

Modeling Macroeconomic Variables Using Principal Component Analysis and Multiple Linear Regression: The Case of Ghana's Economy

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To cite this article:

Eyiah-bediako Francis, Bosson-amedenu Senyefia, Otoo Joseph. Modeling Macroeconomic Variables Using Principal Component Analysis and Multiple Linear Regression: The Case of Ghana's Economy. *Journal of Business and Economic Development*. Vol. 5, No. 1, 2020, pp. 1-9. doi: 10.11648/j.jbed.20200501.11

Received: December 25, 2019; **Accepted:** January 4, 2020; **Published:** January 13, 2020

Abstract: The paper sought to model the relationship between GDP and 29 macroeconomic variables in Ghana using the Principal Component Analysis and multiple linear regression. Economic data with 583 data points were collected from January, 1990 through to May, 2018. The KMO statistics was 0.750 and the Bartlett's Test of sphericity statistic obtained for the data was 24807.231 of p-value 0.000. The variables were found to be powerfully correlated with reference to the correlation matrix. Principal Component Analysis was performed to reduce the factors (using orthogonal varimax technique to produce uncorrelated factor structures to help allocate appropriately loadings to factors) to a minimum without compromising the variability of the original data. Seven factors were retained (explained 74% of the overall variation) after using multiple extraction approaches of Scree test, Kaiser Criterion and parallel analysis to avoid over- and under-extraction errors. Regression analysis was performed where component scores were used to develop a relationship with the uncorrelated components and GDP. The component 2 (Closed Economy without Government Activities) explicitly contained seven indicators consisting of consumer price index-Food, Consumer price index-Nonfood, Consumer Price index (overall), Monetary Policy Rate, 91-Days Treasury Bill, 182-Days Treasury Bill, crude oil, and Core Inflation (Adjusted for Energy and Utility). Component 2 was significant and positively related with GDP ($B = 0.6$, $p < 0.01$). Again, Component 5 (Closed Economy with Government activities) explicitly contained two indicators such as Tax-Equivalent Rate on 28-Days Treasury Bill and Tax-Equivalent Rate on 56-Days Treasury Bill. Component 5 had a positive and significant impact on GDP ($B = 0.386$, $p < 0.01$). However, component 4 (monetary economy; $B = -3.927$, $p < 0.01$), component 6 ($B = -0.577$, $p < 0.01$) and component 7 ($B = -0.256$, $p < 0.01$) were negatively related with GDP but were statistically significant. The R-squared value of 0.304 shows that the regression model explains about 30% of the variance. It was recommended for future researchers to consider increasing the number of macroeconomic variables to increase the predictive power of the model.

Keywords: Principal Component Analysis, Modeling, Macroeconomic Economic Variables, Ghana, Factor Analysis, Eigenvalues, Multiple Linear Regression

1. Introduction

Principal components analysis (PCA) has been extensively applied in diverse fields as a multivariate approach in reducing the dimension of data points so as to facilitate the interpretation and construction of predictive models [1].

Decomposition and dimensional reduction techniques such as Principal component analysis is recommended for a study that involves exploring multidimensional data [2].

Syed *et al.*, (2013), [3] sought to measure the effects of macroeconomic variables on Gross Domestic Product (GDP) in Pakistan using principal component analysis and factor

analysis. From their results, seventeen variables contributing to GDP were retained in three factors. These three factors derived by the method of PCA were the service factor, agriculture & infrastructure factor, and the fishing & mining factor. In the maximum likelihood method the factors were renamed as service factor, agriculture & infrastructure factor, and education factor.

Hussain *et al.* (2016), [4] also examined the impact of macroeconomic variables on GDP in Pakistan using multiple regression model. The study found a significant effect on inflation rate, interest rate, and exchange rate on GDP.

A study by Purnamasari *et al.* (2019), [5] determined the factors capable of affecting learning motivation of students at Yogyakarta State University Postgraduate Program. By the method of Factor analysis, six factors were identified to affect student motivation. These factors were diligence, discipline, discipline and learning frequency, motivation, independence, time management.

A study by Adongo *et al.* (2018), [6] used PCA to determine the extent of redundant information from correlation matrix. Varimax rotation was applied. Three factors were retained by scree test. The first factors had 6 indicator loadings, the second had one indicator loading, and the third factor had one indicator loading between the nine macroeconomic indicators. They established that PCA method was more robust in comparison with maximum likelihood estimate MLE method. However, the researchers failed to consider varied extraction approaches prior to making decisions on the number of components to retain. This is noteworthy as since the Scree test is known to be highly subjective.

Razak *et al.* (2015), [7] applied PCA to socioeconomic variables in Asia and found a strong significant and positive correlation between life expectancy at birth and health expenditures, gross national income, good governance, and a healthy life. The study however did not consider the parallel analysis in decision making with regards to the number of components to retain.

Twenefour *et al.* (2015), [8] in their study sought to identify a yardstick for measuring student performance in a public school in Ghana using PCA. The researchers retained three components using scree test and Kaiser Criterion. The parallel analysis method which has been found to be the most robust method was not incorporated.

