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Enhancing Bipartite Entanglement via Optical Parametric Amplifier in an Optomechanical Device

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Abstract: On this paper, we present an in depth evaluation of the bipartite entanglement of cavity radiation from the optomechanical device with an optical parametric amplifier (OPA) placed inside two cavity mode, which together have interaction with a mechanical resonator. Right here by linearizing the equations of motion, we set the entanglement gift inside the gadget, the use of the logarithmic negativity as a degree. We thereby symbolize the adjustments inside the machine entanglement that result from the addition of an quadratic coupling to a linearly coupled gadget. With the assist of the optical parametric amplifier, the desk bounds macroscopic entanglement between the movable replicate and the hollow space subject can be mainly more suitable, and the degree of entanglement increases while the parametric gain increases as well as input laser electricity will increase. For that reason, while an optical parametric amplifier is delivered inside a hollow space, which leads to extensive improvement of the two-mode entanglement. These results establish a promising theoretical basis for optical parametric amplifier (OPA) more suitable bipartite entanglement of optomechanical device for quantum technology and advanced quantum information processing applications.

Keywords: Optomechanical System, Optical Parametric Amplifier (OPA), Logarithmic Negativity

1. Introduction

In recent years, optomechanical systems have been considered the best tools for studying quantum mechanics. Optomechanical systems act as interfaces for the coupling of photons and phonons due to mechanical oscillations [1]. Optomechanics, exploring the interaction between light and mechanical objects through radiation pressure, is taken into consideration as a super platform to prepare entangled states, in particular of big and big gadgets.These machines investigate the interaction between light and the machine motion (moving objects). One way to improve the optomechanical interaction is to apply a repulsive force to the optical cavity, thus creating a linear optomechanical interaction [2]. Significant progress has been made in the investigation of optomechanics, such as squeezing [3], ultrahigh precision displacement detection [4], mass detection [5], gravational wave detection [6] and the transition between classical and quantum behaviour of a mechanical system [7]. Over the past two decades, numerous initiatives have been undertaken to prepare entangled states in cavity optomechanical systems. In general, a simple optomechanical system contains a mechanical oscillator whose displacement is coupled only through the linear optomechanical coupling (LOC) to the cavity mode [8]. Optomechanical coupling describes the phenomenon of interaction of optical radiation with vibration patterns. It was found that linear optomechanical interactions are responsible for cooling mechanical systems and establishing quantum correlations between cavity light fields and mechanical oscillators. Such kind of linear interaction-based optomechanical systems has been widely used in many interesting studies like optomechanical-induced transparency [16], entanglement [17], squeezing [18], quantum information processing [19] including normal mode splitting (NMS) [20–22].

Cavity optomechanics systems based on the quadratic optomechanical coupling (QOC) involve two phonon process due to the coupling of the cavity mode, studies the radiation pressure interaction between a cavity field and the motion of a mechanical oscillator [8], providing a platform both for fundamental physics of macroscopic quantum systems and for practical applications of precision sensing. On a more fundamental level, these systems represent one of the most promising platforms for experimental verification of physical theories, with applications ranging from gravitational wave detection [38] to the potential observation of quantum gravitational effects [10] and entanglement between nearly-macroscopic mechanical objects [11–14]. The cavity fields exert radiation pressure on the movable mirror, which leads to the changes of both the resonance frequency and damping rate and of the mechanical modes [15]. In these systems, an optical cavity, with a movable end mirror is subjected to mechanical effect caused by light through radiation pressure. The radiation pressure is central interaction in the field of optomechanical systems [24, 25]. So, optomechanical systems with quadratic interaction also show various nonlinear effects such as two-phonon optomechanically induced transparency (OMIT) [26], phonon shot noise [27], photon and phonon blockade [28, 29], cooling and squeezing of the mechanical oscillator [30–32], optomechanically induced opacity and amplification [33], macroscopic nonclassical states [34] and the parametric amplifier [35]. Besides optomechanically induced transparency (OMIT), linear optomechanical interactions have been mainly used in entanglement between optical and mechanical mode [36–38] as well as in studies related to normal-mode splitting [10]. In such systems generally the optical cavities are used for coherent control of the quantum state and dynamics of the mechanical oscillators by their coupling to a light field [39, 40] and to study the entanglement of macroscopic states [41, 42]. The entanglement generated in such systems is significant for quantum mechanics and is a fundamental resource for many quantum technology applications [44, 45] and especially the entanglement of macroscopic objects, is of great interest both for fundamental physics and for possible applications in quantum information processing and quantum communication [46, 47]. In additionally, cavity optomechanical systems have potential applications in sensitive measurement, quantum computing, quantum information, hybrid systems, and the foundation tests of quantum theory [46].

