
Stochastic Integration with Respect to Fractional Brownian Sheets and Applications to Anisotropic SPDEs

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Abstract: This paper develops a comprehensive framework for stochastic integration with respect to multidimensional fractional Brownian sheets, with particular emphasis on the anisotropic setting where each Hurst index exceeds one-half. We introduce a regularization-based approach that accounts for the directional structure of the sheet and establish its equivalence to the Malliavin calculus construction. Building on this foundation, we obtain wellposedness and regularity results for a class of stochastic partial differential equations driven by such sheets. The regularization method provides an intuitive interpretation of the integral while preserving the directional heterogeneity inherent in anisotropic models. We prove that the regularized integral converges in the mean-square sense to the Skorokhod integral, establishing a duality relation that connects our constructive approach to established analytical frameworks. Our well-posedness results are obtained through a fixed-point argument in suitably chosen solution spaces, while the regularity analysis relies on anisotropic versions of the classical Kolmogorov criterion. Notably, the Hölder exponents we obtain directly reflect the directional smoothness of the driving sheet. This feature has practical implications for numerical discretization and statistical estimation. Our findings extend known one-dimensional theories to genuinely multidimensional and anisotropic regimes, offering new tools for modelling systems with long-range dependence and directional heterogeneity. Applications include fluid flows with directional turbulence, financial markets with correlated assets exhibiting different memory properties, and biological tissues with anisotropic diffusion characteristics.

Keywords: Stochastic Partial Differential Equations, Fractional Brownian Sheets, Anisotropic Regularity, Malliavin Calculus, Stochastic Integration, Regularization Method, Hölder Continuity, Long-Range Dependence

1. Introduction

Stochastic partial differential equations (SPDEs) have become indispensable in the mathematical description of randomly evolving spatial systems [1], with applications ranging from turbulent fluid flow and population dynamics to mathematical finance [2]. When the driving noise exhibits long-range temporal or spatial correlations, fractional Brownian motion (FBM) and its multiparameter analogues—fractional Brownian sheets (FBS)—provide a natural modelling framework [3]. These processes capture memory effects that standard Brownian motion cannot [4], making them particularly suitable for systems where past states influence future evolution in a non-Markovian manner [5].

While the theory of stochastic integration with respect to

one-parameter fractional Brownian motion is well understood [6], the passage to the multiparameter setting introduces substantial challenges [7]. These arise chiefly from the anisotropic regularity of the sheet—each coordinate direction may possess a different Hölder exponent—and from the non-trivial interaction between directions encoded in the covariance structure [8]. Much of the existing literature either assumes isotropy or treats the directions independently [9], thereby neglecting important features present in many applied contexts such as fluid flows with directional turbulence, financial markets with correlated assets exhibiting different memory properties [10], or biological tissues with anisotropic diffusion characteristics [11].

Recent advances in the theory of multiparameter stochastic processes have opened new perspectives for studying

anisotropic systems [13]. However, a comprehensive framework for stochastic integration with respect to anisotropic fractional Brownian sheets and its application to SPDEs remains an active area of research [14].

In this work, we focus on the case $H_k > \frac{1}{2}$, where the noise is sufficiently regular to allow a pathwise-type integration theory [15]. This regime is not only mathematically tractable but also physically relevant for many applications where correlations decay slowly [16]. Our primary aims are threefold [17]:

- 1) to construct a stochastic integral with respect to an anisotropic fractional Brownian sheet via a regularisation procedure that respects its directional structure,
- 2) to establish existence, uniqueness, and spatial-temporal regularity of solutions to semilinear SPDEs driven by such a sheet,
- 3) to illustrate how the anisotropy of the sheet influences the regularity and qualitative behaviour of the solution through explicit Hölder exponents [20].

The regularisation approach we adopt has both theoretical and practical appeal [21]: it yields an intuitive interpretation of the integral and suggests natural numerical approximations [22]. We show that it coincides with the Skorokhod integral defined via Malliavin calculus (MC) [23], thereby connecting it to a well-established analytical framework [24]. This connection is non-trivial in the anisotropic case, as the reproducing kernel Hilbert space (RKHS) associated with the sheet has a product structure that must be carefully handled [25].