The ability of researchers to correctly determine the number of components to retain in PCA has been a major problem confronting researchers. Errors arising from such problems may lead to over-extraction or under-extraction. Field (2005), [9] suggested that if the sample size is more than 250 and the average communality is above 0.6 then one can retain all factors having Eigen values beyond 1 (Kaiser's criterion). He further suggested for Scree Plot to be used for sample size of at least 300. A study by Scott *et al.* (1995), [10] reviewed articles that applied PCA (by comparing the three methods such as scree plot, Kaiser Criterion and parallel analysis) from 1987-1993 found that parallel analysis was the most robust method in deciding on the number of

components to retain. Williams (2012), [11] suggested that multiple extraction techniques should be explored before making decisions to retain prospective components.

2. Method

The study examined 30 economic variables (with samples obtained from 583 data points) in Ghana. Data was obtained from the Bank of Ghana (annual reports) panning the years from 1990 through to May of 2018. The analytical methods of PCA and regression analysis were employed.

2.1. Construction of Principal Components

For a random vector, X , with domain \Re^m , will have a mean and covariance matrix of μ_X and Σ_X , respectively. $\lambda_1 > \lambda_2 > \dots > \lambda_m > 0$ for an array of eigenvalues of Σ_X , so that the i -th eigenvalue of Σ_X represents the largest i -th eigenvalue. Suppose a vector α_i denotes the i -th eigenvector of Σ_X corresponding to the i -th eigenvalue of Σ_X . We wish to derive principal components (PCs) form by considering the maximization of $\text{var}[\alpha_i^T X] = \alpha_i^T \Sigma_X \alpha_i$, with respect to $\alpha_i^T \alpha_i = 1$ (a typical optimization problem). The Lagrange multiplier approach is then applied to solve the problem.

To that end,

$$L(\alpha_1, \phi_1) = \alpha_1^T \Sigma_X \alpha_1 + \phi_1 (\alpha_1^T \alpha_1 - 1)$$

$$\frac{\partial L}{\partial \alpha_1} = 2 \Sigma_X \alpha_1 + 2 \phi_1 \alpha_1 = 0 \Rightarrow \Sigma_X \alpha_1 = -\phi_1 \alpha_1 \Rightarrow$$

$$\text{var}[\alpha_1^T X] = -\phi_1 \alpha_1^T \alpha_1 = -\phi_1.$$

Since $-\phi_1$ represent the eigenvalue of Σ_X , with α_1 denoting the respective normalized eigenvector, $\text{var}[\alpha_1^T X]$ is maximized when α_1 chosen as the initial eigenvector of Σ_X . To this end, $z_1 = \alpha_1^T X$ is referred to as the first PC of X , with α_1 representing the vector of coefficients for z_1 , where $\text{var}(z_1) = \lambda_1$.

To get the second PC, $z_2 = \alpha_2^T X$, we shall maximize $\text{var}[\alpha_2^T X] = \alpha_2^T \Sigma_X \alpha_2$ on condition that z_2 is not correlated with z_1 . But $\text{cov}(\alpha_1^T X, \alpha_2^T X) = 0 \Rightarrow \alpha_1^T \Sigma_X \alpha_2 = 0 \Rightarrow \alpha_1^T \alpha_2 = 0$, which we will solve by maximizing $\alpha_2^T \Sigma_X \alpha_2$, on condition that $\alpha_1^T \alpha_2 = 0$, and $\alpha_2^T \alpha_2 = 1$. We again make use of the Lagrange multiplier approach.

To that end,

$$L(\alpha_2, \phi_1, \phi_2) = \alpha_2^T \Sigma_X \alpha_2 + \phi_1 \alpha_1^T \alpha_2 + \phi_2 (\alpha_2^T \alpha_2 - 1)$$

$$\frac{\partial L}{\partial \alpha_2} = 2 \sum_X \alpha_2 + \phi_1 \alpha_1 + 2 \phi_2 \alpha_2 = 0$$

$$\Rightarrow \alpha_1^T (2 \sum_X \alpha_2 + \phi_1 \alpha_1 + 2 \phi_2 \alpha_2) = 0 \Rightarrow \phi_1 = 0$$

$$\Rightarrow \sum_X \alpha_2 = -\phi_2 \alpha_2 \Rightarrow \alpha_2^T \sum_X \alpha_2 = -\phi_2$$

As $-\phi_2$ is the eigenvalue of \sum_X , where α_2 is the respective normalized eigenvector, we are able to maximize $\text{var}[\alpha_2^T X]$ when we select α_2 as the second eigenvector of \sum_X . As a result, $z_2 = \alpha_2^T X$ becomes the second PC of X , where α_2 represents the vector of coefficients for z_2 , and $\text{var}(z_2) = \lambda_2$. Per the above results, we can deduce that the i -th PC $z_i = \alpha_i^T X$ is constructed α_i is chosen as the i -th eigenvector of \sum_X , which will then have the variance λ_i . We can conclude by the above results that PCA are the only set of linear functions of original data that are uncorrelated and have orthogonal vectors of coefficients. PCA relies on either covariance matrix or the correlation matrix. The linear combination weights directly originate from combination eigenvectors of correlation matrix or covariance matrix.

