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Research Article

Using the Logistic Growth Model to Assess Fishing Techniques for Sustainable Tilapia Catch and Management at Omega Farm

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Abstract

Mathematical modeling utilizes a differential equation, either a partial differential equation or ordinary differential equation to depict physical scenarios, such as Tilapia harvesting strategies and other population dynamics models. Fish farming constitutes the cornerstone of the Kenyan economy, notably in Baringo, where it serves as the primary economic activity. Additionally, it holds significance in the health sector due to the nutritious protein provision derived from the harvested fish. Despite the commercialization of Tilapia fish farming, the utilization of mathematical models to determine harvesting strategies remains largely unexplored in Omega Farm. Consequently, this oversight has resulted in a decline in harvest quantity over recent years. The primary objective of this study was to leverage the Logistic Growth Model to implement harvesting and management strategies for Tilapia at Omega Farm, Baringo County. The specific aims were determining the maximum sustainable yield (MSY) of the Tilapia population in the Farm after a six-month duration, employing an adapted logistic growth model to delineate harvesting rates (both constant and periodic), and identifying an efficient harvesting strategy for managing the Tilapia population by comparing the two approaches. The study investigated the existence of equilibrium solutions and their stabilities of the modified Logistic Growth Model under both constant and periodic harvest scenarios. A maximum sustainable yield of 13,000, with a growth rate of 80%, was achieved for optimal harvest, maintaining a carrying capacity of 65,000 without compromising ecological integrity. The obtained results were discussed and presented graphically. By analysing different harvesting strategies constant and periodic using Python simulations, the impact of these approaches was determined on the sustainability and productivity of fish populations. The findings underscore the importance of adaptive management and strategic harvesting to maintain a balance between maximizing yields and ensuring population stability. The study findings suggest that periodic harvesting emerges as the most effective strategy, fostering sustainable fish farm management. Future research endeavors should delve deeper into refining these strategies and exploring additional avenues for enhancing Tilapia farming sustainability.

Keywords

Logistic Growth Model, Harvesting Rates, Maximizing Yields, Population Stability

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1. Aims

1) Utilize the logistic growth model to predict Tilapia population growth under varying harvesting strategies.
2) Determine optimal harvesting rates that maximize yield while maintaining population sustainability.
3) Incorporate machine learning, genetic algorithms, and remote sensing data to enhance model accuracy and decision-making.
4) Evaluate the impact of environmental variables and climate projections on Tilapia population dynamics.
5) Develop actionable guidelines for sustainable Tilapia aquaculture management.

2. Introduction

Aquaculture is considered as one of the fastest-growing food-producing sectors in the world. The annual growth in global aquaculture production reached 5.3% during the 2010 to 2018 period. The production of aquaculture was 82.1 million tonnes in 2018 compared to 57.7 million tonnes in 2010 [7]. Tilapia ranks the fourth in the global aquaculture production [8]. Similarly, Nile tilapia have the potential of becoming the leading farmed fish species in the world [9]. The Nile tilapia is one of the most popular species in aquaculture farming around the world as a means for achieving food security [15]. Coincidentally, the dominant native species of fish in Lake Baringo is Nile tilapia (*O. niloticus*). Omega Farm has also taken an initiative to restock the lake with Nile tilapia [14].

The logistic growth model is a cornerstone of fisheries science, providing a mathematical framework for understanding population dynamics and optimizing harvesting rates. Recent advancements in modeling techniques and data analytics have enhanced the model's applicability to Tilapia aquaculture. For instance, Nazmi, H., et al conducted a pioneering study that employed machine learning algorithms to predict Tilapia growth trajectories and optimize harvesting schedules within aquaponic systems [1]. Leveraging real-time data and predictive analytics, their approach demonstrated remarkable accuracy in forecasting Tilapia yields, enabling precise decision-making and adaptive management strategies.

Similarly, Esmaeili, H. R. and Z. Eslami Barzoki introduced a dynamic modeling framework that integrates environmental variables and climate projections into logistic growth models for Tilapia populations [2]. By accounting for climate variability and uncertainty, their study provided insights into the resilience of Tilapia populations to future environmental stressors, highlighting the importance of adaptive management strategies in mitigating climate-related risks.

