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Finite Time Analysis of Endoreversible Combined Cycle Based on the Stefan-boltzmann Heat Transfer Law

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Abstract: This work examines endoreversible combined cycle based on finite time thermodynamic concepts. In this study, the proposed system is cascade combined cycle have three heat sources. Effects of irreversibility due to the heat transfer at the system boundaries are considered. The study is based on Stephen Boltzmann's heat transfer laws. Based on finite size, this research analyzes the system based on first and second law thermodynamics. Dimensionless power, efficiency, and entropy generation are calculated based on the dimensionless variables. Dimensionless variables are primary and secondary temperature ratios, common temperature ratio, and the ratio of thermal conductance of each heat exchanger. The effects of dimensionless variables on thermodynamic criteria are examined. Also, optimization is performed base on different criteria such as dimensionless power, energy efficiency and entropy generation by genetic algorithm. The optimization results show that the maximum dimensionless power, the maximum energy efficiency and minimum entropy generation are 0.035092393, 61.09% and 8.132 E-07, respectively. The results of this study are very close to the actual results. New thermodynamic criteria bring systems closer to better conditions. Furthermore, the heat transfer mechanism and heat transfer law greatly affect performance and thermodynamic criteria another. These results are used in the design of radiant heat exchangers.

Keywords: Endoreversible Combined Cycle, Stephen Boltzmann's Heat Transfer Laws, Entropy Generation

1. Introduction

Classical thermodynamics is a physical theory that deals with the general characteristics and behavior of macroscopic systems based on four basic laws and some specific concepts.

Classical thermodynamics is generally relies on concepts and types of conversion of microscopic energy into macroscopic energy based on equilibrium. Finite time thermodynamics was developed by Berry, Salamon and Andresen in 1975 [1-4]. Finite time thermodynamics is a result of divergent view to the science of thermodynamics. In terms of heat transfer aspects such as thermal conductivity, finite time thermodynamics is a microscopic extended form, and on the other hand, a combination of classical ideas like exergy, availability, and new concepts such as favorable criteria, thermodynamic limitations and maximum power. Therefore, this properties of finite time thermodynamics lead thermodynamics toward practical. Irreversible Carnot cycle or Curzon-Ahlborn's cycle was the beginning of interest in endoreversible heat engines among researchers. Heat engine, which operates between a high-temperature heat resource with finite heat capacity and a low-temperature reservoir with infinite heat capacity, is studied and presented by Yan and Chen [5], their cycle was consisted of two adiabatic and two constant pressure processes. Their assumptions included temperature difference between working fluid and heat sources and substituting Newtonian heat transfer law by another linear heat transfer law. In mentioned work, they discussed the calculation of favorable efficiency and power output. Moreover, they derived the relation between maximum power output and efficiency. Their other results included a comparison of Carnot cycle efficiency based on Newtonian heat transfer law with extended case. Chen and Yan [6] investigated an endoreversible cycle with two heat sources at different states of heat transfer laws (for different values of n). As a result, they obtained a correlation between optimum efficiency and output power for each value of n in terms of variations in heat transfer coefficients. They found out that the maximum power is dependent on heat transfer coefficient and heat transfer law as well as temperatures of the sources. Wu discussed a finite time Carnot heat engine with finite heat capacity heat sink and source. He calculated maximum power output of as-mentioned heat engine and compared the performance of finite time Carnot heat engine with a real plant. Wu's results indicate that the cycle of finite time Carnot heat engine is more realistic than ideal Carnot cycle [7-9]. Chen and Yan [6] pointed out to the difference between concept of Carnot efficiency and Curzon-Ahlborn efficiency and stated that this difference is due to internal irreversibility of the system. Therefore, in some cases internal irreversibility of the system may lead to higher efficiency in realistic heat engines compared to that of Curzon-Ahlborn, and Curzon-Ahlborn cannot be considered as the upper bound of heat engine. Their results have shown that only Carnot heat engine must be considered as a measure of maximum heat engine efficiency in evaluation of systems. This result is of significant importance in development of finite tie thermodynamics.

Naserian et al. [10-12] discussed regenerative closed Brayton based on Ecology function, power and efficiency in finite time thermodynamics. Their decision making variables included high-temperature heat exchanger thermal conductivity ratio, low-temperature heat exchanger thermal conductivity ratio, and pressure ratio. In their work, size and time limitation is applied based on expression of dimension-less mass flow rate (F). By modification of ecology function concept, it has been used in exergy analysis and exergoeconomic analysis. Based on their study, maximum of ecology function at F=0.1 equals to 72% of maximum power, while at F=0.3, only 24% of exergy is dissipated and cost reduction by 60% compared to the case of maximum power is one of their most important results. De Vos [13] has thermoeconomic discussed an endoreversible plant. He has optimized an endoreversible plant in terms of investment cost and fuel cost. Results of this research indicate that optimum efficiency ranges between Carnot efficiency and maximum power efficiency.