Recall that for m variables, the $m \times m$ covariance or correlation matrix will contain the following sets:

m eigenvalues $-\{l_1, l_2, \dots, l_p\}$

m eigenvectors $-\{e_1, e_2, \dots, e_p\}$.

Each principal component (PC) formed when we considering the values of the elements of the eigenvalues as the weights of the linear combination [6, 12].

Assuming that the k -th eigenvector $e_k = (e_{1k}, e_{2k}, \dots, e_{pk})$, then the PCs Y_1, \dots , are produced by

$$Y_1 = e_{11}X_1 + e_{21}X_2 + \dots + e_{m1}X_m$$

$$Y_2 = e_{12}X_1 + e_{22}X_2 + \dots + e_{m2}X_m \dots$$

$$Y_m = e_{1m}X_1 + e_{2m}X_2 + \dots + e_{mm}X_m$$

2.2. Multiple Linear Regressions

The data $(Y_1, z_{11}, z_{12}, \dots, z_{1r}), (Y_2, z_{21}, z_{22}, \dots, z_{2r}), \dots, (Y_n, z_{n1}, z_{n2}, \dots, z_{nr})$ will have the following multiple linear regression model:

$$Y_i = \beta_0 + \beta_1 z_{i1} + \beta_2 z_{i2} + \dots + \beta_r z_{ir} + \varepsilon_i, i = 1, \dots, n,$$

The terms satisfy the following properties:

$$1. E(\varepsilon_i) = 0; \quad 2. \text{Var}(\varepsilon_i) = \sigma^2; \quad 3. \text{Cov}(\varepsilon_i, \varepsilon_j) = 0, i \neq j$$

The matrix form of the above data is:

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} \beta_0 + \beta_1 z_{11} + \dots + \beta_r z_{1r} + \varepsilon_1 \\ \beta_0 + \beta_1 z_{21} + \dots + \beta_r z_{2r} + \varepsilon_2 \\ \vdots \\ \beta_0 + \beta_1 z_{n1} + \dots + \beta_r z_{nr} + \varepsilon_n \end{bmatrix} = \begin{bmatrix} \beta_0 + \beta_1 z_{11} + \dots + \beta_r z_{1r} \\ \beta_0 + \beta_1 z_{21} + \dots + \beta_r z_{2r} \\ \vdots \\ \beta_0 + \beta_1 z_{n1} + \dots + \beta_r z_{nr} \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

Or

$$\begin{bmatrix} 1 & z_{11} & \dots & z_{1r} \\ 1 & z_{21} & \dots & z_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & z_{n1} & \dots & z_{nr} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_r \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix} = Z\beta + \varepsilon$$

Where;

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix}, Z = \begin{bmatrix} 1 & z_{11} & \dots & z_{1r} \\ 1 & z_{21} & \dots & z_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & z_{n1} & \dots & z_{nr} \end{bmatrix}, \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}, \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_r \end{bmatrix}.$$

The error terms are;

$$1. E(\varepsilon) = 0; \text{ and } 2. \text{Cov}(\varepsilon) = E(\varepsilon \varepsilon^T) = \sigma^2 I;$$

Table 1. Interpretation of the KMO as characterised by Kaiser, Meyer, and Olkin.

KMO Value	Degree of Common Variance
0.90 to 1.00	Marvelous
0.80 to 0.89	Meritorious
0.70 to 0.79	Middling
0.60 to 0.69	Mediocre
0.50 to 0.59	Miserable
0.00 to 0.49	Not Appropriate for Factor Analysis

(Schwarz, 2011).

3. Results

William (2010), [11] suggested that the sample size of at least 300 and the sample-variable ratio of 10: 1 are adequate

for factor analysis. The sample size for this study was made up of 583 data points from 30 economic variables in Ghana.

Table 2. Truncated Correlation Matrix.