Any other manner to decorate entanglement among cavity modes is setting optical parametric amplifier (OPA) within the Fabry Perot hollow space. The Optical Parametric Amplifiers give rise to single-mode squeezing of the cavity fields, which results in significant improvement of the bipartite entanglement [48–51]. It gives rise to the coupling between the movable reflect and the hollow space mode increase. This indicates that the bipartite entanglement increases with the increasing parametric gain. In the context of optomechanical systems, an optical parametric amplifier (OPA) can improve optomechanical cooling and the qualities of the nonlinear crystals, optical parametric amplifiers [52, 53] and modify the normal-mode splitting behavior of the coupled movable mirror and the cavity field [54] and as well as enhance the optomechanical interaction strength into the single-photon strong-coupling regime [55, 56].In addition, a degenerate optical parametric amplifier (OPA) can be used to generate a flash via a degenerate parametric down-conversion process. It has been noted that placing a degenerate optical parametric amplifier (OPA) in a Fabry-Perot optical cavity can improve cooling of the resonator and create quantum correlations between the dental lamp lines Fields and mechanical oscillator. Optical parametric amplifier (OPA) plays a more important role in generating entangled and squeezed states of light [55, 57, 58] Zhang et al. [58] showed that a squeezed vacuum field can be amplified and quantum fluctuations can be manipulated in the optical parametric amplifiers (OPAs).

Currently, the dynamic driving of optomechanical systems with the cavity of mechanical oscillators affected by thermal noise has been neglected in the last few years. Therefore, the novelty of this work is based on the development of the integrated system created by optical parametric amplifier (OPA), which is used to reduce the effect of loud noises. With this idea, we create a two-part entanglement of the cavity in an optomechanical system and a moving mirror filled with optical parametric amplifier interacting with a nonlinear lead in the cavity. In the presence of optical parametric amplifiers (such as degenerate optical parametric amplifier (DOPA)), the effect of macroscopic systems is increased compared to the case without optical parametric amplifiers.

1.1. Model and Hamiltonian Formulation

For a given parametric gain, the larger entanglement between cavity modes and movable mirrors and the interaction between the driving field and the cavity mode and parametric interaction by larger input laser power as shown in figure 1.

Figure 1. *Schematic representation of a cavity optomechanical system with OPA.*

The two cavity modes interact with each other through the typical optomechanical interaction with the mechanical resonator. The total Hamiltonian for such a system is given by;

$$\hat{H} = \hbar(\omega_c - \omega_L)\hat{a}^\dagger\hat{a} - \hbar g_o\hat{a}^\dagger\hat{a}\hat{q} + \frac{1}{2}\hbar\omega_m(\hat{p}^2 + \hat{q}^2) + i\hbar\varepsilon(\hat{a}^\dagger - \hat{a}) + i\hbar G(e^{i\theta}\hat{a}^{\dagger 2} - e^{-i\theta}\hat{a}^2), \quad (1)$$

where $\hat{a}(\hat{a}^\dagger)$ are the annihilation(creation) operator of the cavity fields with frequency $\omega_c = 2\pi c/L$ and L is the cavity length, ω_L is the driving laser frequency, κ is the cavity decay constant. In addition, for mechanical oscillator $q(p)$ is the dimensionless position (momentum) operator of the mechanical oscillators with mechanical frequency ω_m and g_o is the single optomechanical coupling strength between the cavity field and mechanical mirror

$$g_o = \frac{\omega_c}{\ell}\sqrt{\frac{\hbar}{2m\omega_m}}, \quad (2)$$

The fourth term gives the coupling between the input laser field and the cavity mode, ε is the amplitude of the field driving laser $\varepsilon = \sqrt{\frac{2\kappa p}{\hbar\omega_L}}$ and ω_L is driving and pump mode frequency. Moreover, G is a nonlinear gain coefficient of the optical parametric amplifier (OPA), which is proportional to the power of the pump mode, and θ is the phase of the field driving the optical parametric amplifier (OPA) [60].