Additionally, applying genetic algorithms to optimize harvesting strategies for Tilapia populations has shown high effectiveness [3]. This approach identifies optimal harvesting rates that maximize yield while maintaining population sustainability, demonstrating the potential of genetic algorithms to enhance the efficiency and sustainability of Tilapia aquaculture operations.

In parallel, recent advancements in technology have revolutionized MSY estimation and resource management in Tilapia aquaculture. Chen et al. leveraged machine learning algorithms to forecast Tilapia growth trajectories and optimize harvesting schedules in aquaponic systems [10]. Their data-driven approach showcased enhanced accuracy in yield projections and operational decision-making, highlighting the transformative potential of digital technologies in aquaculture optimization.

The integration of remote sensing technologies is also emerging as a promising tool for monitoring Tilapia populations and optimizing harvesting schedules [4]. UAVs equipped with multispectral sensors collect real-time data on Tilapia biomass distribution, enabling precision harvesting practices that optimize resource utilization and minimize environmental impacts [11].

According to Omondi et al., applying TOC in financial management can help fish farms prioritize investments that yield the highest return [12]. Ngugi et al. highlighted those inefficiencies in feed utilization, such as overfeeding or improper feed composition, can lead to significant financial losses and environmental degradation [13]. Mugo and Kim (2021) noted that aligning production cycles with market demand through careful planning and forecasting can help mitigate this constraint. By optimizing the timing of harvesting and adjusting production levels based on market trends, fish farms can ensure a steady supply of fish to meet consumer demand, stabilize prices, and maximize revenue.

Socio-economic considerations significantly influence harvesting strategies and aquaculture management policies [12]. For example, a socio-economic analysis of Tilapia farming communities examined the impact of market dynamics and policy interventions on production outcomes [5], providing valuable insights for designing targeted interventions that enhance the resilience and economic viability of aquaculture enterprises.

3. Methodology

3.1. Logistic Growth Model

The logistic growth model for population dynamics can be described by the differential equation as [6];

$$\frac{dT}{dt} = kT\left(1 - \frac{T}{N}\right) \quad (1)$$

Where T- population size
k- intrinsic growth rate
N- carrying capacity of the environment
The logistic equation predicts how a population grows

rapidly initially, the slows as it approaches the carrying capacity, N.

3.2. Maximum Sustainable Yield for Constant Harvesting

When a constant harvesting rate H is applied, the system is described as:

$$Change\ \ in\ \ population = Growth - Harvest$$

$$\frac{dT}{dt} = kT\left(1 - \frac{T}{N}\right) - H(t) \quad (2)$$

Where H(t) is the harvesting rate

3.3. Equilibrium Analysis

For equilibrium stability, the rate of change is set to Zero because population does not change.

$$0.8T\left(1 - \frac{T}{65{,}000}\right) - H = 0 \quad (3)$$

$$0.8T - 0.0000123077T^2 - H = 0 \quad (4)$$

Solving the quadratic equation, we find;

$$T \ = \frac{(-0.8) \pm \sqrt{(-0.8)^2 - 4 \times 0.0000123077H}}{2(0.0000123077)} \quad (5)$$

Setting the discriminant to zero. We obtain the maximum sustainable yield (MSY) for constant harvesting as a viable solution below.

$$H = 12999.99188$$

$$H \approx 13000$$

The sustainable harvesting amount, H at equilibrium would not exceed growth rate at 65000.

3.3.1. Harvesting Scenarios

1) Constant Harvesting: Analysed for $H < 13000H$ and $H\ \ > 13000H$
2) Periodic Harvesting: Simulated harvesting every 6 months.

3.3.2. Computational Analysis

Simulations were conducted using Python to visualize population trajectories and determine sustainable harvesting rates.

a) Logistic growth model Curve

Our analysis of the fish population growth at Omega Fish Farm suggests a trajectory following a logistic curve. Initially, the fish population will experience rapid growth as there are ample resources available. However, this growth will slow down as the fish population approaches its maximum size – the carrying capacity. This carrying capacity is determined by the limitations of the farm's environment, such as space and available food.