		CPIF	CPINF	CPIO	INFF	INFNF	INFYAV	INTYOY	M1	M2	M2PLUS	MPR	TotDep	TotCred
Correlation	CPIF	1.000	.971	.929	-.444	-.428	-.480	-.462	.361	.372	.363	-.733	-.003	.098
	CPINF	.971	1.000	.935	-.461	-.422	-.493	-.464	.481	.490	.482	-.710	.003	.086
	CPIO	.929	.935	1.000	-.433	-.402	-.463	-.440	.417	.426	.417	-.674	-.013	.085
	INFF	-.444	-.461	-.433	1.000	.790	.747	.930	-.375	-.377	-.373	.576	.123	-.102
	INFNF	-.428	-.422	-.402	.790	1.000	.800	.957	-.227	-.229	-.224	.604	.046	-.040
	INFYAV	-.480	-.493	-.463	.747	.800	1.000	.837	-.392	-.393	-.390	.711	.068	-.037
	INTYOY	-.462	-.464	-.440	.930	.957	.837	1.000	-.297	-.299	-.294	.645	.095	-.068
	M1	.361	.481	.417	-.375	-.227	-.392	-.297	1.000	.997	.997	-.323	.161	.035
	M2	.372	.490	.426	-.377	-.229	-.393	-.299	.997	1.000	.999	-.324	.164	.034
	M2PLUS	.363	.482	.417	-.373	-.224	-.390	-.294	.997	.999	1.000	-.318	.168	.033
	MPR	-.733	-.710	-.674	.576	.604	.711	.645	-.323	-.324	-.318	1.000	.145	-.003
	TotDep	-.003	.003	-.013	.123	.046	.068	.095	.161	.164	.168	.145	1.000	.084
	TotCred	.098	.086	.085	-.102	-.040	-.037	-.068	.035	.034	.033	-.003	.084	1.000
	TBR91day	-.294	-.302	-.288	.191	.247	.267	.238	-.149	-.149	-.147	.433	-.184	.021
	TBR182DAY	-.039	-.017	-.029	-.283	-.331	-.273	-.320	.149	.149	.153	-.013	-.050	.050
	TBRIYEAR	-.067	-.062	-.059	.019	.003	.020	.011	-.038	-.038	-.037	.031	-.060	.033
	STD	.027	.041	.031	-.135	-.059	-.069	-.099	.253	.245	.250	-.059	.046	.072
	RMPGRATE	-.005	.000	-.006	.045	.009	.056	.026	-.018	-.018	-.020	.005	.027	-.185
	RM	-.227	-.174	-.182	.325	.238	.176	.293	.065	.065	.068	.183	.538	.057
	QM	-.144	-.132	-.128	.412	.330	.275	.384	-.059	-.058	-.058	.214	.558	.067
	PSCREDIT	-.169	-.127	-.112	.217	.212	.287	.239	-.030	-.036	-.027	.374	.108	-.071
	PETROLEUM	.619	.529	.503	-.209	-.262	-.280	-.259	-.060	-.050	-.057	-.569	.006	.040
	CIC	-.231	-.180	-.187	.308	.208	.149	.267	.077	.076	.079	.178	.517	.066
	BNCG	-.228	-.212	-.205	.319	.249	.198	.292	-.123	-.122	-.121	.201	.516	.051
	BCROIL	.235	.210	.221	-.151	-.011	-.044	-.083	.099	.094	.093	-.072	.042	.153
	DDR28	-.013	-.017	-.016	-.018	-.017	.025	-.017	-.016	-.017	-.017	.033	-.011	.028
	DIREq28	.115	.149	.135	-.207	-.040	-.117	-.117	.233	.235	.235	.036	.341	.161
	DIREQ56	.154	.185	.170	-.221	-.036	-.108	-.121	.255	.256	.257	-.009	.386	.174
	CIAER	-.316	-.256	-.246	.076	.105	.124	.108	.147	.142	.146	.396	-.357	-.065
Sig. (1-tailed)	CPIF		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.474	.009
	CPINF	.000		.000	.000	.000	.000	.000	.000	.000	.000	.000	.473	.020
	CPIO	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000	.373	.020
	INFF	.000	.000	.000		.000	.000	.000	.000	.000	.000	.000	.002	.007
	INFNF	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000	.133	.165
	INFYAV	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000	.050	.184
	INTYOY	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000	.011	.050
	M1	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000	.202
	M2	.000	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000	.204
	M2PLUS	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000	.000	.210
	MPR	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000	.472
	TotDep	.474	.473	.373	.002	.133	.050	.011	.000	.000	.000	.000		.021
	TotCred	.009	.020	.020	.007	.165	.184	.050	.202	.204	.210	.472	.021	
	TBR91day	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.305
	TBR182DAY	.173	.341	.242	.000	.000	.000	.000	.000	.000	.000	.373	.114	.116
	TBRIYEAR	.055	.069	.077	.323	.475	.317	.399	.182	.182	.184	.229	.073	.211
	STD	.257	.162	.231	.001	.077	.048	.009	.000	.000	.000	.079	.136	.041
	RMPGRATE	.457	.499	.445	.138	.416	.090	.265	.330	.335	.318	.450	.261	.000
	RM	.000	.000	.000	.000	.000	.000	.000	.059	.060	.050	.000	.000	.085
	QM	.000	.001	.001	.000	.000	.000	.000	.077	.080	.082	.000	.000	.052
	PSCREDIT	.000	.001	.004	.000	.000	.000	.000	.238	.195	.256	.000	.005	.044
	PETROLEUM	.000	.000	.000	.000	.000	.000	.000	.075	.116	.083	.000	.445	.168
	CIC	.000	.000	.000	.000	.000	.000	.000	.031	.034	.028	.000	.000	.056
	BNCG	.000	.000	.000	.000	.000	.000	.000	.002	.002	.002	.000	.000	.108
	BCROIL	.000	.000	.000	.000	.397	.147	.023	.009	.012	.012	.043	.157	.000
	DDR28	.380	.337	.354	.335	.345	.273	.340	.347	.341	.344	.211	.399	.252
	DIREq28	.003	.000	.001	.000	.165	.002	.002	.000	.000	.000	.191	.000	.000
	DIREQ56	.000	.000	.000	.000	.190	.005	.002	.000	.000	.000	.411	.000	.000
	CIAER	.000	.000	.000	.034	.006	.001	.005	.000	.000	.000	.000	.000	.059