1.2. Dissipation and Quantum Noise of Open Quantum System

In a real system, dissipation of both photons and phonons is present. Below this circumstance, the full dynamics of the machine are quantized using the standard method of the Heisenberg-Langevin equation of motion, and taking the corresponding damping and noise effects of the reservoir, optical cavity as properly as mechanical modes. Due to fluctuation-dissipation processes [61], the nonlinear quantum Langevin equations can be obtained as

$$i\hbar\dot{\hat{Z}} = [\hat{Z}, \hat{H}] + N - H_{dis}, \quad (3)$$

where H_{dis} describes the dissipation of each mode as well as the driving of the cavity mode in the Langevin approach and $\hat{Z}$ is an arbitrary operator for $\hat{p}, \hat{q}$ and $\hat{a}$. The corresponding nonlinear Quantum Langevin equations (QLEs), which include various noises entering into the system, in the interaction picture with cavity mode From Eq. 3 is given by

$$\begin{aligned}
\dot{\hat{q}} &= \omega_m\hat{p} \\
\dot{\hat{p}} &= \omega_m\hat{q} + g_o\hat{a}^\dagger\hat{a} - \gamma_m\hat{p} + \xi \\
\dot{\hat{a}} &= -(\kappa - i\Delta)\hat{a} + ig_o\hat{a}\hat{q} + \varepsilon + 2Ge^{i\theta}\hat{a}^\dagger + \sqrt{2\kappa}\hat{a}_{in}, \quad (4)
\end{aligned}$$

where $\Delta = \omega_c - \omega_L$ is effective cavity detuning and $\xi, \hat{a}_{in}$ is quantum noise and γ_m is the mechanical damping rate. The force ξ is the Brownian noise operator resulting from the coupling of the movable mirror to the thermal bath, whose mean value is zero and has the following correlation function $\frac{1}{2}\langle\xi_i(t)\xi_j(t') + \xi_j(t')\xi_i(t)\rangle = \gamma_m(2\bar{n}+1)\delta(t-t')$ [62], where $\bar{n} = [exp(\hbar\omega_m/K_BT) - 1]^{-1}$ is the mean thermal excitation number of the resonator. The noise operator $\hat{a}_{in}$ and the nonzero time-domain correlation functions satisfies equation $\langle\hat{a}_{in}(t)\hat{a}^\dagger_{in}(t')\rangle = \delta(t - t')$.

2. Linearization of Langevin Equations

A good way to study the steady state entanglement of the system, we adopt the standard methods of quantum optics [63] to solve Eq. 4, specifically, the steady state solutions of the system can be obtained through putting all the time derivatives of Eq. 4 to zero. Therefore, we acquire the steady state values of the system

$$\begin{aligned}
p_s &= 0 \quad (5) \\
q_s &= -\tfrac{g_o|\alpha_s|^2}{\omega_m} \\
\alpha_s &= \tfrac{\kappa - i\Delta + 2Ge^{i\theta}}{\kappa^2 + \Delta^2 - 4G^2}\varepsilon \quad (6)
\end{aligned}$$

We express each operator of the system as the average value it maintains over time, plus a small variation, When the mean value of the operator is zero, it can be expressed as a $\hat{Z} = Z_s + \delta Z(\hat{Z} = \hat{a}, \hat{q}, \hat{p})$ [64, 65] and the dynamics of the small fluctuations are around the steady state of the system [62]. This can be expanded as a sum of its coherent amplitudes semi-classical steady-state value plus a small fluctuation operator. We have expressed langevin equations in a linear form as follows:

$$\begin{aligned}
\delta\dot{\hat{q}} &= \omega_m\delta\hat{p} \\
\delta\dot{\hat{p}} &= -\omega_m\delta\hat{q} + g_o(\alpha_s\delta\hat{a}^\dagger + \alpha_s^*\delta\hat{a}) - \gamma_m\delta\hat{p} + \xi(t) \\
\delta\dot{\hat{a}} &= -(\kappa + i\Delta)\delta\hat{a} + ig_o\alpha_s\delta\hat{q} + 2Ge^{i\theta}\delta\hat{a}^\dagger + \sqrt{2\kappa}\hat{a}_{in} \quad (7)
\end{aligned}$$