By understanding this logistic growth model, farm managers can anticipate population trends and make informed decisions. Monitoring the fish population and ensuring it stays below the carrying capacity is crucial to prevent resource competition, stunted growth, and fish health issues.

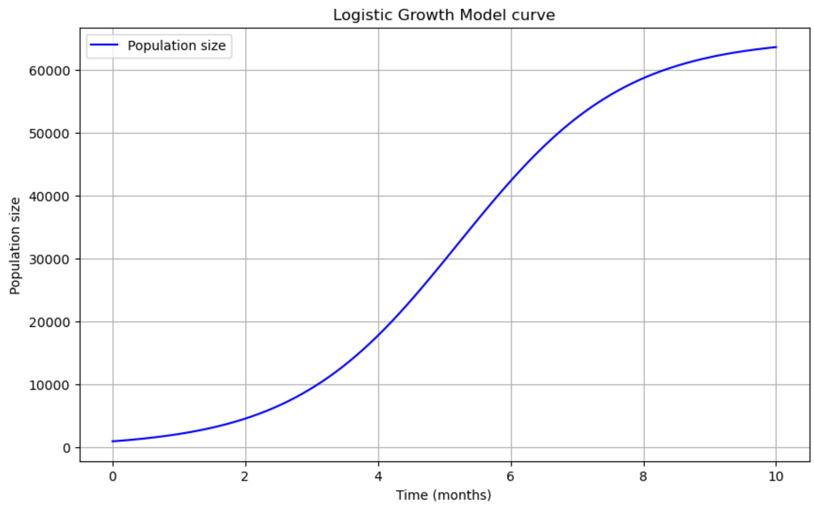

Figure 1. *Logistic Growth Model Curve.*

b) Logistic Growth Model with Constant Harvesting (H < 13000)

The logistic growth model predicts a fascinating pattern for the omega fish farm's population. Imagine the fish population starting small, with plenty of space and food to thrive. This initial period translates to rapid growth in the model, visualized by a steep upward curve. However, this growth can't continue forever. As the fish population increases, resources become limited. There's less food and space per fish, causing the growth to slow down. This slowdown is reflected in the curve flattening out. Eventually, the population reaches a stable equilibrium point – the carrying capacity. This represents the maximum number of fish the farm's environment can support in a healthy way.

Understanding this logistic model empowers farm managers to make informed decisions. By monitoring the fish population and ensuring it stays below the carrying capacity, they can prevent competition for resources. This proactive approach helps to avoid stunted growth and potential health issues, ultimately promoting a thriving fish population within the farm's limitations.

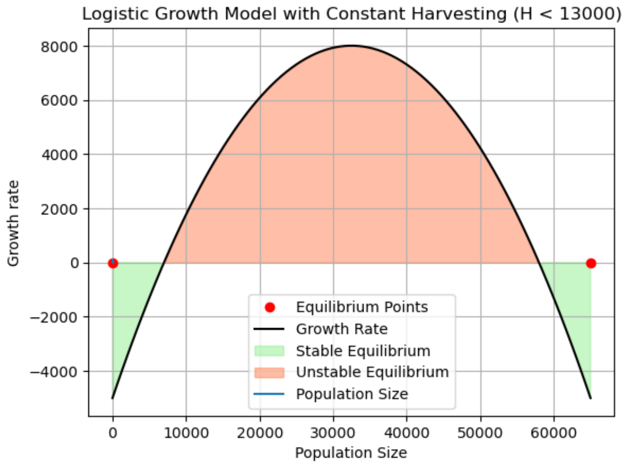

Figure 2. *Logistic Growth Model with Constant Harvesting H<13000.*

c) Logistic Growth Model with Constant Harvesting (H > 13000)

The logistic growth model with harvesting applied to the omega fish farm paints a concerning picture. While the model predicts an initial period of rapid growth due to abundant resources, the high harvesting rate (H > 13,000) throws a wrench in the works. This constant removal of fish prevents the population from ever reaching its natural carrying capacity, the sustainable maximum number of fish the farm can support.

The curve depicts two key points: a stable equilibrium points at a lower population level, and an unreachable unstable point at a higher level. The high harvesting rate ensures the population gets stuck at the lower level, potentially leading to overfishing and a collapse if not managed carefully.