Chen and Wu [14] have discussed an irreversible combined cycle, they have shown that combined cycle efficiency at maximum power may be equal to Curzon-Ahlborn efficiency and also discussed optimum temperature of working fluid in the heat exchanger. They calculated that maximum power could be a criterion in determination of working fluid temperature and designing heat exchangers in combined cycles. Wu has discussed an irreversible combined cycle which is an extended form of endoreversible Curzon-Ahlborn cycle and, has also calculated the upper limit of its power. His results could be used as a proper verification criterion for analyzing realistic combined cycles. Wu has analyzed endoreversible combined cycle. This is a cascade cycle consisting of several endoreversible cycles. Evaluation has been performed based on maximum power in finite time thermodynamics. In practice, efficiency of a Rankine steam is closer to Carnot efficiency than that of other cycles. Since a heat engine with a working fluid operating at a wide temperature range is limited by metallurgical issues and leak in boiler and condenser, thus no working fluid in a real heat engine could operate at a wide temperature range, resulting in low heat engine efficiency. His results has shown that efficiency of a cascade cycle having more than one working fluid is higher than that of a cycle with one working fluid. Sahin and Kodal [15] have studied endoreversible combined cycle at steady state based on finite time thermodynamics, they demonstrated the irreversibility at the highest possible power by means of two parameters corresponding to entropy difference ratio. They studied the effects of these two irreversibility parameters in terms of thermal efficiency and power and also showed that the maximum power of irreversible combined Carnot cycle cannot exceed that of endoreversible cycle in same temperature range. Finally, they showed that efficiency at maximum power for an endoreversible combined cycle is same as the Curzon-Ahlborn efficiency. Ghasemkhani et al. [16-20] evaluated the irreversible combined cycle by assuming the same heat exchangers, and their optimization results showed that the maximum dimensionless total power and thermal efficiency associated with it are 0.086102 and 47.81%, respectively.

2. Description of the System Under Study

Investigated system is a combined cycle consisted of two endoreversible heat engines. Heat is transferred from heat sources at Th1 and Th2 to a high-temperature reversible heat engine, low-temperature heat engine heat is provided by the heat dissipating from the high-temperature reversible engine and the heat transferred from the heat source at 3th, and the low-temperature heat is transferred to the low-temperature heat sink. Effects of irreversibility due to the heat transfer at the system boundaries, the effects of size confinement are included during the analysis. Thermal conductivity is assumed to be constant throughout the system. The equation C=UA is used for simplification of algebraic equations. System input is taken from references [21-23]. Specifications of the system under study show in Table 1.

Table 1. *Specifications of the system under study.*

T_{l1} (°C)	T_{h3} (°C)	T_{h2} (°C)	T_{h1} (°C)
25	259.93	562.11	1126.85

Newton's law of heat transfer applies to issues that convection heat transfer is dominant. Other heat transfer laws need to be considered. Thus, the laws of Stephen Boltzmann and Dulong–Petit have been used [24 890, 25 898, 26 917]. Stephen Boltzmann's heat engine is in fact based only on the law of thermal radiation (solar system). Stephen Boltzmann's law states:

$$\dot{Q}_b = UA\left(T_2^4 - T_1^4\right) \tag{1}$$

Heat transfer rate from heat source to the high-temperature endoreversible cycle at Th1 is obtained from equation (1).

$$\dot{Q}_{h1} = C_{h1} \cdot \left(T_{h1}{}^4 - T_{a1}{}^4\right) \tag{2}$$

Heat transfer rate from heat source to the high-temperature endoreversible cycle at Th2 is obtained from equation (3).

$$\dot{Q}_{h2} = C_{h2} \cdot \left(T_{h2}{}^{4} - T_{c1}{}^{4} \right) \tag{3}$$

Heat loss at high-temperature cycle to the low-temperature cycle is illustrated, heat transfer rate between two endoreversible cycles is expressed as equation (4).

$$\dot{Q}_{m} = C_{m} \cdot \left(T_{b1} - T_{a2} \right) \tag{4}$$

Therefore, generated power at high-temperature endoreversible cycle is in the form of equation (5).

$$\dot{W}_{1} = \dot{Q}_{h1} + \dot{Q}_{h2} - \dot{Q}_{m} \tag{5}$$

Heat transfer rate from the heat source at 3th to the low-temperature endoreversible cycle is written as equation (6).

$$\dot{Q}_{h3} = C_{h3} \cdot \left(T_{h3}{}^{4} - T_{c2}{}^{4} \right) \tag{6}$$

Heat transfer rate to the heat sink at Tl1 is written as equation (7).

$$\dot{Q}_{l1} = C_{l1} \cdot \left(T_{b2} - T_{l1} \right) \tag{7}$$

Power generation at the low-temperature endoreversible cycle is calculated from equation (8).

$$\dot{W}_{2} = \dot{Q}_{m} + \dot{Q}_{h3} - \dot{Q}_{l1} \tag{8}$$

The combined cycle is consisted of two endoreversible subsystems. Thus the second thermodynamic law for each subsystem is expressed as equations (8) and (9).

$$\dot{Q}_{h1} / T_{a1} + \dot{Q}_{h2} / T_{c1} = \dot{Q}_{m} / T_{b1} \tag{9}$$

$$\dot{Q}_{m} / T_{a2} + \dot{Q}_{h3} / T_{c2} = \dot{Q}_{l1} / T_{b2} \tag{10}$$

