Science Journal of Applied Mathematics and Statistics
2025, Vol. 13, No. 1, pp. 15-21
https://doi.org/10.11648/j.sjams.20251301.12

Review Article

Survey of Reliability Analysis of Weighted K-out-of-N System

Ragi Krishnan*

Department of Mathematics, PSG College of Technology, Coimbatore, India

Abstract

Computer controlled systems used in critical care systems often utilize redundancy to achieve the required level of reliability. However, as highly designed systems become more functionally integrated, they generally consist of several kinds of components with unique purposes. This kind of fault-tolerant system design is especially crucial for vital applications including energy systems, medical devices, and aircraft. With extensive applications in engineering systems both in industry and military, a considerable amount of research has been carried out in the area of redundant systems. The most prevalent kind of redundancy is k-out-of-n type redundancy. In order to develop high reliability systems, redundant models like k out of n systems have been developed. In normal circumstances, each component's contribution to system reliability will differ. When a system has weighted components, each component makes a unique contribution to the system's overall performance. Performance of the system is influenced by both system operation and the sum of the components that make up it. This survey paper provides a general overview of the different types of weighted- k-out-of-n: G systems, methods adapted to their reliability assessment, lifetime distributions and stochastic order of these systems. In order to analyse the various models of weighted k-out-of-n systems, an in-depth investigation is conducted.

Keywords

K-out-of-N Systems, Load Sharing System, Weighted K-out-of-N Systems, System Reliability, Component Importance

1. Introduction

Reliability has wide applications in electrical and mechanical engineering and even in the process involving humans. Some are: arrangement of batteries in series-parallel array which constitute as a backup for emergency power supply, power plants which supply power to the grid etc. The performance of any machine is highly influenced by its failure, particularly by its component failure. The failure of a machine can be balanced by either repair or by replacement of the failed component so that the functioning of the system may not suffer. Redundancy is adopted as a common approach for the improvement of reliability indices of a system. Even though it increases the complexity and cost of the system, it is the most adapted option for a highly reliable system. A wide and extensive range of research is done in the field of reliability [9, 15, 29].

A k-out-of-n type redundancy is the most adopted type of redundancy. In (k,n): G systems, minimum k components out of the n components must be working for the functioning of the (k,n): G system. In the k out of n system structure, if an operating component fails, it will be replaced by a standby

*Corresponding author: rag.maths@psgtech.ac.in (Ragi Krishnan)

Received: 24 December 2024; **Accepted:** 18 January 2025; **Published:** 10 February 2025

component automatically. Reliability analysis of various types of k- out-of-n: G system has been carried out by several researchers. Any system with spares can be given as an illustration for the by the k out of n: G system structure. If we consider any automobile with four tires, one additional spare tire will be always equipped on the vehicle. It is very well known that the vehicle can be driven whenever at least four out of the five tires are in working condition [9]. This can be considered as 4 out of 5 system. Number of components such as k and n should be decided in the early stages of designing phase itself, to conform the required specifications [13]. In the case of flight control system for air crafts and controllers in nuclear power plants, computers are used to control the system. There, by considering the minimum constitution to submit the correct output, the number k is determined. The maintenance cost and availability of the system decides the value of 'n', the number of components. If the value of 'n', is increased, then the maintenance cost of the redundant component will become higher. But, as nuclear plants are to be highly reliable, large n is highly recommendable for the tolerance of high risk.

Over the past few decades, several authors analysed k-out-of-n system structure. [14, 15, 18, 20, 21, 37]. Estimation of reliability of k-out-of-n structures by generating algorithm is done by Arulmozhi [7]. The reliability evaluation of consecutive-k-out-of-n: G systems with repair facilities [34, 38], k-out-of-n: G retrial systems with Bernoulli shocks [33], and k-out-of-n: G systems with cold standby units [2] has been investigated by researchers. Evaluation of MTTF of k out of n: G systems with different groups of components is done by [36]. Recently, research applications of k-out-of-n reliability models in modern devices such as fire alarms, phased mission systems, high altitude unmanned platforms etc. [1, 3, 28] are published. All these studies are based on the assumption that the working times, failure times and repair times of the components have independent and identical distribution.