Our well-posedness results are proved using a fixed-point argument in a suitably chosen solution space [26], while the regularity analysis relies on anisotropic versions of the classical Kolmogorov criterion [27]. Notably, the Hölder exponents we obtain reflect the directional smoothness of the driving sheet, a feature that cannot be captured by isotropic models [28]. For instance, if the sheet is smoother in the temporal direction than in some spatial directions, this asymmetry propagates to the solution of the SPDE, with implications for numerical discretization and statistical estimation [29].

The paper is organised as follows. Section 2 recalls the definition and fundamental properties of fractional Brownian sheets, emphasizing their anisotropic nature [30]. In Section 3, we construct the stochastic integral via regularisation and establish its key properties, including isometry and duality relations. Section 4 is devoted to the study of a class of semilinear SPDEs, where we prove existence, uniqueness, and anisotropic regularity of solutions through careful estimates of the stochastic convolution term. We conclude in Section 5 with a discussion of potential applications and directions for future research, particularly regarding rough sheets and non-Gaussian extensions.

2. Fractional Brownian Sheets: Basic Properties

Definition 2.1. Let $H = (H_1, \dots, H_N) \in (0, 1)^N$. A fractional Brownian sheet (FBS) $B^H = \{B^H(t), t \in \mathbb{R}_+^N\}$ is a centered Gaussian random field whose covariance function is given by

$$\mathbb{E}[B^H(t)B^H(s)] = \prod_{k=1}^N \frac{1}{2} (|t_k|^{2H_k} + |s_k|^{2H_k} - |t_k - s_k|^{2H_k}).$$

When $N = 1$ this reduces to the usual fractional Brownian motion (FBM) with Hurst parameter H_1 . For $N > 1$ the field can be viewed as a tensor product of independent one-parameter fractional Brownian motions, though correlated constructions are also possible in certain applications [24].

The covariance structure immediately reveals two important features. First, the sheet is separable in the sense that the covariance factors as a product over directions. Second, when $H_k = \frac{1}{2}$ for all k we recover the standard Brownian sheet. The following properties are direct consequences of this structure [25].

Proposition 2.1 (Self-similarity and regularity). The fractional Brownian sheet satisfies:

- 1) Self-similarity: for any $a > 0$

$$B^H(at) \stackrel{d}{=} a^{\sum_{k=1}^N H_k} B^H(t).$$

This scaling property reflects the different roughness indices in each direction.

- 2) Anisotropic Hölder regularity: with probability one, the sample paths are locally Hölder continuous of order $\alpha_k < H_k$ in the k -th direction. This means that for almost every sample path, there exists a constant $C(\omega) > 0$ such that

$$|B^H(t + he_k) - B^H(t)| \leq C(\omega)|h|^{\alpha_k}$$

for small $|h|$ where e_k is the unit vector in the k -th direction [26].

- 3) Non-stationarity of increments: unless $H_1 = \dots = H_N = \frac{1}{2}$, the increments of B^H are not stationary. This complicates the analysis but reflects realistic scenarios where scaling properties vary with location [27].

The anisotropic nature of the regularity is particularly important for our purposes: it means that the sheet may be rougher in some directions than in others. This directional heterogeneity must be carefully accounted for in both the integration theory and the analysis of SPDEs [28]. For instance, when $H_1 \neq H_2$, the sample paths exhibit different oscillatory behaviors along the two axes, which influences how we approximate integrals and estimate solutions [29].

3. Stochastic Integration via Regularisation

For $H \in (\frac{1}{2}, 1)^N$ the fractional Brownian sheet is sufficiently regular to allow a pathwise-type construction of stochastic integrals [21]. We present here a regularisation approach that is well-suited to the anisotropic setting and provides an intuitive understanding of the integration mechanism.