The dimensionless total power of combined cycle is obtained from the summation over the powers of endoreversible cycles. Since the total thermal conductivity is assumed to be constant. The dimensionless variables are defined as the following temperature ratios [16-20]:

$$\tau_{1} = T_{b1} / T_{a1} \tag{11}$$

$$\tau_{2} = T_{b2} / T_{a2} \tag{12}$$

$$k = T_{b2} / T_{b1} \tag{13}$$

$$\sigma_{1} = T_{c1} / T_{b1} \tag{14}$$

$$\sigma_{2} = T_{c2} / T_{b2} \tag{15}$$

$$x = (T_{h1} - T_{a1}) / T_{h1} \tag{16}$$

$$y = (T_{h2} - T_{c1}) / T_{h1} \tag{17}$$

$$z = (T_{h3} - T_{c2}) / T_{h1} \tag{18}$$

$$l = (T_{b2} - T_{l1}) / T_{h1} \tag{19}$$

$$o = (T_{b1} - T_{a2}) / T_{h1} \tag{20}$$

Dimensionless total power combined cycle based on a function of heat sources temperature, heat sink temperature and thermal conductivities achieved in the heat exchange between the boundaries.

$$\begin{aligned}\dot{W}_{CC} / C_{T} T_{h1} = &-(-r_{1} \cdot \sigma_{1} \cdot \tau_{1} \cdot x^{4} \cdot \tau_{2} \cdot T_{h1} + 4 \cdot r_{1} \cdot \sigma_{1} \cdot \tau_{1} \cdot x^{3} \cdot \tau_{2} \cdot T_{h1} - 6 \cdot r_{1} \cdot \sigma_{1} \cdot \tau_{1} \cdot x^{2} \cdot \tau_{2} \cdot T_{h1} + 4 \cdot r_{1} \cdot \sigma_{1} \cdot \tau_{1} \cdot x \cdot \tau_{2} \cdot T_{h1} + r_{1} \\ &\cdot x^{4} \cdot \tau_{2} \cdot T_{h1} - 4 \cdot r_{1} \cdot x^{3} \cdot \tau_{2} \cdot T_{h1} + 6 \cdot r_{1} \cdot x^{2} \cdot \tau_{2} \cdot T_{h1} - 4 \cdot r_{1} \cdot x \cdot \tau_{2} \cdot T_{h1} - r_{4} \cdot \sigma_{2} \cdot \tau_{2} \cdot T_{l1} - r_{4} \cdot \sigma_{2} \cdot \tau_{2} \cdot l \cdot T_{h1} - r_{5} \cdot \sigma_{2} \cdot \tau_{2} \cdot l \cdot T_{h1} + r_{4} \\ &\cdot \sigma_{1} \cdot T_{l1} + r_{4} \cdot \sigma_{1} \cdot l \cdot T_{h1} + l \cdot T_{h1} \cdot r_{5} \cdot \tau_{2} - T_{h1} \cdot r_{4} \cdot \sigma_{1} \cdot \tau_{1} \cdot \tau_{2} + T_{h1} \cdot r_{4} \cdot \sigma_{2} \cdot \tau_{1} \cdot \tau_{2}{}^{2} - T_{h1} \cdot r_{4} \cdot \sigma_{2} \cdot \tau_{1} \cdot \tau_{2}{}^{2} \cdot x + T_{h1} \cdot r_{4} \cdot \sigma_{1} \cdot \tau_{1} \cdot \tau_{2} \cdot x) / T_{h1} . \tau_{2}\end{aligned} \tag{21}$$

In combined cycle, heat is transferred between heat sources, heat sink and high-temperature heat engines by five heat exchangers, in other words, the combined cycle has five sources of heat transfer irreversibility. The ratio of thermal conductivity of each heat exchanger to the total thermal conductivity, in fact reflects mostly the size of thermal heat. Finite time thermodynamics is a combination of analyses based on thermodynamic and heat transfer concepts. In present study, the heat exchangers are assumed to be identical, hence the ratios of thermal conductivities of the heat exchangers in the entire system equals to 0.2. Unknown parameters of the irreversible combined cycle include independent thermodynamic variables of the system, T_{a1}, T_{a2}, T_{b2}, T_{c1}, T_{c2} and the heat transfer independent variables at the heat exchangers, r_1, r_2, r_3, r_4, and r_5. Heat transfer-thermodynamic analysis leads to an increase in the system degrees of freedom.