Relationship patterns were examined by critical analysis of the respective Pearson correlation coefficient (from the correlation matrix) between all pairs of the economic variables as well as their one-tailed significance of the coefficients. Multicollineality was a problem since the

determinant of 1.119×10^{-19} was smaller in value than the necessary value of 0.00001. Therefore PCA application to the data will produce uncorrelated output which will deal with the issue of some economic variables being highly

correlated with each other. A critical look at the correlation matrix (truncated) shows some highly correlated variables such as CPIF and CPIFN, M1 and M2 among others. There was a case of singularity of the data since numerous data sets yielded significance above 0.5 whilst a number of the correlation coefficients were more than 0.9.

Table 3. Kaiser-Meyer-Olkin (KMO) Measure of Sampling Adequacy/Bartlett's Test of Sphericity.

KMO and Bartlett's Test		
Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.750
Approx. Chi-Square		24807.231
Bartlett's Test of Sphericity	df	406
	Sig.	.000

Null Hypothesis: The inter-correlation matrix of the variables is not different from an identity matrix.

Alternate Hypothesis: The inter-correlation matrix of the variables is different from an identity matrix.

Test Results

$\chi^2 = 24807.231$; $df = 406$; $p < 0.0001$

Statistical Decision

The inter-correlation matrix of the variables is significantly different from an identity matrix. In other words, the sample inter-correlation matrix did not come from a population in which the inter-correlation matrix is an identity matrix. The KMO Statistic was 0.750. The degree of common variance among the ten variables is Middling.

In factor analysis, the factors extracted will account for a substantial amount of variance.

Factor analysis is recommended for analysis when the

Bartlett's test of sphericity is statistically significant (≤ 0.05), and the KMO statistic exceeds 0.6 [14]. Kaiser-Meyer-Olkin Measure of Sampling Adequacy values which are greater than 0.7 by rule of thumb approach is considered a good indication that PCA will be useful for the variables under study (Williams, 2010). The KMO statistics of 0.750 is an indication of the appropriateness of the correlation matrix for component analysis. The Bartlett's test of Sphericity tests the difference between the correlation matrix for variables and the identity matrix. Bartlett's Test of Sphericity obtained for the data was 24807.231 and p-value was 0.000; an indication of a significant difference which makes it inferable that our correlation matrix for our measured variables is significantly different from an identity matrix which is consistent with the assumption that the matrix should be treated as factorable. This shows that the Bartlett's test of sphericity is highly sufficient for the data under study.

Table 4. The eigenvalues greater than one of the correlation matrix.

Eigenvalue	Total variance (%)	Cumulative Eigenvalue	Cumulative (%)
7.353	25.356	7.353	25.356
4.629	15.961	11.982	41.317
2.963	10.216	14.945	51.533
2.159	7.445	17.104	58.979
1.859	6.411	18.963	58.979
1.4	4.827	20.363	65.389
1.211	3.852	21.574	70.216
1.117	4.176	22.691	74.329

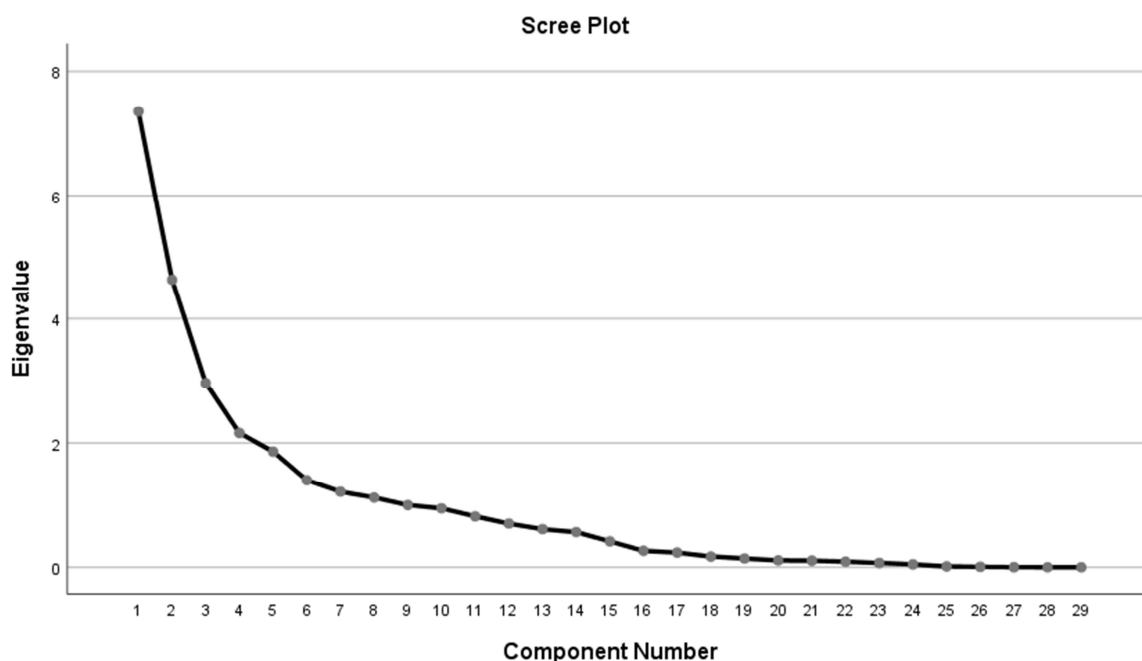


Figure 1. The Scree Plot.