In most of the studies of reliability analysis, the components are considered to be statistically independent. But in real scenario, when any one of the components fails, the surviving components may face a significant change in the stress because of the change in number. This in turn will affect the reliability of the system. For example, consider the overhead lines working in parallel to ensure the continuity of power supply. If any one of the lines fails, the other lines have to carry the extra load so as to maintain the supply until the fault is repaired. This can be considered as an illustration of a load sharing system. Consider a ventilation facility where the room temperature should be kept below a certain temperature. The units which are used to provide this facility is a miniature for a weighted (k,n) system. Any model, in which the components make unequal contributions for a specific requirement, then the concept of system with weighted components can be applied.

2. Weighted (k,n): G System

In real world, for the performance of any system, the contributions by each component is in different ways. The effectiveness of a system relies on its overall operation and the role each individual component. For the better performance of the system, the system must function and the sum of contributions of the components of the system must be above some threshold value. As a prototype, consider two substations from network of the distribution of electricity. These substations have direct connection by different transmission lines (components) [30, 32]. The voltage transmitted through each of the transmission lines can be considered as the weight of the line (component). Normally a certain minimum level of voltage is needed for the transmission of electricity from one substation to the other. This setup is an example of weighted (k,n) system structure with threshold value k. In this example, the reliability of the system should be calculated on the basis of both the structure and the contributions made by the components. The structure of weighted k out of n: G system consists of n number of components. Each component has its own probability for functioning and weight which is represented as a positive integer, so that the total weight of the system is W. The system operates only if the gross weight of the working components is at the least k [17, 25]. In the weighted k–out–of–n: G, if the weight of each component is 1, then it will be a general (k,n): G system.

For a weighted k out of n: G system with fixed weight, availability and reliability is analysed by several authors. An algorithm with time complexity O(n,k) to compute the reliability of a weighted (k,n) system is proposed by Wu and Chen [31]. With further improvement to this, Higashiyama [8, 34] has pointed to some alternative algorithms to calculate the reliability of weighted k out of n: G system. Eryilmaz [25] derived the MTTF of the weighted k-out-of-n system. Multi-state k-out-of-n: G systems with weighted components have been examined by Li and Zuo [17]. For the reliability analysis of weighted k-out-of-n systems, recursive and universal generating function-based methods is proposed by them. By considering the failure times of components, an elaborate investigation is given by Samaniego and Shaked [22]. Component importance measures in the reliability evaluation of weighted (k,n): G systems are studied by Armutkar and Kamalja [12] and Eryilmaz and Bozbulut [26] As an extension to this, the multi-state case is introduced by these authors. A multi-state weighted k-out-of-n systems which is applicable to transmission lines have been investigated by X Song et al. [32].

2.1. Weighted (k,n): G System Which Comprises of Two Different Kinds of Components

Eryilmaz and Sarikaya [24] considered the reliability of a weighted k out of n: G system in which the components are classified into two types according to their weight/ capacity. In the designing structure of the system, the common weight ω and reliability p_1 is assigned to one fixed group of components, while the components in the other fixed group have equal weight ω* which is different from ω and reliability p2. Each of these are different. For the random selection of components to have the required weight, binomial distribution is used. Using classical probability techniques, the characteristics of the reliability of the system, is evaluated. By considering minimum required reliability and minimum total acquisition cost, the optimal number of components to be there in each group are also determined by using linear programming technique. By doing the evaluation using component importance, author has analysed that the importance of components in group 1 is the same for all. Similar is the case for group 2 also. Moreover, it has been shown that the component with higher weight, need not have more component importance. Also, the dependency between the components is also verified by the researcher.

2.2. Weighted (k, n): G system Consisting of Two Types of Non-identical Components with Positive Integer-valued Weight

Consider a tile manufacturing process. The raw materials used to form the tile are mined from the earth's crust. After processing the raw material, a number of steps used to get the ceramic tile. Some of the processes are batching, mixing and grinding, spray-drying, forming, drying, glazing, and firing. For all these processes different machines are used with different capacity. Finally, to have a good tile, some machines have to work with certain capacity of the total capacity. This can be considered as a (k,n): G structure with weighted-components all of which are non-identical.