And the thermal efficiency of the system is written as equation (22).

$$\begin{aligned}\eta_{CC} = &(-r_{1} \cdot \sigma_{1} \cdot \tau_{1} \cdot x^{4} \cdot \tau_{2} \cdot T_{h1} + 4 \cdot r_{1} \cdot \sigma_{1} \cdot \tau_{1} \cdot x^{3} \cdot \tau_{2} \cdot T_{h1} - 6 \cdot r_{1} \cdot \sigma_{1} \cdot \tau_{1} \cdot x^{2} \cdot \tau_{2} \cdot T_{h1} + 4 \cdot r_{1} \cdot \sigma_{1} \cdot \tau_{1} \cdot x \cdot \tau_{2} \cdot T_{h1} + r_{1} \cdot x^{4} \cdot \tau_{2} \cdot T_{h1} - 4 \\ &\cdot r_{1} \cdot x^{3} \cdot \tau_{2} \cdot T_{h1} + 6 \cdot r_{1} \cdot x^{2} \cdot \tau_{2} \cdot T_{h1} - 4 \cdot r_{1} \cdot x \cdot \tau_{2} \cdot T_{h1} - r_{4} \cdot \sigma_{2} \cdot \tau_{2} \cdot T_{l1} - r_{4} \cdot \sigma_{2} \cdot \tau_{2} \cdot l \cdot T_{h1} - r_{5} \cdot \sigma_{2} \cdot \tau_{2} \cdot l \cdot T_{h1} + r_{4} \cdot \sigma_{1} \cdot T_{l1} + r_{4} \cdot \sigma_{1} \cdot l \cdot \\ &T_{h1} + T_{h1} \cdot r_{5} \cdot \tau_{2} - T_{h1} \cdot r_{4} \cdot \sigma_{1} \cdot \tau_{1} \cdot \tau_{2} + T_{h1} \cdot r_{4} \cdot \sigma_{2} \cdot \tau_{1} \cdot \tau_{2}{}^{2} - T_{h1} \cdot r_{4} \cdot \sigma_{2} \cdot \tau_{1} \cdot \tau_{2}{}^{2} \cdot x + T_{h1} \cdot r_{4} \cdot \sigma_{1} \cdot \tau_{1} \cdot \tau_{2} \cdot x) / (-r_{1} \cdot \sigma_{1} \cdot \tau_{1} \cdot x^{4} \cdot \tau_{2} \cdot \\ &T_{h1} + 4 \cdot r_{1} \cdot \sigma_{1} \cdot \tau_{1} \cdot x^{3} \cdot \tau_{2} \cdot T_{h1} - 6 \cdot r_{1} \cdot \sigma_{1} \cdot \tau_{1} \cdot x^{2} \cdot \tau_{2} \cdot T_{h1} + 4 \cdot r_{1} \cdot \sigma_{1} \cdot \tau_{1} \cdot x \cdot \tau_{2} \cdot T_{h1} + r_{1} \cdot x^{4} \cdot \tau_{2} \cdot T_{h1} - 4 \cdot r_{1} \cdot x^{3} \cdot \tau_{2} \cdot T_{h1} + 6 \cdot r_{1} \cdot \\ &x^{2} \cdot \tau_{2} \cdot T_{h1} - 4 \cdot r_{1} \cdot x \cdot \tau_{2} \cdot T_{h1} - r_{4} \cdot \sigma_{2} \cdot \tau_{2} \cdot T_{l1} - r_{4} \cdot \sigma_{2} \cdot \tau_{2} \cdot l \cdot T_{h1} - r_{5} \cdot \sigma_{2} \cdot \tau_{2} \cdot l \cdot T_{h1} + r_{4} \cdot \sigma_{1} \cdot T_{l1} + r_{4} \cdot \sigma_{1} \cdot T_{h1} - T_{h1} \cdot r_{4} \cdot \sigma_{1} \cdot \tau_{1} \\ &\cdot \tau_{2} + T_{h1} \cdot r_{1} \cdot \tau_{1} \cdot \tau_{2} \cdot x)\end{aligned} \tag{22}$$

Entropy generation is calculated as follows.

$$\dot{S}_{gen}/C_T T_{h1} = -T_{h1}\cdot T_{h3}\cdot T_{l1}\cdot\sigma_1\cdot\tau_1\cdot r_1\cdot x^4\cdot\tau_2 + 4\cdot T_{h1}\cdot T_{h3}\cdot T_{l1}\cdot\sigma_1\cdot\tau_1\cdot r_1\cdot x^3\cdot\tau_2 - 6\cdot T_{h1}\cdot T_{h3}\cdot T_{l1}\cdot\sigma_1\cdot\tau_1\cdot r_1\cdot x^2\cdot\tau_2 + 4\cdot T_{h1}\cdot T_{h3}\cdot T_{l1}\cdot\sigma_1\cdot\tau_1\cdot r_1\cdot\tau_2\cdot x + T_{h1}\cdot T_{h3}\cdot T_{l1}\cdot r_4\cdot\sigma_1\cdot\tau_1\cdot\tau_2\cdot x - T_{h1}\cdot T_{h3}\cdot T_{l1}\cdot r_4\cdot\sigma_1\cdot\tau_1\cdot\tau_2 + T_{h3}\cdot T_{l1}\cdot r_4\cdot\sigma_1\cdot l\cdot T_{h1} + T_{h3}\cdot T_{l1}^{\,2}\cdot r_4\cdot\sigma_1 - T_{h1}\cdot T_{h2}\cdot T_{l1}\cdot r_4\cdot\sigma_2\cdot\tau_1\cdot\tau_2^{\,2}\cdot x + T_{h1}\cdot T_{h2}\cdot T_{l1}\cdot r_4\cdot\sigma_2\cdot\tau_1\cdot\tau_2^{\,2} - T_{h2}\cdot T_{l1}\cdot r_4\cdot\sigma_2\cdot\tau_2\cdot l\cdot T_{h1} + T_{h2}\cdot T_{h3}\cdot T_{l1}\cdot r_1\cdot x^4\cdot\tau_2 - 4\cdot T_{h2}\cdot T_{h3}\cdot T_{l1}\cdot r_1\cdot x^3\cdot\tau_2 + 6\cdot T_{h2}\cdot T_{h3}\cdot T_{l1}\cdot r_1\cdot x^2\cdot\tau_2 - 4\cdot T_{h2}\cdot T_{h3}\cdot T_{l1}\cdot r_1\cdot\tau_2\cdot x - l\cdot T_{h1}\cdot T_{h2}\cdot T_{l1}\cdot r_5\cdot\sigma_2\cdot\tau_2 - T_{h2}\cdot T_{l1}^{\,2}\cdot r_4\cdot\sigma_2\cdot\tau_2 + l\cdot T_{h1}\cdot T_{h2}.T_{h3}\cdot r_5\cdot\tau_2/(T_{h1}\cdot T_{h2}\cdot T_{h3}\cdot T_{l1}\cdot\tau_2) \tag{23}$$