Cattell's Scree test requires visual analysis of a graphical representation of eigenvalues for point of inflection. The observations above the point of inflection (including the point itself) will inform the number of factors to be retained.

Researcher's judgment is required in the decision for the number of factors to retain which introduces subjectivity. The subjectivity is reduced for larger sample size [15].

There appears to be a marked decrease in downward slope

after the fourth principal component implying that the preceding five principal components can be summarized as being representative of the variables in totality. This is in not in line with the Kaiser criterion (eigenvalue greater than one rule) which extracted 8 factors out of the 29.

Table 5. Parallel Analysis.

Mean Eigenvalue from Parallel Analysis	Eigen values from dataset (Kaisor eigen value >1 rule)	Eigenvalues retained
1.436368	7.353	7.353
1.377602	4.629	4.629
1.329983	2.963	2.963
1.287357	2.159	2.159
1.252328	1.859	1.859
1.221142	1.400	1.400
1.188662	1.211	1.211
1.162285	1.117	

The Kaiser's eigenvalue >1 rule requires factors with eigenvalues exceeding 1 to be the only ones to be retained.

Parallel analysis (although less applied) has been pointed out by many researchers as the most robust method for retaining factors [15].

Table 5 (second column), the Kaiser's rule output suggests retaining of 8 factors. The parallel analysis was performed with parameters of 29 economic variables and 583 data points. Percentile Eigen value was set at 95 and the default to generate 100 correlation matrices was maintained. The Eigen values computed from the randomly generated correlation matrices of the parallel analysis were compared with the Eigen values extracted from the data set. The factors which had their Eigen values from the data set exceeding that from that parallel analysis were retained with those failing the threshold considered as spurious as shown in Table 5. To this end, 7 factors were retained. Whereas the Kaiser criterion extracted 8 factors, the parallel analysis suggested retaining only 7 of these out of the 29 variables. To this end, 7 factors were retained for further analysis since the parallel analysis method has been noted as the most robust compared to Scree test and Kaiser criterion (Scott, *et al.* 1995).

Table 6. Total Variance Explained.

Total Variance Explained									
Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative%	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	7.353	25.356	25.356	7.353	25.356	25.356	4.498	15.511	15.511
2	4.629	15.961	41.317	4.629	15.961	41.317	4.253	14.666	30.177
3	2.963	10.216	51.533	2.963	10.216	51.533	4.121	14.210	44.387
4	2.159	7.445	58.979	2.159	7.445	58.979	3.448	11.890	56.277
5	1.859	6.411	65.389	1.859	6.411	65.389	2.106	7.262	63.539
6	1.400	4.827	70.216	1.400	4.827	70.216	1.875	6.464	70.003
7	1.211	4.176	74.392	1.211	4.176	74.392	1.273	4.390	74.392
8	1.117	3.852	78.245						
9	.997	3.439	81.684						
10	.943	3.253	84.937						
11	.816	2.813	87.750						
12	.700	2.412	90.162						
13	.610	2.103	92.266						
14	.563	1.943	94.208						
15	.415	1.432	95.640						
16	.264	.909	96.549						
17	.235	.812	97.360						
18	.172	.595	97.955						
19	.141	.485	98.440						
20	.110	.380	98.820						
21	.107	.368	99.188						
22	.090	.311	99.499						
23	.069	.239	99.738						
24	.048	.164	99.902						
25	.015	.052	99.954						
26	.008	.028	99.982						
27	.003	.011	99.993						
28	.001	.005	99.998						
29	.001	.002	100.000						

Extraction Method: Principal Component Analysis.

The first component explains about 25.4% of the total variance. The second component explains about 16% of the overall variance. The third component explains about 10.2% of the total variation and so on and so forth. It can be observed that the first component recorded the greatest

variance followed by the second component with the remaining components following the same decreasing trend of variance. Orthogonal Varimax technique was employed to produce uncorrelated factor structures. The summarized overall variation in the original set of variables per the 7

retained variables was about 74.4%.

Table 7. Rotated Component Matrix^a.

