 10^{-2}$
Azizi et al. [39]	$f = \left(1.805 \log \left(\frac{\left(\frac{\varepsilon}{D} \right)^{1.108}}{4.267} + \frac{5.164}{Re^{0.966}} \right) \right)^{-2}$	Explicit correlation.	$2 \times 10^3 \leq Re \leq 10^8$ and $10^{-6} \leq (\varepsilon/D) \leq 5 \times 10^{-2}$
López-Silva et al. [40]	$f = 0.219 \left(\left(\frac{0.028\varepsilon}{D} \right) + \left(\frac{0.896}{Re} \right) \right)^{0.25}$	Model formulated using genetic expression programming (GEP).	$4000 < Re < 10^8$ and $10^{-6} < \varepsilon/D < 10^{-2}$.

4.2. Expressions for Smooth Pipes

Similar to rough pipes, a wide variety of expressions have been developed to estimate the friction coefficient in smooth pipes ($\varepsilon/D=0$). To provide a concise and relevant overview, Table 4 highlights the most significant explicit relationships

found in the literature. This selection covers both the classic fundamental models for laminar and turbulent flow regimes and more specific correlations, notably those adapted to Bingham plastic fluids. As in the previous section, the exhaustive list of all explicit formulas identified for smooth pipes is available in the Supplementary Material.

Table 4. Selected explicit approximations for rough pipes.

Author and Year	Formula	Method / Main characteristic	Range of application
Hazen and Poiseuille cited by [41]	$f = \frac{64}{Re}$	Standard basic formula for laminar flow.	$Re \leq 2100$
Blasius [42]	$f = \frac{0.3164}{Re^{0.25}}$ for $2100 \leq Re \leq 2 \times 10^4$ $f = \frac{0.184}{Re^{0.2}}$ for $2 \times 10^4 \leq Re$	The simplest numerical equation for smooth turbulent flow.	$3 \times 10^3 \leq Re \leq 10^5$
Colebrook-White cited by [32]	$\frac{1}{\sqrt{f}} = 1.8 \log_{10} \left(\frac{Re}{6.9} \right)$	A specialized design developed specifically for smooth pipes.	$4 \times 10^3 \leq Re \leq 10^8$
Moody [41]	$f = \frac{0.184}{Re^{0.2}}$	Classical approximation for smooth pipes.	$3 \times 10^3 \leq Re \leq 10^7$