Let ϕ_ϵ be a smooth mollifier on $\mathbb{R}^N$ (for instance, a Gaussian kernel with variance ϵ), and define the regularised noise

$$\dot{B}_\epsilon^H(t) = \int_{\mathbb{R}^N} \phi_\epsilon(t-s) \dot{B}^H(s) ds,$$

where $\dot{B}^H$ denotes the formal derivative of B^H a distribution-valued Gaussian process. For a deterministic or random field

$$\mathbb{E}[|I_\epsilon(f)|^2] = \int_{\mathbb{R}^N \times \mathbb{R}^N} \left(\int_{\mathbb{R}^N} f(t) \phi_\epsilon(t-s) dt \right) \left(\int_{\mathbb{R}^N} f(u) \phi_\epsilon(u-s) du \right),$$

where R_H is the covariance kernel of the fractional white noise. For $H_k > \frac{1}{2}$, this kernel has the explicit form

$$R_H(s, s') = \prod_{k=1}^N H_k(2H_k - 1) |s_k - s'_k|^{2H_k-2}.$$

As $\epsilon \rightarrow 0$, the mollifiers converge to Dirac masses, and we obtain

$$\lim_{\epsilon \rightarrow 0} \mathbb{E}[|I_\epsilon(f)|^2] = \int_{\mathbb{R}^N \times \mathbb{R}^N} f(t) f(u) R_H(t, u) dt du = \|f\|_{\mathcal{H}}^2,$$

where $\mathcal{H}$ is the reproducing kernel Hilbert space (RKHS) associated with B^H [30].

The identification with the Skorokhod integral follows from a duality argument: for any smooth random variable F , one shows that

$$\lim_{\epsilon \rightarrow 0} \mathbb{E}[I_\epsilon(f)F] = \mathbb{E}[(D^H F, f)_{\mathcal{H}}],$$

where D^H denotes the Malliavin derivative. Since smooth random variables are dense, this characterizes the limit uniquely as $\delta^H(f)$ [13].

Theorem 3.2 (Properties of the integral). The integral operator I satisfies:

$$\begin{cases} \partial_t u(t, x) = \mathcal{L}u(t, x) + F(t, x, u(t, x)) + \sigma(t, x, u(t, x)) \dot{B}^H(t, x), \\ u(0, x) = u_0(x), \quad x \in \mathbb{R}^N, \end{cases}$$

where $\mathcal{L}$ is a second-order elliptic operator (for example, the Laplacian or a more general diffusion operator), F and σ are the diffusion coefficients satisfying growth and regularity conditions, and the noise term is interpreted in the sense of

f satisfying appropriate regularity conditions, we then set

$$I_\epsilon(f) = \int_{\mathbb{R}_+^N} f(t) \dot{B}_\epsilon^H(t) dt.$$

This expression is well-defined pointwise for each $\epsilon > 0$ since $\dot{B}_\epsilon^H$ is a smooth function [22].

Theorem 3.1 (Convergence of the regularised integral). Let $H \in (\frac{1}{2}, 1)^N$ and assume that f belongs to the Sobolev-type space $\mathbb{D}^{1,2}(\Omega)$ with suitable directional regularity. Then the limit

$$I(f) = \lim_{\epsilon \rightarrow 0} I_\epsilon(f)$$

exists in $L^2(\Omega)$ and coincides with the Skorokhod integral $\delta^H(f)$ defined via Malliavin calculus (MC) [23]. Moreover, the convergence is uniform over compact sets of integrands.

Main arguments. The proof proceeds by analyzing the L^2 norm of $I_\epsilon(f)$ as $\epsilon \rightarrow 0$. A direct computation shows that

- 1) Isometry: $\mathbb{E}[I(f)^2] = \|f\|_{\mathcal{H}}^2$, which generalizes the Itô isometry to the anisotropic fractional setting [14].
- 2) Duality: for any smooth random variable F

$$\mathbb{E}[I(f)F] = \mathbb{E}[(D^H F, f)_{\mathcal{H}}],$$

providing an integration-by-parts formula essential for analysis [15].

- 3) Linearity and continuity: The mapping $f \mapsto I(f)$ is linear and continuous from $\mathcal{H}$ to $L^2(\Omega)$ [16].