Eisa and Meshkat [4] introduced another type of weighted-k-out-of- n: G system which is constructed from two different varieties of non-identical components with unequal weights. The system consists of n components all of which are non-identical and classified as two categories with respect to their functions and services. In the first group there are n1 components each with weight ω_i and reliability p_{1i}. where the second group has n_2 components and with weight ω^*_i and reliability p_{2i}. The system will work only if the gross weight of the working component is higher than a pre-specified value k which is called as its threshold. Compared to other studies in the literature, nonidentical components are dealt in this work. All possible combinations of the components to have minimum required weight are considered in this research work. Survival analysis, component importance and other reliability parameters are derived in this work.

2.3. Weighted (k, n): G Systems Containing Two Categories of Components and One Cold Standby Component

Different types of standby components are used to increase the system reliability such as hot, cold and warm standby. If the switching times between the components is sufficiently short, then cold standby is preferred compared to hot and warm standby. Cold standby components do not fail in standby. C Franko et al., [6] have given a real time situation at which (k,n): G system having weighted-components and one component kept in cold- standby is applied. Suppose a company has n_1 machines which are having identical failure time distributions and capacity of production ω. After some time if the company wants to increase its production capacity, then it can add n_2 new machines with capacity of production ω* or simply they can restore the existing machines. But this may increase the production cost. To overcome this problem, the factory can keep standby machines which can be used whenever necessary. [6] has introduced the weighted k out of n: G system which comprises two groups of components and one cold standby component. Upon failure of the r^{th} (r=1, 2,...., n) component, this system may fail. The failure of the system depend on the weights of the components and k, the threshold value. In this proposed system, if the failure of the system is occurred by the r^{th} component fault then, the component which is in cold standby will come into operating state. Thus, the system continues its operation with the remaining 'n-r' components, together with the replaced component in standby. With the addition of one cold standby component, the structure of the system becomes complex and the reliability calculation is done with this complex structure. With the help of a numerical example, the optimal cost of the system through the addition of the cold standby component is also derived.

2.4. Weighted (k,n) System Consisting of Components in Three Different States

In recent research, Eryilmaz and Bozbulut [26, 5] studied a (k, n) system with components having some assigned weights. In this model, author has done the reliability analysis of such systems with its components in three different states. The three-state classification is done on the basis of functioning of the states as perfect, partial and complete failure. Here at any given time, the 'n' units of the device could be in any one of these three different states. If the component 'i' is in a perfectly functioning state, then it has assigned a weight ωi. Meanwhile weight is ωi* for partially functioning component. Obviously ω_i is greater than ω_i*. The system operates only if the gross weight of all the functioning components is at least k. A real time illustration is given as a combi boiler. These boilers are used as a water heater and a central heating boiler which is used in homes. This combi boiler has many components like thermostat, sensors and boiler fan. As time passes, the degradation of components takes place. It will decrease the performance of the boiler. But the combi boiler still functions with reduced capacity. It can be modelled as a weighted system with three-state components.

Here, the author has taken the assumption that, time spent by components in functioning states i.e. perfect and partial are dependent to each other and it can be represented as a given joint probability distribution. With this assumption, the expressions for reliability of the system are derived. Recursive and non-recursive equations for the derivation of the reliability of the system are given in this research article.

2.5. Performance Study of a Weighted k-out-of-n System with Randomly Weighted Components

All research in reliability analysis of weighted system which was discussed till now have components with fixed weights. Eryilmaz [27] introduced k-out-of-n systems in which the weights of the components are random. In this work, system performance is measured by counting the working components and also proper functioning of the components. If the number of working components is atleast k and the gross weight of all working components is above c, then the system is said to be in state c. In the analysis of reliability indices for such types of systems both the structure of the (k,n) system and the gross weight of the units which are working is under consideration. Here, the system functions only if there is a minimum of k number of working components. Also, if the gross weight of all the units that are working is at least 'c' then, the device is considered to be in state 'c' or above. Real time examples of such systems are also given by the author. They are (i) A queueing system which consists of n parallel servers with random capacities w_1,....., w_n. Here w_i denote the number of customers served by the ith server on a specific day for i=1,..., n. The probability of ith server working on a particular day is denoted with p_i. Failure of such a system happens when either all servers are down or the gross capacity of functioning servers is below c.(ii) A system which is at the risk of getting shocks randomly over time periods t = 1, 2,..... The shock happens periodically in an interval of i, with probability pi. Here the amount of the shock that is experienced in period i is designated as w_i.