2.1. Quantification of Bipartite Entanglement

The mechanical entanglement will be quantitatively analyzed using logarithmic negativity on the approximated Quantum Langevin equations (QLEs). We first introduce the amplitude and phase fluctuations for a cavity mode. Where we have defined quadrature operators for the cavity field quadratures and input noise quadratures

$$\delta\hat{x} = \frac{\delta\hat{a}^\dagger + \delta\hat{a}}{\sqrt{2}}, \delta\hat{y} = i\frac{\delta\hat{a}^\dagger - \delta\hat{a}}{\sqrt{2}} \quad (8)$$

$$\delta\hat{x}_{in} = \frac{\delta\hat{a}^\dagger_{in} + \delta\hat{a}_{in}}{\sqrt{2}}, \delta\hat{y}_{in} = i\frac{\delta\hat{a}^\dagger_{in} - \delta\hat{a}_{in}}{\sqrt{2}} \quad (9)$$

have been defined, respectively. After substitute Eq. 8 and Eq. 9 into Eq. 7, the linearized Quantum Langevin equations (QLEs) for the quantum fluctuation can be rewritten as 10

$$
\begin{aligned}
\delta\dot{\hat{q}} &= \omega_m \delta\hat{p} \\
\delta\dot{\hat{p}} &= -\omega_m \delta\hat{q} + g_o\alpha_s\sqrt{2}\delta\hat{x} - \gamma_m\delta\hat{p} + \xi \\
\delta\dot{\hat{x}} &= g_o\alpha_s\sqrt{2}\delta\hat{p} + (-\kappa + 2G\cos\theta)\delta\hat{x} + (\Delta + 2G\sin\theta)\delta\hat{y} + \sqrt{2\kappa}\delta\hat{x}_{in} \\
\delta\dot{\hat{y}} &= (-\Delta + 2G\sin\theta)\delta\hat{x} + (-\kappa - 2G\cos\theta)\delta\hat{y} + \sqrt{2\kappa}\delta\hat{y}_{in}
\end{aligned} \tag{10}
$$

Equation 10 can be rewritten as the matrix form

$$\dot{f}(t) = Af(t) + \eta(t) \tag{11}$$

In which, $f(t)$ is the column vector of the fluctuations, and $\eta(t)$ is the column vector of the noise sources can be expressed as $f(t)^T = (\delta\hat{q}, \delta\hat{p}, \delta\hat{x}, \delta\hat{y})$ and $\eta(t)^T = (0, \xi, \sqrt{2\kappa}\delta\hat{x}_{in}, \sqrt{2\kappa}\delta\hat{y}_{in})$, respectively. The drift matrix A for the above equation is given by

$$A = \begin{pmatrix} 0 & \omega_m & 0 & 0 \\ -\omega_m & \gamma_m & g_o\alpha_s\sqrt{2} & 0 \\ 0 & g_o\alpha_s\sqrt{2} & \nu_1 & \nu_2 \\ 0 & 0 & \nu_3 & \nu_4 \end{pmatrix} \tag{12}$$

where abbreviation notations are defined as $\nu_1 = -\kappa + 2G\cos\theta$, $\nu_2 = \Delta + 2G\sin\theta$, $\nu_3 = -\Delta + 2G\sin\theta$ and $\nu_4 = -\kappa - 2G\cos\theta$. When the real parts of all the eigenvalues of matrix A are negative, the system is stable and reaches its steady state. Then we can describe the asymptotic state of the quantum fluctuations by a 4×4 covariance matrix $M_{i,j}(\infty) = \langle f_i(\infty)f_j(\infty) + f_j(\infty)f_i(\infty)\rangle/2$ is used to investigate the nature of the steady state of the cavity optomechanical system. The steady state covariance matrix M(t) can be obtained by solving the Lyapunov equation [66]

$$AM + MA^T = -Q \tag{13}$$

Where, Q is the diffusion matrix from noise correlations, which is defined as $Q_{i,j}\delta(t-t') = \frac{1}{2}\langle\varsigma_i(t)\varsigma_j(t') + \varsigma_j(t')\varsigma_i(t)\rangle$. We can directly solve the covariance matrix. In vacuum, $\hat{a}_{in}$ is the input vacuum noise operator for the cavity, of which the only nonzero correlation function is $\langle\hat{a}_{in}(t)\hat{a}^\dagger_{in}(t')\rangle = \delta(t-t')$ and ξ is the Brownian noise operator resulting from the coupling of the movable mirror to the thermal bath, whose mean value is zero, and its correlation defined as $\frac{1}{2}\langle\xi_i(t)\xi_j(t') + \xi_j(t')\xi_i(t)\rangle = \gamma_m(2\bar{n}+1)$ and where $\bar{n}$ is the mean thermal phonon number. This implies that, $Q = diag[0, \gamma_m(2\bar{n}+1), \kappa, \kappa]$ is the noise correlation matrix.