3. Parametric Study

In the parametric study of the behavior of the defined decision variables, including the primary temperature ratio and secondary temperature ratio and thermal conductivity, etc., the relation to the objective functions such as energy efficiency, dimensionless power and entropy production are examined. Based on Figure 1 of energy efficiency, the dimensionless power of the entropy generated relative to the thermal conductivity of the heat exchanger of the high temperature source is examined and shows that each of the criteria has a maximum and the maximum of different variables does not correspond to each other. The maximum dimensionless power is equal to 0.04473 $(r_1 = 0.1263)$, the maximum efficiency is 59.62% $(r_1 = 0.1919)$ and the minimum entropy generation has occurred in $r_1 = 0.0808$.

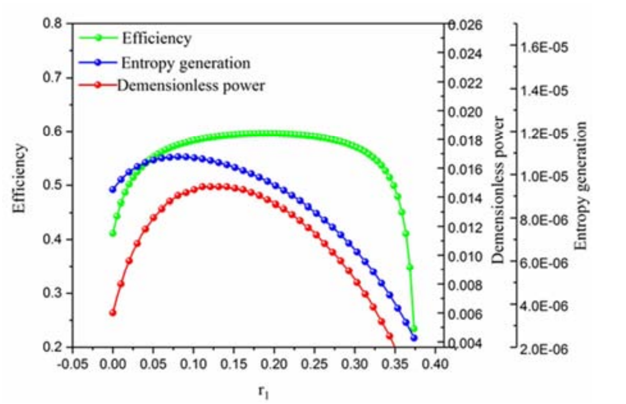

Figure 1. *Changes r_1 to energy efficiency, dimensionless power and entropy production.*

Regarding the behavior of the thermodynamic criteria analyzed relative to the variable r_2, it shows that increasing the thermal ratio of the second heat exchanger leads to a decrease in all criteria.

The changes in efficiency, dimensionless power, and entropy produced in the figure 3 below are similar to the figure 2, and it points out that with increasing r_3 increases energy efficiency and dimensionless power and produced entropy. The second and third heat exchangers have been added to help the concept of integrating systems with auxiliary resources to increase system availability.

The heat exchanger located between the top and bottom cycles is very important (HRSG), as shown in the figure 4, this converter has the maximum value based on different criteria.

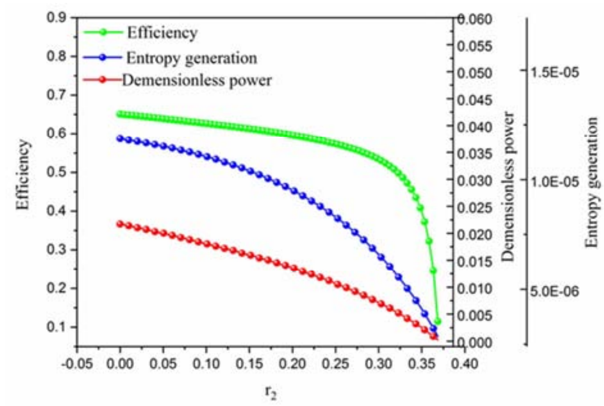

Figure 2. *Changes r_2 to energy efficiency, dimensionless power and entropy production.*

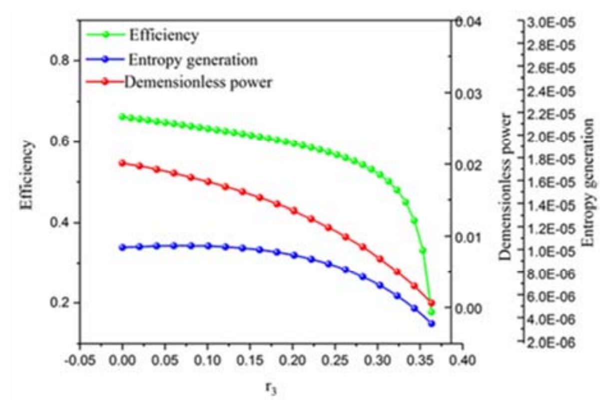

Figure 3. *Changes r_3 to energy efficiency, dimensionless power and entropy production.*