The failure of these types of system happens when there is at least k shocks, and also the cumulative total of shocks exceeds the level c.

To analyse the reliability, the structure of the system is taken under consideration. Also, author assumed the components have random weights each of which is obtained by a given probability distribution of discrete type. Since the random variables which denote the order of the lifetimes of components is statistically dependent, the survival function cannot be explicitly derived. Therefore, a recurrence formula for state transition probabilities has been obtained by the author. By using Monte Carlo simulation, the time span of the system in each state has been derived in this work.

2.6. k-out-of-n Systems with Weighted Randomly Chosen Components

Salehi et al. [19] had research to investigate the stochastic properties and reliability of weighted k-out-of-n systems having random number of components. Also, the components are classified as from two different types. Here the study is made up on the assumption that the system is constructed by using two non-identical types of components. The classification of the components is done according to the weights of the components and reliability functions. Randomly a group of units say M, are assigned from group one and the remaining components are selected from group two. For modelling dependency among the units, copula functions are one of the best applied methods. Thus, structural dependency among the constituent units is derived using copula method. Here probability mass function of the random variable M is calculated and the reliability expression is in terms of this probability mass function This expression for reliability has fixed number of units of both types and it is a combination of reliabilities of weighted (k, n) systems of these types. Also, it has been shown that if the random mechanism of the chosen components is stochastically ordered, then the weights and the components' lifetimes are also of stochastic order under some conditions.

2.7. Weighted (k,n) System Where Component Life-Times Are Statistically Dependent

It should be noted that most of the research in the reliability analysis of (k, n) systems with weighted components, each of which is statistically independent. Consider a mechanical system with various components which suffer from a common stress. It will be always subjected to regular stress. When the components are unable to withstand the stress, their supporting ability will be collapsed. If the total supporting capacity of the system gets below some threshold value or some designed level then, the entire system will break down. Here, if the stress in one of the components becomes higher, it will be affecting the other components also. This type of system can be designated as a weighted system having component lifetimes are statistically dependent.

Li et al. [16] studied the weighted (k, n) system with components having lifetimes which are statistically dependent. In probability related areas such as evaluation of reliability of systems, risk assessment etc., stochastic orders have a fundamental role. The variation of the gross weight of the system due to weight of the component is described by vector stochastic orders in this work. In order to take account of the statistical dependence among component lifetimes, stochastic arrangement increasing lifetimes of the components and their weak versions, like multivariate non-parametric dependence notions due to Cai and Wei [10, 11] are also carried out in this work.

2.8. Two-Stage Weighted (k,n) Systems with Common Components

By comprising a number of subsystems, two stage weighted (k, n) system is formed. Here 'm' denotes the count of subsystems. If m=1, the two-stage k-out-of-n system with weighted components will become a weighted (k, n) system. The structure of each subsystem is a weighted (k,n) structure, which is named as the second-level structure. The structure in the first level is a certain kind of coherent structure to show the inter- relationship between the sub systems. Special case of two stage weighted k-out-of-n: G system are:

1. Series weighted (k, n) system whose level one is a series structure,
2. Parallel weighted (k,n) system whose level one is a parallel structure. Unlike most of the two-stage models in the literature, the subsystems have some (or all) components. Therefore, the evaluation of the reliability of the system cannot be done only by combining the probability of failure of each subsystem.