Rotated Component Matrix^a							
	Component						
	1	2	3	4	5	6	7
CPIF	-.128	.898	-.202	.224	.095	.107	.014
CPINF	-.102	.851	-.201	.359	.110	.088	.036
CPIO	-.113	.840	-.177	.305	.114	.096	.042
INFF	.232	-.219	.805	-.197	-.169	-.129	.028
INFNF	.128	-.211	.894	-.063	-.005	-.014	-.052
INFYAV	.072	-.284	.815	-.235	.003	-.012	.024
INTYOY	.181	-.234	.909	-.115	-.072	-.072	-.015
M1	.046	.139	-.177	.957	.069	.077	-.022
M2	.046	.150	-.176	.956	.072	.071	-.022
M2PLUS	.048	.140	-.174	.958	.072	.072	-.021
MPR	.050	-.686	.551	-.148	.174	-.011	.059
TotDep	.672	.057	.051	.093	.206	.143	.020
TotCred	.041	.061	.024	-.011	.255	.201	-.590
TBR91day	-.404	-.423	.308	-.012	.037	.307	.205
TBR182DAY	-.063	-.302	-.491	.108	.425	-.178	-.089
TBRIYEAR	-.055	-.076	.023	.021	-.101	-.146	-.164
STD	-.007	-.133	-.137	.202	-.140	.791	-.108
RMPGRATE	.046	.021	-.044	-.054	-.021	.008	.719
RM	.936	-.132	.127	.093	.020	-.091	.035
QM	.884	-.015	.274	-.036	.028	.066	.036
PSCREDIT	.031	-.174	.353	.069	.188	.184	.504
PETROLEUM	-.048	.767	-.125	-.196	-.107	-.177	-.058
CIC	.914	-.159	.096	.107	.074	-.132	.035
BNCG	.909	-.093	.128	-.121	.064	-.038	.035
BCROIL	-.099	.127	.053	.020	.145	.889	.032
DDR28	-.088	-.029	.022	-.016	.205	-.140	-.016
DIREq28	.261	.039	-.102	.121	.890	.120	-.008
DIREQ56	.300	.096	-.097	.131	.866	.134	-.033
CIAER	-.465	-.506	.185	.344	.103	-.189	.204

Extraction Method: Principal Component Analysis.

Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 7 iterations.

Component 1 which is highly correlated (Table 7) with 5 original variables (Total Deposit, Reserve Money, Quasi Money, Consumption index component, and Bank of Ghana Composite Index of Economic Activity (Nominal Growth) was more resembling of a monetary economy.

Component 2 was powerfully correlated (Table 7) with 7 original variables (Consumer price index-food, consumer price index-nonfood, consumer price index overall, Monetary policy rate, crude oil, core inflation adjusted for energy and utility, and 91-day Treasury Bill was typical of a closed economy without government activity. This is because the indicators are made of mainly investment and consumption factors.

Component 3 which is highly correlated with 5 original variables (Inflation-food, Inflation-nonfood, inflation-average of year, Inflation-year-on-year, 182-Day Treasury

Bill) is a more representative of an inflationary economy.

Component 4 is highly correlated with three variables (Narrow Money, Broad Money and Total Liquidity) and a clear case of monetary economy.

Component 5 was highly correlated with 2 original factors (tax-equivalent on treasury bill 28 days, and tax equivalent on Treasury bill 56 days) is a representation of a closed economy with government activity. This is because the indicators are made of investment and government activity (eg. Tax on treasury).

Component 6 has two variables with high correlation (Savings & Time deposits, and International Brent Crude Oil (US\$/Barrel) - Monthly Average).

Component 7 has three highly correlated variables (Total Credit, Reserve Money Policy Rate and Private sector credit.

Table 8. Model Summary.

Model Summary				
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.551 ^a	.304	.289	1.97120

a. Predictors: (Constant), REGR factor score 7 for analysis 1, REGR factor score 5 for analysis 1, REGR factor score 1 for analysis 1, REGR factor score 6 for analysis 1, REGR factor score 3 for analysis 1, REGR factor score 2 for analysis 1, REGR factor score 4 for analysis 1.

Table 9. ANOVA.

ANOVA ^a						
Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	563.394	7	80.485	20.714	.000 ^b
	Residual	1290.026	332	3.886		
	Total	1853.420	339			

a. Dependent Variable: GDP.

b. Predictors: (Constant), REGR factor score 7 for analysis 1, REGR factor score 5 for analysis 1, REGR factor score 1 for analysis 1, REGR factor score 6 for analysis 1, REGR factor score 3 for analysis 1, REGR factor score 2 for analysis 1, REGR factor score 4 for analysis 1.

There was a statistically significant difference among the 7 components.

Table 10. Coefficients.

Coefficients ^a						
Model		Unstandardized Coefficients		Standardized Coefficients		Sig.
		B	Std. Error	Beta	t	
1	(Constant)	4.564	.239		19.085	.000
	REGR factor score 1 for analysis 1	-.051	.091	-.028	-.556	.579
	REGR factor score 2 for analysis 1	.600	.135	.224	4.436	.000
	REGR factor score 3 for analysis 1	-.021	.120	-.011	-.177	.860
	REGR factor score 4 for analysis 1	-3.927	.571	-.468	-6.882	.000
	REGR factor score 5 for analysis 1	.386	.104	.193	3.720	.000
	REGR factor score 6 for analysis 1	-.577	.114	-.260	-5.063	.000
	REGR factor score 7 for analysis 1	-.256	.104	-.116	-2.475	.014

a. Dependent Variable: GDP.