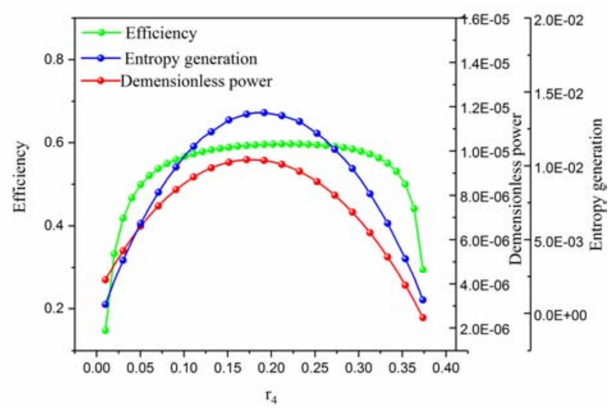

Figure 4. *Changes r_4 to energy efficiency, dimensionless power and entropy production.*

Obviously, in places where there is a lot of dimensionless power and efficiency, the production entropy is also high. According to the figure, the maximum energy efficiency is 59.67% in $r_4 = 0.2222$, the maximum dimensionless power is 0.0136 in $r_4 = 0.1919$ and the minimum entropy generation is 0.000002464 in $r_4 = 0.3737$.

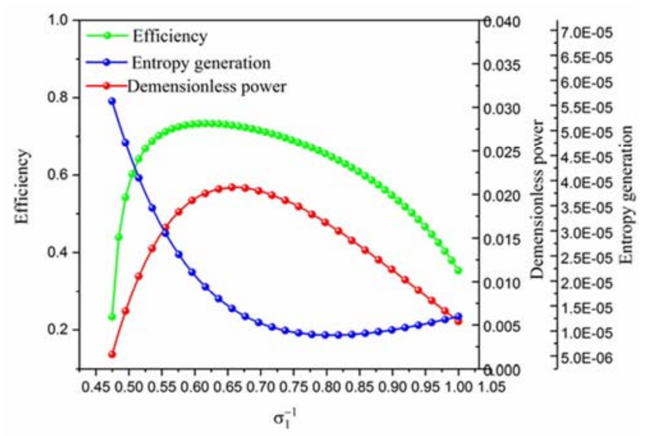

Figure 5. *Changes* σ_1^{-1} *to energy efficiency, dimensionless power and entropy production.*

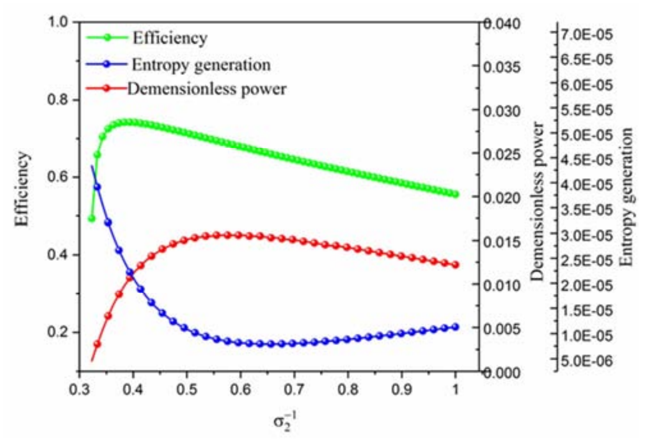

Figure 6. *Changes* σ_2^{-1} *to energy efficiency, dimensionless power and entropy production*

Also, figure 5 shows the behavior of the secondary temperature ratio of the high temperature cycle to the thermodynamic criteria. By increasing the secondary temperature ratio, the dimensionless power and energy efficiency have the maximum value and the produced entropy has minimum value. The maximum dimensionless power occurs in $\sigma_1^{-1} = 0.5758$. In terms of yield analysis, energy efficiency has occurred in $\sigma_1^{-1} = 0.3939$ and the minimum produced entropy in $\sigma_1^{-1} = 0.6566$ has occurred. σ_1^{-1} behavior has a strong effect on $\dot{Q}_{h2}$.