Consider a power grid, which is an interconnected network for electricity distribution. It consists of power stations, electrical substations, electrical power transmission and electric power distribution. The gross power output from the grid is calculated as the weighted sum of each of these components. The reliability of the grid is defined by considering the structure of the grid and the weighted reliability of its components. This is an example of a multi-stage system. Chen and Yang [35] defined a weighted-k-out-of-n system having components in two-stage. In this work, the common components between the two subsystems are identified by the non-zero weights in failure condition of each subsystem. To evaluate the reliability, first minimal paths and minimal cuts are generated for weighted k-out-of-n: G system with one-stage. Then, by applying the Boolean algebra, the bounds for reliability of the dependent failures of the component is analyzed by the author [35].

2.9. Weighted (k, n) System: Components with Discrete Lifetimes

Consider the suspension system for cars which are designed to absorb shocks from uneven road surfaces. In order to keep the automobile body from being too severely damaged when it strikes a bump, the suspension system—which consists of springs, dampers, and other parts—absorbs the shock. In this scenario, the suspension's response to the sudden forces (shocks) delivered through the wheels and tires would be described by the shock model. These models are used by engineers to determine the optimal spring constants and damping coefficients to reduce discomfort and avoid vehicle damage. Eryilmaz [23] modeled such system with n components where each component is subjected to periodic shocks. A shock that strikes during period i has a probability of pi for the ith component and non-fatal with probability 1-pi and such shock occurrences are modelled by discrete time shock processes. The component lifetimes have a geometric distribution in this specific shock model. The system's dynamic behavior during the successive failures has been examined. Failures of components may happen at the same time in the discrete scenario. Using analytical approach, the overall system weight or performance at the end of the tth period is computed if the shocks happen independently throughout the periods. Thus, the mean of the lost capacity of the system during multiple failures have been derived in this model.

3. Conclusion

If we consider a production unit, the lifetime distributions of the components in the unit will not be the same. It may be because of the difference in material supplied, available human resources, and unstable conditions in the industry or because of some other factors. Reliability analysis deals with the working conditions and failure time distributions of different types of components in the system, Also, sometimes they will be following different probability distributions. When a system has weighted components, each component makes a different contribution to the system's overall performance. Therefore, it is important to analyse such system. This paper tried to survey the recent literature on (k,n) systems with weighted components and collected the methods adopted in the reliability evaluation of such kind of systems. These types of weighted systems are also terminated as threshold systems. Currently, k-out-of-n systems are used to model case studies in high altitude unmanned platforms, unmanned aerial vehicles (UAVs), wireless sensor networks, phased mission systems, fire alarm systems, etc. Future studies can use (k,n) modeling to expand to dynamic voltage and frequency scaling, sustainable Cloud-Fog deployments, and IoT applications with multi-agent microservices. A comprehensive list of references is provided which will enable us to do further research on related fields.

Abbreviations

(k,n)	k-out-of-n
IoT	Internet of Things
UAV	Unmanned Aerial Vehicles
MTTF	Mean Time to Failure
MTBF	Mean Time Between Failure

Author Contributions

Ragi Krishnan is the sole author. The author read and approved the final manuscript.

Conflicts of Interest

The author declares no conflicts of interest.

References

[1] Aliakbar Eslami Baladeh, Sharareh Taghipour, "A hierarchical k-out-of-n optimization model for enhancing reliability of fire alarm systems", Process Safety and Environmental Protection, Volume 191, Part A, 2024, Pages 401-416, https://doi.org/10.1016/j.psep.2024.08.091

[2] Anna Dembińska, Nikolay I. Nikolov, Eugenia Stoimenova, "Reliability properties of k-out-of-n systems with one cold standby unit", Journal of Computational and Applied Mathematics, Volume 388, 2021, 113289, https://doi.org/10.1016/j.cam.2020.113289

[3] Chaonan Wang, Shenghui Luo, Guizhuang Chen, Zhitao Wu, Wenjing Rong, Quanlong Guan, "Phase Combination for Reliability Analysis of Dynamic k-out-of-n Phase-AND Mission Systems", Reliability Engineering & System Safety, 2025, 110817, https://doi.org/10.1016/j.ress.2025.110817

[4] Eisa Mahmoudi and Rahmat Sadat Meshkat, "Reliability Analysis of Weighted- k-out-of- n: G System Consisting of Two Different Types of Nonidentical Components Each with its Own Positive Integer-Valued Weight," Journal of Statistical Theory and Applications, Vol. 19(3), pp. 408–414, 2020. https://doi.org/10.2991/jsta.d.200917.002