The mechanical entanglement can be measured by the logarithmic negativity E_N [67, 68], which can be easily calculated from Eq. 11 covariance matrix $M_{i,j}(t)$ for a mechanical modes. The purpose of the present paper is to study the mechanical entanglement that can be enhanced by an optical parametric amplifier added inside a cavity. We employ the logarithmic negativity to quantify the entanglement can be defined as [68]

$$E_N = max\{0, -ln2\eta\} \tag{14}$$

Where $\bar{\eta} = \sqrt{\frac{\Gamma - \sqrt{\Gamma^2 - 4\det V}}{2}}$ is the smallest possible eigenvalue of the covariance matrix with invariant matrix $\Gamma = \det V_A + \det V_B - 2\det V_C$ and the correlation matrix is defined as

$$V = \begin{pmatrix} V_A & V_C \\ V_C^T & V_B \end{pmatrix} \tag{15}$$

The elements of the covariance matrix V_A, V_B and V_C are sub-block matrices of 2×2 from the drift matrix A Eq. 12. According to the definition of Eq. 14 the system is entangled if E_N>0.

3. Numerical Results and Discussions

In this section, we numerically calculate the logarithmic negativity E_N given by Eq. 14 to show mechanical entanglement under the effect of the optical parametric amplifier (OPA) coupled to squeezed vacuum reservoir. In the numerical calculations, the following system parameters are taken, which are based on current experiment conditions [69–71]: $\omega_m = 2\pi\times10^7 Hz$, $\gamma = 2\pi\times100Hz$, $\lambda = 810\times10^{-9}m$, $c = 3\times10^8 m/s$ and $f = 1.07\times10^4$. In figure 2, the logarithmic negativity E_N is plotted as a function of the normalized detuning Δ and one can observe that the entanglement with the optical parametric amplifier can be particularly enhanced compared to increasing the value of parametric gain G (0.5κ, 0.8κ and 1.8κ), respectively between the movable mirror and the cavity field.

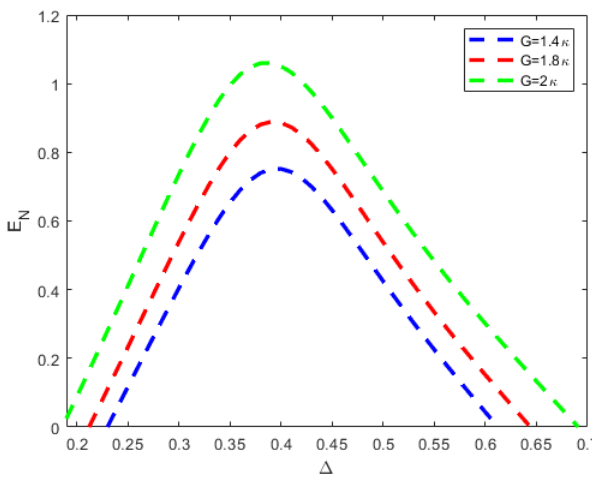

Figure 2. *Plots of entanglement E_N versus detuning Δ/ω for $L = 3mm$, $\omega_m = 2\pi\times10^7 Hz$, $\gamma = 2\pi\times100Hz$, T= 100$\times10^{-3}K$, $\lambda = 810\times10^{-9}m$, $c = 3\times10^8 m/s$, P=10mW, $f = 1.07\times10^4$ and for different values of parametric gain of OPA γ.*

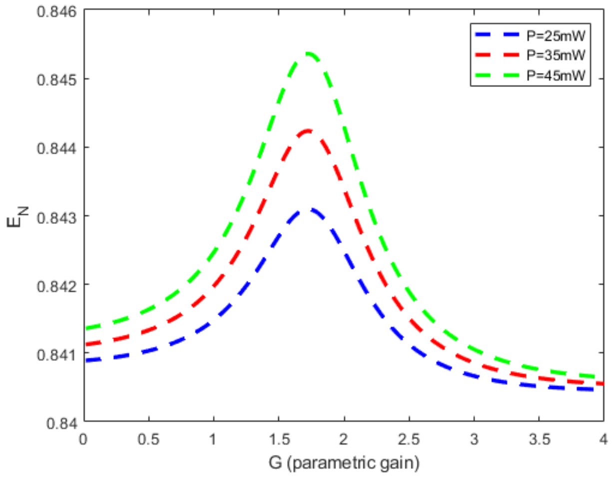

Figure 3. *Plots of entanglement E_N versus detuning Δ/ω for G=2κ and for different values of input laser (P).*