Farm managers need to take action. By understanding this model and the impact of harvesting, they can adjust the harvesting rate to a more sustainable level. This will allow the fish population to reach a healthy equilibrium closer to the carrying capacity, ensuring a thriving fish farm in the long run.

d) Maximum Sustainable Yield for Periodic Harvesting

Maximum sustainable yield occurs when the population is at half the carrying capacity ($T = N/2$). At this point, the population is growing most rapidly. Given the logistic growth model, we can express the growth rate when $T = N/2$

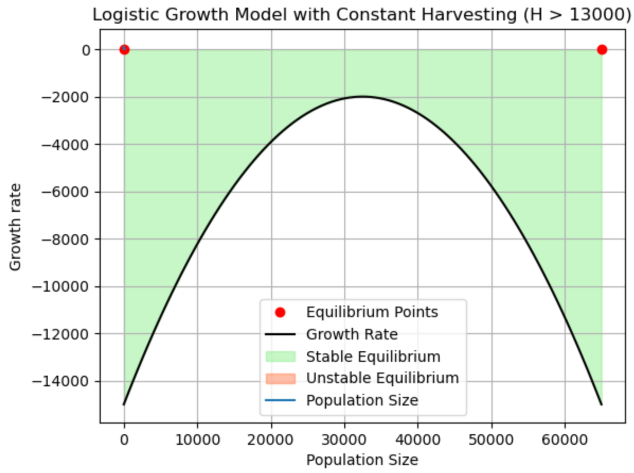

Figure 3. *Logistic Growth Model with Constant Harvesting H>13000.*

$$\frac{dT}{dt} = k(N/2)\left(1 - \frac{N/2}{N}\right) \quad (6)$$

$$\frac{dT}{dt} = k\left(\frac{N}{2}\right)(1/2) = \left(\frac{kN}{4}\right) \quad (7)$$

Thus the maximum sustainable yield is;

$$\left(MSY = \frac{KN}{4}\right) \quad (8)$$

This means that for sustainable harvesting, the periodic harvesting rate should not exceed maximum sustainable yield.

Since $k = 0.8$ and $N = 65000$,

$$MSY = \frac{0.8 \times 65000}{4} = 13000 \quad (9)$$

In this Harvesting Scenario, one would harvest 13000 of the total population at regular intervals of 6 months.

e) Logistic growth model with periodic harvesting

The simulation of the Omega Fish Farm using the logistic growth model with periodic harvesting reveals valuable insights into the dynamics of fish population management. Over the course of 80 months, the population size of fish in the farm exhibits a cyclic pattern, characterized by periodic fluctuations corresponding to harvesting events occurring every 6 months. These fluctuations indicate a delicate balance between the natural growth of the fish population and the harvesting activities implemented by the fish farm management. Notably, the oscillations in population size suggest that the fish farm is operating within sustainable limits, with the population able to recover between harvesting intervals. This sustainable dynamic underscore the importance of effective resource management practices in maintaining the long-term viability of the fish farm.

In essence, the results of the Omega Fish Farm simulation underscore the complex interplay between natural growth processes and human intervention in resource management. By leveraging insights from population dynamics, the fish farm can enhance its operational efficiency, promote sustainability, and achieve its goals of providing a reliable source of fish while preserving the health of the ecosystem.