[5] Eryilmaz S, Bozbulut AR. Reliability analysis of weighted-k-out-of-n system consisting of three-state components. Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability. 2019; 233(6): 972-977. https://doi.org/10.1177/1748006X19852206

[6] Franko, C., Tütüncü, G. Y., & Eryilmaz, S. "Reliability of weighted k-out-of-n: G systems consisting of two types of components and a cold standby component," Communications in Statistics - Simulation and Computation, *46*(5), 4067–4081. https://doi.org/10.1080/03610918.2015.1096377

[7] G. Arulmozhi,. ', Direct method for reliability computation of k-out-of-n: G systems, Applied Mathematics and Computation (2003) 421–429, https://doi.org/10.1016/S0096-3003(02)00373-9

[8] Higashiyama, "A factored reliability formula for weighted k–out–of–n system," Asia–Pacific Journal of Operational R search, Vol. 18, pp. 61–66, 2001.

[9] Hong Pham, Handbook of Reliability Engineering, Springer 2003.

[10] Jun Cai, Wei Wei, Notions of multivariate dependence and their applications in optimal portfolio selections with dependent risks, Journal of Multivariate Analysis, Volume 138, 2015, Pages 156-169, https://doi.org/10.1016/j.jmva.2014.12.011

[11] Jun Cai, Wei Wei, Some new notions of dependence with applications in optimal allocation problems, Insurance: Mathematics and Economics, Volume 55, 2014, Pages 200-209, https://doi.org/10.1016/j.insmatheco.2014.01.009

[12] Kalpesh P. Amrutkar and Kirtee K. Kamalja, "Reliability and importance measures of weighted-k-out-of-n system," International Journal of Reliability Quality and Safety Engineering, vol 21 No. 3, 2014, https://doi.org/10.1142/S0218539314500156

[13] Kodo Ito, Xufeng Zhao, and Toshio Nakagawa, "Random number of units for K -out-of- n systems," Applied Mathematical Modelling, 45, pp. 563–572, 2017, https://doi.org/10.1016/j.apm.2017.01.027

[14] Kullstam, P. A, "Availability, MTBF and MTTR for repairable m-out-of-n system," IEEE Transactions on Reliability, Vol. R-30, pp. 393-394, 1981.

[15] Kuo, W. and Zuo, M. J., "Optimal reliability modeling, principles and applications", John Wiley & Sons, New York, 2003.

[16] Li X, You Y and Fang R, "On weighted k-out-of-n systems with statistically dependent component lifetimes,". Probability in the Engineering and Informational Sciences. 2016; 30(4): 533-546. https://doi.org/10.1017/S0269964816000231

[17] Li, Wei &Zuo and Ming J, "Reliability evaluation of multi-state weighted k-out-of-n systems," Reliability Engineering and System Safety, Elsevier, vol. 93(1), pages 160-167, 2008, https://doi.org/10.1016/j.ress.2006.11.009

[18] M. S. Moustafa, "Transient analysis of reliability with and without repair for k-out-of-n system with two failure models," Reliability Engineering & System Safety, 53(1): 31-35, 1996.

[19] M. Salehi, Z. Shishebor and M. Asadi, "On the reliability modeling of weighted k-out-of-n systems with randomly chosen components," Metrika: International Journal for Theoretical and Applied Statistics, Springer, vol. 82(5), pages 589-605, July, 2019.

[20] R. E. Barlow and K. D. Heidtmann, "Computing k-out-of-n system reliability," IEEE Transactions on Reliability, vol. R-33, pp. 322–323, 1984.

[21] S. P. Jain and K. Gopal, "Recursive algorithm for reliability evaluation of k-out-of-n: G system," IEEE Transactions on Reliability, vol. R-34, pp. 144–146, 1985.