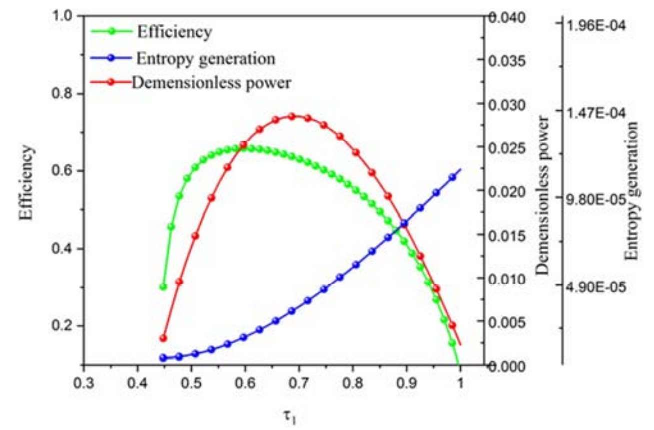

Figure 7. *Changes* τ_1 *to energy efficiency, dimensionless power and entropy production.*

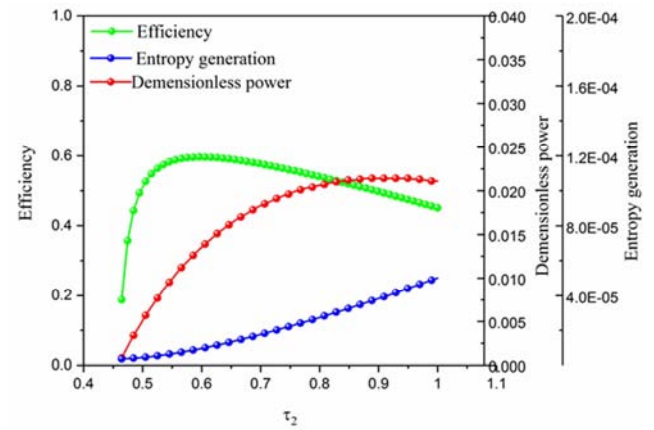

Figure 8. *Changes* τ_2 *to energy efficiency, dimensionless power and entropy production.*

The primary temperature ratios in the high and low temperature source are shown in the table 2.

Table 2. *Result of primary temperature ratio of the high and low temperature source.*

	Based on the minimum entropy production	**Based on maximum energy efficiency**	**Based on maximum dimensionless power**
The primary temperature ratio of the high temperature source	0.6866	0.5224	0.4478
The primary temperature ratio of the low temperature source	0.9192	0.5960	0.4646

4. Optimization

The goal of thermodynamics is to limit the time to assess the limitations of work and heat conversion and to optimize different thermodynamic criteria. Optimization in thermodynamics is performed in order to maximize or minimize the thermodynamic criterion. The optimization is based on the power criterion as follows. Accordingly, the optimization bounds include in Table 3. Optimization results based on different criteria show in Table 4.

Table 3. *Bound of optimization.*

Range	Variable
r_1	[0, 1]
r_2	[0, 1]
r_3	[0, 1]
r_4	[0, 1]
σ_1^{-1}	[0, 1]
σ_2^{-1}	[0, 1]
τ_1	[0, 1]
τ_2	[0, 1]
k	[0, 1]

5. Conclusion

This study was conducted to investigate the cascade combined cycle according to Stephen Boltzmann's law of heat transfer. The application of this research is the first in the solar system that the sun is used as a heat source. In addition, the calculated results of this study can be used to improve the performance of thermodynamic systems. The results of this study are very close to the actual results. The calculated operating fluid temperature and thermal conductivity can be very useful in the design and development of heat exchangers. The optimization results show that the maximum dimensionless power, the maximum energy efficiency and minimum entropy generation are 0.035092393, 61.09% and 8.132 E-07, respectively. The results show that converters based on radiant heat transfer have a good future, and future heat exchangers are a combination of all heat transfer mechanisms.

Table 4. *Optimization results based on different criteria.*

Variable	Based on dimensionless power	Based on energy efficiency	Based on entropy generation
r_1	0.1703	0.5198	0.5198
r_2	0.1255	0.1395	0.1395
r_3	0.0543	0.0600	0.0600
r_4	0.3044	0.2516	0.2516
r_5	0.3454	0.0291	0.0291
σ_1^{-1}	0.8028	0.9540	0.9540
σ_2^{-1}	0.8860	0.7060	0.7060
k	0.6933	0.4613	0.4613
$T_{a1}(k)$	1238.6663	1233.3705	1399.6745
$T_{a2}(k)$	464.7456	572.8918	760.4318
$T_{b1}(k)$	631.8762	646.2826	772.7495
$T_{b2}(k)$	438.0494	318.7359	356.4970
$T_{c1}(k)$	787.0963	757.6172	810.0237
$T_{c2}(k)$	494.3922	374.2541	504.9881
τ_1	0.5101	0.5240	0.5521
τ_2	0.9426	0.5564	0.4688
$\dot{Q}_{h1}/T_{l1}\cdot C_T$	0.0660	0.0132	0.0005
$\dot{Q}_{h2}/T_{l1}\cdot C_T$	0.0034	0.0046	0.0020
$\dot{Q}_{h3}/T_{l1}\cdot C_T$	0.0003	0.0019	0.0002
$\dot{W}_{CC}/T_{l1}\cdot C_T$	0.035092393	0.012099428	0.001558578
η	0.5041	0.6109	0.5627
S_{gen}	0.00006408	0.000002801	8.132E-07

References

[1] Curzon F., Ahlborn B. Efficiency of a Carnot engine at maximum power output. American Journal of Physics. 1975; 43 (1): 22-4.