The regularisation approach has several advantages: it is intuitive, it preserves the directional structure of the sheet, and it suggests straightforward numerical approximations [17]. Moreover, Theorem 3.1 ensures that it is consistent with the more abstract Malliavin calculus framework, allowing us to leverage established tools such as the Clark-Ocone formula and Meyer's inequalities when needed [20].

4. SPDEs Driven by Fractional Sheets

4.1. Problem Formulation

We consider the semilinear stochastic partial differential equation (SPDE)

the integral constructed in Section 3. The spatial variable x belongs to $\mathbb{R}^N$, though bounded domains with appropriate boundary conditions could also be considered [18].

This class of equations arises naturally in several contexts

[19]. For instance, when $\mathcal{L} = \Delta$ and σ is constant, (1) models heat propagation in a medium with random heat sources exhibiting long-range correlations. The anisotropy of the noise allows different correlation structures in different spatial directions, which is physically realistic for many materials

[11].

4.2. Well-posedness

To analyze (1), we work with the mild formulation

$$u(t) = S(t)u_0 + \int_0^t S(t-s)F(s, u(s))ds + \int_0^t S(t-s)\sigma(s, u(s))dB^H(s),$$

where $S(t)$ denotes the semigroup generated by $\mathcal{L}$. The last term is a stochastic convolution, defined using the integral from Section 3.

Let us introduce the solution space

$$E = L^2(\Omega; C([0, T]; L^2(\mathbb{R}^N))),$$

equipped with the norm

$$\|u\|_E = \left(\mathbb{E} \left[\sup_{t \in [0, T]} \|u(t)\|_L^2 \right] \right)^{1/2}.$$

This space captures both the spatial regularity (L^2 in space) and the temporal regularity (continuous paths in time), while also accommodating the random nature of the solution [1].

Theorem 4.1 (Existence and uniqueness). Assume that:

1. $H_k > \frac{1}{2}$ for all $k = 1, \dots, N$,
2. F and σ are globally Lipschitz in the third variable, uniformly in (t, x) ,
3. $u_0 \in L^2(\mathbb{R}^N)$.

Then equation (1) admits a unique mild solution $u \in E$ for any $T > 0$ [2].

Key steps. The proof follows a fixed-point argument in the space E . Define the mapping $\Gamma : E \rightarrow E$ by

$$\Gamma(u)(t) = S(t)u_0 + \int_0^t S(t-s)F(s, u(s))ds + \int_0^t S(t-s)\sigma(s, u(s))dB^H(s).$$

We need to show that Γ is a contraction on E for sufficiently small T , and then extend the solution to arbitrary time intervals by patching [3].

The deterministic terms are estimated using standard semigroup theory and the Lipschitz condition on F . For the stochastic convolution term, the crucial estimate is

$$\mathbb{E} \left[\left\| \int_0^t \Phi(s)dB^H(s) \right\|_L^2 \right] \leq C_H \int_0^t (t-s)^{2\bar{H}-2} \mathbb{E} [\|\Phi(s)\|_L^2] d_s^{2H-2},$$

where $\bar{H} = \min_k H_k$ and $\Phi(s) = S(t-s)\sigma(s, u(s))$. This inequality exploits the regularising effect of the semigroup and the fact that $H_k > \frac{1}{2}$ [4]. The constant C_H depends on the Hurst indices and reflects the anisotropic nature of the noise [5].

Combining these estimates, we obtain

$$\|\Gamma(u)\|_E^2 \leq C_1 \|u_0\|_{L^2}^2 + C_2 T^2 (1 + \|u\|_E^2) + C_H \frac{T^{2\bar{H}-1}}{2\bar{H}-1} (1 + \|u\|_E^2).$$

For small T , the coefficients of $\|u\|_E^2$ on the right-hand side become less than 1, making Γ a contraction [6]. The solution is then extended to arbitrary T by considering successive intervals of small length [7].

4.3. Anisotropic Regularity

The next result describes how the directional regularity of the sheet influences the solution. This is where the anisotropy plays a crucial role: different Hurst indices lead to different regularity properties in different directions [8].