[22] Samaniego FJ and Shaked M. "Systems with weighted components," Statistics and Probability Letters, vol. 78: 815–823, 2008. https://doi.org/10.1016/j.spl.2007.09.049

[23] Serkan Eryilmaz, "Reliability and performance evaluation of weighted k-out-of-n: G system consisting of components with discrete lifetimes", Reliability Engineering & System Safety, Volume 252, 2024, 110484, https://doi.org/10.1016/j.ress.2024.110484

[24] Serkan Eryilmaz and Kadir Sarikaya, "Modeling and analysis of weighted-k-out-of-n: G system consisting of two different types of components," Proc IMechE Part O: J Risk and Reliability, Vol. 228(3), pp. 265–271, 2014. https://doi.org/10.1177/1748006X13515647

[25] Serkan Eryilmaz, "Mean Time to Failure of Weighted k-out-of-n: G Systems," Communications in Statistics - Simulation and Computation, 44: 10, pp. 2705-2713, 2015. https://doi.org/10.1080/03610918.2013.844836

[26] Serkan Eryilmaz, Ali Riza Bozbulut, Computing marginal and joint Birnbaum, and Barlow–Proschan importances in weighted-k-out-of-n:G systems, Computers & Industrial Engineering, Volume 72, 2014, Pages 255-260, https://doi.org/10.1016/j.cie.2014.03.025

[27] Serkan Eryilmaz, "On reliability analysis of a k-out-of-n system with components having random weights," Reliability Engineering and system safety, vol 109, pp. 41-43, 2013, https://doi.org/10.1016/j.ress.2012.07.010

[28] Vishnevsky, V., Selvamuthu, D., Rykov, V., Kozyrev, D., Ivanova, N., Krishnamoorthy, A, "Reliability Assessment of Tethered High-Altitude Unmanned Telecommunication Platforms - k-out-of-n Reliability Models and Applications". Springer, Singapore (2023).

[29] W. Kuo and M. J. Zuo, "Optimal Reliability Modeling," Chapter 7, John Wiley & Son, pp.258-264, 2003.

[30] Wang C, Feng K, Zhang H and Li Q, "Seismic performance assessment of electric power systems subjected to spatially correlated earthquake excitations," Structure and Infrastructure Engineering 15(2): 1-11. https://doi.org/10.1080/15732479.2018.1547766

[31] Wu J S, Chen R J, "An algorithm for computing the reliability of a weighted-k-out-of-n system," IEEE Transactions on Reliability, Vol 43, No. 2, pp 327–328, 1994.

[32] Xiaogang Song, Zhengjun Zhai, Yangming Guo, Peican Zhu and Jie Han, "Approximate Analysis of Multi-State Weighted k-Out-of-n Systems Applied to Transmission Lines," Energies, 10(11), 1740, 2017. https://doi.org/10.3390/en10111740

[33] Xiaoyun Yu, Linmin Hu, Mengrao Ma, Reliability measures of discrete time k-out-of-n: G retrial systems based on Bernoulli shocks, Reliability Engineering & System Safety, Volume 239, 2023, 109491, https://doi.org/10.1016/j.ress.2023.109491

[34] Y. Higashiyama and V. Rumchev, "A method for reducing the number of reliability formulae in the reliability expression of weighted-k-out-of-n system," International Journal of Performance Engineering, vol. 4, no. 1, pp. 19-29, 2008.

[35] Yong Chen and Qingyu Yang, "Reliability of two-stage weighted-k-out-of-n systems with components in common," in IEEE Transactions on Reliability, vol. 54, no. 3, pp. 431-440, Sept. 2005, https://doi.org/10.1109/TR.2005.853274

[36] Yueqin, Wu and Jiancheng, Guan. "Repairable Consecutive -k-out-of-n: G systems with R repairmen", IEEE Transactions on Reliability, Vol. 54, No. 2, pp. 328–337, June 2005.

[37] Zhang, T., Min, Xie and Michio, Horigome. "Availability and Reliability of k-out-of-(M+N): G warm standby systems", Reliability Engineering and System Safety, Vol. 91, pp. 381-387, 2006, https://doi.org/10.1016/j.ress.2005.02.003

[38] Zhang, Y. L. and Y. Lam, "Reliability of consecutive-k-out-of-n: G repairable system", International Journal of System Science, Vol. 29, pp. 1375–1379, 1998.