[2] Andresen B., Berry R. S., Nitzan A., Salamon P. Thermodynamics in finite time. I. The step-Carnot cycle. Physical Review A. 1977; 15 (5): 2086-93.

[3] Andresen B., Salamon P., Berry R. S., F. P. Thermodynamics in finite time: extremals for imperfect heat engines. The Journal of Chemical Physics. 1977; 66 (4): 1571-7.

[4] Salamon P., Andresen B., Berry R. S. Thermodynamics in finite time. II. Potentials for finite-time processes. Physical Review A. 1977; 15 (5): 2094-102.

[5] Yan Z., Chen J. Optimal performance of a generalized Carnot cycle for another linear heat transfer law. The Journal of Chemical Physics. 1990; 92 (3): 1994-8.

[6] Chen J., Yan Z., Lin G., Andresen B. On the Curzon - Ahlborn efficiency and its connection with the efficiencies of real heat engines. Energy Conversion and Management. 2001; 42 (2): 173-81.

[7] Wu C. Finite-time thermodynamics and its potential naval shipboard application. Naval engineers journal. 1989; 101 (1): 35-9.

[8] Wu C. Power performance of a cascade endoreversible cycle. Energy Conversion and Management. 1990; 30 (3): 261-6.

[9] Wu C. Maximum obtainable power of a carnot combined power plant. Heat Recovery Systems and CHP. 1995; 15 (4): 351-5.

[10] Naserian M. M., Farahat S., Sarhaddi F. Finite time exergy analysis and multi-objective ecological optimization of a regenerative Brayton cycle considering the impact of flow rate variations. Energy Conversion and Management. 2015; 103 (Supplement C): 790-800.

[11] Naserian M. M., Farahat S., Sarhaddi F. Exergoeconomic multi objective optimization and sensitivity analysis of a regenerative Brayton cycle. Energy Conversion and Management. 2016; 117 (Supplement C): 95-105.

[12] Naserian M. M., Farahat S., Sarhaddi F. New exergy analysis of a regenerative closed Brayton cycle. Energy Conversion and Management. 2017; 134 (Supplement C): 116-24.

[13] De Vos A. Endoreversible thermoeconomics. Energy conversion and management. 1995; 36 (1): 1-5.

[14] Chen J., Wu C. Maximum specific power output of a two-stage endoreversible combined cycle. Energy. 1995; 20 (4): 305-9.

[15] Şahin B., Kodal A. Steady-state thermodynamic analysis of a combined Carnot cycle with internal irreversibility. Energy. 1995; 20 (12): 1285-9.

[16] Ghasemkhani A., Farahat S., Naserian M. M. Evaluation combined cycle irreversible on the criteria of maximum power in finite time thermodynamics. Modares Mechanical Engineering. 2018; 17 (11): 333-42.

[17] Ghasemkhani A., Farahat S., Naserian M. M. Performance analysis and optimization Tri-Generation system using Finite-Time Thermodynamics concepts. Modares Mechanical Engineering. 2018; 18 (02): 61-72.

[18] Ghasemkhani A., Farahat S., Naserian M. M. Multi-objective optimization and decision making of endoreversible combined cycles with consideration of different heat exchangers by finite time thermodynamics. Energy Conversion and Management. 2018; 1711052-62.

[19] Ghasemkhani A., Farahat S., Naserian M. M. Thermodynamic investigation and optimization Tri-generation system for the provision of power, heating, and cooling: A case study of Zahedan, Iran. International Journal of Heat and Technology. 2018; 36 (3): 904-12.

[20] Ghasemkhani A., Farahat S., Naserian M. M. The development and assessment of solar-driven Tri-generation system energy and optimization of criteria comparison. Energy Equipment and Systems. 2018; 6 (4): 367-79.

[21] Baghernejad A., Yaghoubi M. Genetic algorithm for multi-objective exergetic and economic optimization of parabolic trough collectors integration into combined cycle system (ISCCS). ASME 2010 10th Biennial Conference on Engineering Systems Design and Analysis. American Society of Mechanical Engineers2010. pp. 289-97.

[22] Baghernejad A., Yaghoubi M., Jafarpur K. Exergoeconomic optimization and environmental analysis of a novel solar-trigeneration system for heating, cooling and power production purpose. Solar Energy. 2016; 134165-79.

[23] Baghernejad A., Yaghoubi M., Jafarpur K. Exergoeconomic comparison of three novel trigeneration systems using SOFC, biomass and solar energies. Applied Thermal Engineering. 2016; 104 (Supplement C): 534-55.

[24] Barranco-Jimenez M. A., Sanchez-Salas N., Angulo-Brown F. On the optimum operation conditions of an endoreversible heat engine with different heat transfer laws in the thermal couplings. Revista mexicana de fisica. 2008; 54 (4): 284-92.

[25] De Parga G. A., Angulo-Brown F., Navarrete-Gonzalez T. A variational optimization of a finite-time thermal cycle with a nonlinear heat transfer law. Energy. 1999; 24 (12): 997-1008.

[26] O'Sullivan C. T. Newton's law of cooling—a critical assessment. American Journal of Physics. 1990; 58 (10): 956-60.