Theorem 4.2 (Hölder regularity). Under the assumptions of Theorem 4.1, if in addition u_0 belongs to the anisotropic Sobolev space $W_H^{n,p}(\mathbb{R}^N)$ for some $\alpha > 0$ and $p > 2$, then for any $\epsilon < \min_k (H_k - \frac{1}{2})$, the solution u is almost surely Hölder continuous of order ϵ in time and in each spatial direction.

More precisely, there exists a random constant $C(\omega)$ such that

$$|u(t, x) - u(s, y)| \leq C(\omega) \left(|t-s|^\epsilon + \sum_{k=1}^N |x_k - y_k|^{\epsilon_k} \right)$$

for all $(t, x), (s, y)$ in a compact set, where $\epsilon_k < H_k - \frac{1}{2}$ [9].

Main ideas. The proof relies on an anisotropic version of Kolmogorov's continuity criterion. We estimate moments of increments of u in different directions. For the temporal increments, we have

$$\mathbb{E}[|u(t, x) - u(s, x)|^p] \leq C |t-s|^{p \min(1, \bar{H} - \frac{1}{2})},$$

where $\bar{H} = \min_k H_k$ [10]. For spatial increments in the k -th

direction,

$$\mathbb{E}[|u(t, x + he_k) - u(t, x)|^p] \leq C|h|^{p(H_k - \frac{1}{2})}.$$

These estimates combine the regularising effect of the semigroup $S(t)$ with the anisotropic regularity of the smoothest integral [12]. The Kolmogorov criterion then yields the stated Hölder regularity, with exponents that directly reflect the Hurst indices of the driving noise [30].

The theorem shows that the solution inherits the anisotropic smoothness of the fractional sheet. If the sheet is smoother in some directions than others (i.e., H_k varies with k), this asymmetry propagates to the solution [1]. This has practical implications: for numerical methods, one might use different discretization steps in different directions; for statistical estimation, the rates of convergence may vary with direction [2].

5. Concluding Remarks

We have developed a coherent theory of stochastic integration with respect to anisotropic fractional Brownian sheets and applied it to establish well-posedness and regularity for a class of semilinear SPDEs [3]. The regularisation method we employed offers a transparent way to handle the directional structure of the sheet and can serve as a basis for numerical schemes [4]. The connection with Malliavin calculus provides a rigorous foundation and access to powerful analytical tools [5].

Several directions for future research appear promising [6]:

- 1) *Rough sheets*: extending the theory to the case $H_k \leq \frac{1}{2}$ would require tools from rough paths theory or regularity structures [7]. This is particularly challenging in the anisotropic setting, as different directions may have different roughness levels [8].
- 2) *Non-Gaussian drivers*: replacing the fractional sheet by a Levy sheet or another non-Gaussian field would broaden the range of applications, especially in finance where jump processes are common [9].
- 3) *Applications*: concrete models in fluid dynamics (e.g., anisotropic turbulence), finance (multi-asset models with long memory), or biology (diffusion in stratified media) could benefit from the anisotropic framework developed here [10]. For instance, in porous media flow, different geological layers often exhibit different correlation structures [11].
- 4) *Numerical methods*: developing efficient numerical schemes that respect the anisotropic structure could leverage the regularisation approach presented here, using different approximation strategies in directions with different Hurst indices [12].

The interplay between the anisotropy of the noise and the spatial operator $\mathcal{L}$ also deserves further investigation [13], as it may lead to new phenomena not present in isotropic settings [14]. For example, certain directional couplings might enhance or suppress regularity in specific ways, with implications for

both theory and applications [15].

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Abbreviations

SPDE	Stochastic Partial Differential Equation
FBS	Fractional Brownian Sheet
FBM	Fractional Brownian Motion
RKHS	Reproducing Kernel Hilbert Space
MC	Malliavin Calculus

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Author Contributions

Bou Diop: Conceptualization, Formal Analysis, Investigation, Methodology, Project Administration, Resources, Supervision, Validation, Visualization, Writing – original draft, Writing – review & editing

Conflicts of Interest

The author declares no conflicts of interest.

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