This demonstrates that the integration of the two types can facilitate the exchange and transmission of messages between them. On the other hand, it is seen from figure 2 and figure 3 that the entanglement of the coupled cavity optomechanical systems increases when the laser power and parametric gain G are increased.

As a result of the optical parametric amplifier, the mechanical entanglement was observed, first reaches its peak and then goes down as see figure 2 and figure 3. We observe that the entanglement has a maximum peak value. This implies that the peaks positively of the bipartite see figure 3 show entanglement sharing and exchange of information between each other. It is because of the coupling between the mirror and the cavity field is enhanced as the input laser power increases due to the rise in the number of photons.

Figure 4 displays that the coupling comes from the cavity frequency, which actually depends on the position of the mirror.

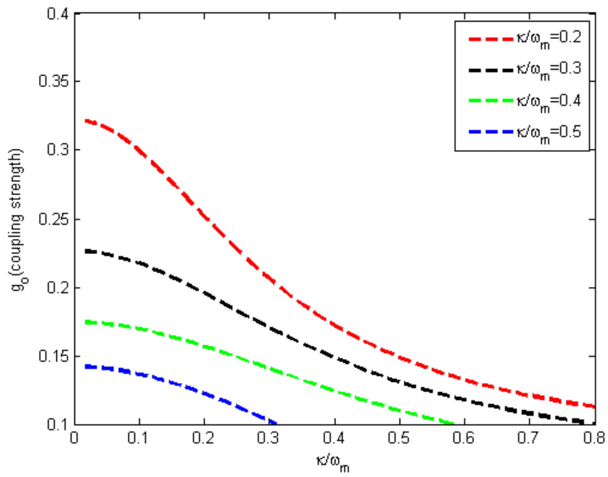

Figure 4. *Plots of entanglement E_N versus detuning Δ/ω for G=2κ and for different values of input laser (P).*

This indicates with increasing coupling strength of the coupled optomechanical system the entanglement is increasing as cavity decay decreases. Indeed, such couplings can be made very large as the effective length in the coupling can be much smaller than the physical dimensions of the cavity involved. Generally, above figure (2, 3, and 4) are our significant finding reveals how the macroscopic entanglement is influenced by the input laser power, cavity decay rate κ, parametric gain G, coupling strength g, squeezing parameters r, and temperature T etc. Furthermore, by placing the optical parametric amplifier (OPA) inside two cavities, it has been shown that the entanglement between the two cavities can be significantly enhanced.

4. Conclusions

In this work, we study the two-part entanglement of the electric cavity via an optomechanical device with an optical parametric amplifier (OPA). To measure the improvement of the integration in the steady state, we present a moving mirror filled with a cavity model and a parametric optical generator interacting with the nonlinear crystal in the cavity. Additionally, by placing the optical parametric amplifier (OPA) inside two cavity fields to become single-mode squeezed, leading to a substantial enhancement of the two-mode entanglement, which produces the highest level of entanglement. Consequently, the device with a tunable nonlinear gain, achieved by controlling the power of the driving field, is crucial for maximizing the entanglement. Although this study primarily focuses on optical entanglement, to improve other aspects of the experiment and interaction between a light wave. Studies show that the improvement of the couplings created by the optical parametric amplifier (OPA) can reduce the effect of noise pollution. New properties of the combination of our model using the Heisenberg equation will be derived from the linearized quantum Langevin equation. The improvement of the index is measured using logarithmic negativity [67].

Coupling strength and gap length. In fact, due to the introduction of optical parametric amplifier (OPA), the entanglement of the cavity optomechanical system can be improved, and more entanglement corresponds to more input laser power and parametric gain. We also show that degenerate parametric amplifiers in optomechanical systems can become entangled above a certain value.

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Author Contributions

Sisay Belachew: Conceptualization, Data curation, Formal Analysis, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing - original draft, Writing - review & editing

Conflicts of Interest

There is no conflict of interest.

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