Author and Year	Formula	Method / Main characteristic	Range of application
Filonienko [43]	$f = \frac{1}{(1.82 \log(R_e) - 1.64)^2}$	An explicit correlation widely used in the literature.	$3 \times 10^3 \leq Re \leq 10^7$
Darby and Melson [44]	$f_T = 10^a Re^{-0.193}$ $a = -1.47(1 + 0.14e^{(-2.9 \times 10^{-5})H_e})$	An approximation for Bingham-type plastic fluids covering all flow regimes.	Not specified.
McKeon et al. [45]	$\frac{1}{\sqrt{f}} = 1.930 \log(R_e \sqrt{f}) - 0.537$	High-precision modern correlation.	Not specified.
Danish et al. [46]	$f_L = \frac{K_1 + \frac{4K_2}{\left(K_1 + \frac{K_1 K_2}{K_1^4 + 3K_2}\right)^3}}{1 + \frac{3K_2}{\left(K_1 + \frac{K_1 K_2}{K_1^4 + 3K_2}\right)^4}}$ Where: $K_1 = \frac{16}{Re} + \frac{16H_e}{6Re^2}$ et $K_2 = -\frac{16H_e^4}{3Re^8}$	Explicit approximations using Adomian's decomposition method.	Not specified.
Swamee-Agarwal [47]	$f_L = \frac{64}{Re} + \frac{10.67 + 0.1414 \left(\frac{H_e}{Re}\right)^{1.143}}{\left(1 + 0.0149 \left(\frac{H_e}{Re}\right)^{1.16}\right) Re} \left(\frac{H_e}{Re}\right)$	Direct approximation of the implicit Buckingham-Reiner equation.	Not specified.
Fang et al. [48]	$f = 0.25 \left(\log \left(\frac{150.39}{Re^{0.98865}} - \frac{152.66}{Re} \right) \right)^{-2}$ New 1: $\frac{1}{\sqrt{f}} = -2 \log \left(-\frac{5.0272}{Re} \log \left(-\frac{4.567}{Re} \log \left(\left(\frac{5.3326}{208.815 + Re} \right)^{0.9345} \right) \right) \right)$ New 2: $\frac{1}{\sqrt{f}} = -2 \log \left(-\frac{5.02}{Re} \log \left(-\frac{5.02}{Re} \log \left(\frac{13}{Re} \right) \right) \right)$ New 3: $\frac{1}{\sqrt{f}} = -2 \log \left(\frac{95}{Re^{0.983}} - \frac{96.82}{Re} \right)$ New 4: $f = \left(S_1 - \frac{(S_2 - S_1)^2}{S_3 - 2S_2 + S_1} \right)^{-2}$	Correlation for smooth pipes.	Not specified
Brkic [6]	$\frac{1}{\sqrt{f}} = -2 \log \left(-\frac{5.02}{Re} \log \left(-\frac{5.02}{Re} \log \left(\frac{13}{Re} \right) \right) \right)$ New 3: $\frac{1}{\sqrt{f}} = -2 \log \left(\frac{95}{Re^{0.983}} - \frac{96.82}{Re} \right)$ New 4: $f = \left(S_1 - \frac{(S_2 - S_1)^2}{S_3 - 2S_2 + S_1} \right)^{-2}$	Based on combinations of previous approximations (Romeo, Serghides, etc.).	Not specified
Morrison [49]	$f = \left(\frac{0.0076 \left(\frac{3170}{Re} \right)^{0.165}}{1 + \left(\frac{3170}{Re} \right)^{7.0}} \right) + \frac{16}{Re}$	A simple explicit correlation that matches Prandtl's equation at high Re numbers.	$4000 \leq Re \leq 10^6$
Genic and Jacimovic [50]	$f = \begin{cases} \frac{0.3164}{Re^{1/4}} \\ \frac{0.118}{Re^{1/6}} \end{cases}$	Statistical expansion based on the form of the Blasius equation.	$Re < 150 \times 10^3$

5. Conclusions

This document provides an exhaustive presentation of the expressions that have been developed for determining the coefficient of friction. Formulas have been presented according to two approaches: the implicit approach and the explicit approach. Some of these formulas for calculating the coefficient of friction can be up to 65% inaccurate, as noted by Cipra [51].

This occurs when comparing theoretical values obtained from the formulas with the practical data taken from the pipe under investigation and a large discrepancy is observed between both values. This discrepancy exists with all formulas, whether obtained using mathematical methods or artificial intelligence. Today, with the evolution of computer technology, we can easily solve the Colebrook-White equation. However, this universally used equation also produces significant errors. Yoo and Singh found that the Colebrook equation produced

an average error of more than 11%, noting that, the roughness of commercial pipes varies considerably, depending on the size and type of pipe [52, 53]. Since roughness is a variable factor and should not be considered constant. We recommend that the friction coefficient value be calculated for a specific pipe and for well-defined fluids in a well-defined space-time. Haddad proposes in his work study that sediment deposits on the pipe walls be taken into account for a better assessment of friction [54]. In the same light, Tchawe recommends a graphical determination of this evolving parameter at every moment and at different points of the pipe including an improved analytical method to minimise errors in determining the coefficient of friction and head loss [55].

Abbreviations

f	Friction Coefficient (or Friction Factor)
Re	Reynolds Number
D	Pipe Diameter
ε	Absolute Roughness
ε/D	Relative Roughness
ν	Kinematic Viscosity
ANN	Artificial Neural Networks
GEP	Genetic Expression Programming
He	Hedstrom Number
Pr	Prandtl Number

Acknowledgments

First and foremost, we would like to express our gratitude to Professor Djeumako Bonaventure (may he rest in peace), who supervised this work. We would also like to thank all the individuals and organizations that assisted us in any way with the completion of this project.

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Data Availability Statement

The data supporting the findings of this research have been reported in this manuscript.

Conflicts of Interest

The authors declare no conflicts of interest.

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