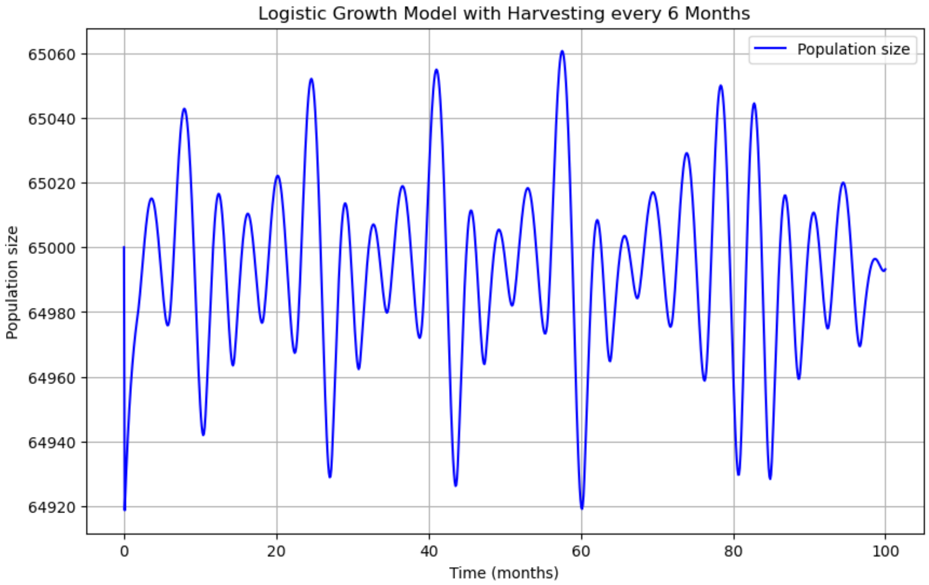

Figure 4. *Logistic Growth Model with Harvesting every 6 months.*

The ponds have full carrying capacity of 65000 tilapia fish in the ponds as an initial population. For the first six months, 16250 tilapia fish is assumed for harvesting until the population of tilapia remains 48750.0 and followed by no harvesting for the next 6 months and this pattern repeats for several years.

4. Summary of Results

4.1. Logistic Growth Model Curve

The logistic growth model for Omega Fish Farm shows an initial rapid increase in population, slowing as it approaches the carrying capacity of 65,000 Tilapia.

4.2. Constant Harvesting (H < 13000)

The population reaches a stable equilibrium below the carrying capacity, ensuring resource availability and fish health.

4.3. Constant Harvesting (H > 13000)

The population is unable to reach the carrying capacity, resulting in potential overfishing and population collapse risking extinction if not managed properly.

4.4. Periodic Harvesting

The population exhibits cyclic fluctuations, recovering between harvesting intervals since it follows the growth dynamics. This indicates a balance between natural growth and harvesting activities, promoting sustainability.

4.5. Simulation Outcomes

4.5.1. Constant Harvesting

At H≈13000H approximately 13000H≈13000, the maximum sustainable yield is achieved, balancing yield and population stability.

4.5.2. Periodic Harvesting

Harvesting 16,250 fish every 6 months, followed by no harvesting, allows the population to stabilize around 52,000, ensuring long-term sustainability.

5. Discussion

5.1. Technological Integration

1) Machine Learning: Enhances predictive accuracy for

growth trajectories and harvesting schedules.
2) Genetic Algorithms: Optimizes harvesting strategies for maximal yield and population sustainability.
3) Remote Sensing: Provides real-time biomass data, enabling precision harvesting and reduced environmental impact.

5.2. Environmental and Socio-economic Considerations

1) Climate Projections: Accounting for environmental variability is crucial for adaptive management.
2) Socio-economic Factors: Market dynamics and policy interventions significantly influence harvesting strategies and aquaculture sustainability.

6. Conclusion

This study demonstrates the effectiveness of logistic growth models in optimizing harvesting strategies for Tilapia aquaculture. By integrating advanced technologies and considering environmental and socio-economic factors, sustainable management practices can be developed. The findings emphasize the need for adaptive strategies to balance yield and population health, ensuring the long-term viability of aquaculture enterprises. Regular monitoring and adjustment of harvest strategies are essential for sustainable management.

7. Future Study

Future studies should investigate the impact of nonlinear dynamics on population growth and harvesting strategies, including potential chaos and bifurcations in population models. Additionally, it would be valuable to test the model's applicability to other aquaculture species with varying growth patterns and environmental requirements.

Abbreviations

H	Harvesting Rate
MSY	Maximum Sustainable Yield
N	Carrying Capacity of the Environment
K	Growth Rate Coefficient
T	Population Size
T	Period

Author Contributions

Dolphine Okeri: Methodology, Writing – original draft
Koech Wesley: Supervision, Writing – review & editing
Kweyu Cleophas: Supervision, Writing – review & editing

Conflicts of Interest

The authors declare no conflicts of interest.

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