The components 2 (closed economy without government activity) and 5 (a closed economy with government activity) were significant and positively related with GDP. However, components 4 (monetary economy), component 6, and 7 are negatively related with GDP and statistically significant. Components 1 (monetary economy) and 3 (inflationary economy) however were not statistically significant and had negative impact on GDP.

4. Conclusion

The paper aimed at modeling the relationship between GDP and 29 macroeconomic variables in Ghana using the PCA and multiple linear regressions methods. Economic data with 583 data points were collected from January, 1990 through to May, 2018. Seven factors were retained (explained 74% of the overall variation) after using multiple extraction approaches of scree test, Kaiser Criterion, and parallel analysis to avoid over- and under-extraction. Regression analysis was performed where component scores were used to develop a relationship with the uncorrelated components and GDP. Closed Economy without Government Activities explicitly contained seven indicators consisting of consumer price index-Food, Consumer price index-Nonfood, Consumer Price index overall, Monetary Policy Rate, 91-Days Treasury, 182-Days Treasury Bill, crude oil, and Core Inflation (Adjusted for Energy & Utility was significant and positively related with GDP ($B = 0.6$, $p < 0.01$). Closed Economy with Government activities explicitly contained two indicators such as Tax-Equivalent Rate on the 28-Day Treasury Bill and Tax-Equivalent Rate on 56-Day Treasury Bill had a significant

impact on GDP ($B = 0.386$, $p < 0.01$).

5. Recommendation

Future researchers should consider increasing the number of macroeconomic variables to increase the predictive power of the model.

References

- [1] Armeanu D, Lache L. (2018): Application of the Model of Principal Components Analysis on Romanian Insurance Market, Theoretical and Applied Economics.
- [2] Fan J, Sun Q, Wen-Xin Z, and Ziwei Z. (2018): Principal component analysis for big data arXiv: 1801.01602v1 [stat. ME].
- [3] Syeda F, Muhammad S. and Shah G. A. A (2013). Effects of Macroeconomic Variables on Gross Domestic Product (GDP) in Pakistan. International Conference on Applied Economics (ICOAE) 2013. Procedia Economics and Finance 5 (2013) 703–711.
- [4] Hussain A, Hazaar M. Sabir and Kashif M (2016). Impact of macroeconomic variables on GDP: evidence from Pakistan European Journal of Business and Innovation Research Vol. 4, No. 3, pp. 38-52, June 2016.
- [5] Purnamasari D. U, Surawidarto M., Dedek Hadi A. S, and Istiyono E., (2019), "Exploratory Factor Analysis: Motivation for Learning" in The First International Conference on Education, Science and Training: Empowering Educational Human Resources for Global Competitiveness, KnE Social Sciences, pages 58–65. DOI 10.18502/kss.v3i15.4354.

- [6] Adongo F. A, John Amo Jr. L, Chikelu C. J, Osei M, (2018): Principal Component and Factor Analysis of Macroeconomic Indicators, IOSR Journal Of Humanities And Social Science (IOSR-JHSS) Volume 23, Issue 7, Ver. 10 (July. 2018) PP 01-07 e-ISSN: 2279-0837, p-ISSN: 2279-0845. www.iosrjournals.org.
- [7] Razzak H, Ali M, (2015): Principal Component Analysis Of Socioeconomic Factors And Their Association With Life Expectancy At Birth In Asia, International Journal of Multidisciplinary Academic Research Vol. 3, No. 1, 2015, ISSN 2309-3218.
- [8] Twenefour F. B. K., Nortey E. N. N., Baah, E. M. (2015): Principal Component Analysis of Students Academic Performance International Journal of Business and Social Research, Volume 05, Issue 02, 2015.
- [9] Field, A. P. (2005). *Discovering statistics using SPSS* (2nd edition). London: Sage.
- [10] Scott F. B., Gibson, David J., Robertson, Philip A., Pohlmann, John T. and Fralish, James S. (1995): "Parallel Analysis: a Method for Determining Significant Principal Components." (Feb 1995).
- [11] Williams B. (2012): Exploratory factor analysis: A five-step guide for novices. *Australasian Journal of Paramedicine* Volume 8 | Issue 3.
- [12] Hans P. F & Janssens E. (2019). Spurious principal components, *Applied Economics Letters*, 26: 1, 37-39, DOI: 10.1080/13504851.2018.1433292.
- [13] Schwarz J. (2011): *Research Methodology: Tools, Applied Data Analysis (with SPSS), Lecture 03: Factor Analysis*.
- [14] Mehmedinovic S. (2017) *Fundamentals Of Application Factor Analysis In Education And Rehabilitation*, DOI: 10.21554/hrr.041708.
- [15] Taherdoost H., Sahibuddin S., Jalaliyoon N. (2010) *Exploratory Factor Analysis; Concepts and Theory, Advances in Applied and Pure Mathematics* ISBN: 978-960-474-380-3.