Journal of Finance and Accounting
2021; 9(6): 236-248
http://www.sciencepublishinggroup.com/j/jfa
doi: 10.11648/j.jfa.20210906.14
ISSN: 2330-7331 (Print); ISSN: 2330-7323 (Online)

Explanatory Elements of the Deformations of the Term Structure of Interest Rates

Assoumou Menye Oscar

Department "Finance and Accounting", ESSEC of the Douala University, Douala, Cameroon

Email address:
oscar_assoumou@yahoo.fr, oscar.assoumou@gmail.com

To cite this article:
Assoumou Menye Oscar. Explanatory Elements of the Deformations of the Term Structure of Interest Rates. *Journal of Finance and Accounting*. Vol. 9, No. 6, 2021, pp. 236-248. doi: 10.11648/j.jfa.20210906.14

Received: October 19, 2021; **Accepted**: November 8, 2021; **Published**: November 17, 2021

Abstract: The interest rate curve has often been defined as a graphical representation of the yield offered by bonds of a single issuer according to their maturity, from shortest to longest. The best-known yield curve, which serve as a benchmark for the entire bond market in a given country, is that of the government bonds. Indeed, the shape of the interest rate curve on sovereign bonds provides information on investor's expectations on the risk of default of the issuing state as well as on the level of inflation and future interest rates. As a result, it is a good indicator of the economic and financial health of the issuing country. In a stable economic environment marked by low inflation and sustainable public debt, bond yields increase with the maturity of securities. This is explained by the fact that the longer the maturity, the greater the risk of events occurring that could adversely affect the value of the bond security. Indeed, the further in time one goes, the greater the uncertainty about the issuer's repayment capacity or about the level of interest rates. Under these conditions, investors require a risk premium to lend on long maturities compares to short maturities. However, as government loans are generally considered to be loans for which repayment is certain, the associated risk premiums are low or almost zero. On the longest maturities (20 to 50 years), the default risk as well as the interest rate or inflation risks can be considered to be broadly identical. This is the reason why the yield curves on government bonds of most "so-called" developed countries have an increasing and concave shape. This article highlights the explanatory factors of the structure of forward interest rates by proposing a multifactorial model of asset valuation that is at the same time exhaustive, simple, intelligible and realistic. The underlying objective is to propose techniques hedging against the risk of interest rates more effective than traditional techniques, especially since we live in an extremely sensitive and changing environment because of the consequences of covid-19 on economies. The contribution is part of the research movement aimed at improving the multifactorial models of the yield curve and to overcome the shortcomings of the techniques traditionally used.

Keywords: Yield Curve, Structure of Forward Rates, Principal Component Analysis (PCA), Multifactor Model, Risk Hedging, Duration

1. Introduction

This work aims to identify the factors that explain the forward structure of interest rates in order to subsequently propose a model multifactorial valuation of assets linked to interest rates that is at the same time exhaustive, simple, realistic, understandable and accessible. Such a model would eventually lead to techniques for hedging against interest rate risk that are more effective than existing techniques, especially since we live in an increasingly whimsical, changing or even kaleidoscopic environment whose covid-19 reinforces the devastating and distorting effects, companies find themselves under an obligation to be proactive and to make major changes. This contribution, which is largely inspired by the work carried out in the United States by Litterman and Scheinkman [13] then applied in France by the "Caisse des Dépôts et Consignations", is part of the research movement aimed at improving the multifactorial models of the curve. Rates and to compensate for the shortcomings of commonly used techniques. Thus, the first section sets out the classic approaches to hedging against interest rate risk, emphasizing their limits; a second section describes the empirical methods usually used, a third presents the results and their interpretations, and a fourth and final section summarizes the main results.

2. Interest of a Principal Components Analysis on the Forward Rate Structure

2.1. Presentation of the Interest Rate Risk Hedging Techniques Currently Used and Their Limits

Interest rate risk is the risk of changes in the market value of a portfolio, due to unanticipated movements in interest rates [1, 6]. Hedging against this risk consists of holding a second portfolio, the variations of which will best offset those of the first. The whole problem is to determine this optimal cover portfolio [16]. Several methods are already used for this purpose.

2.1.1. Simple Duration

The simplest method consists of hedging against an unanticipated variation in rates, corresponding to a vertical translation of the forward rate structure. We only consider one source of risk: the overall level of interest rates. For this method, the hedging will not, a priori, be effective, if the yield curve undergoes a deformation other than an overall vertical shift.

Literman and Scheinkman [13] evaluate the percentage of unexplained variance of profitability for portfolios of Treasury bills hedged in two ways: once using three factors, once using simple duration. The percentage of unexplained variance in profitability in the second case is always significantly higher.

2.1.2. Polynomial Duration

This technique consists in modeling the forward structure of rates by a polynomial of degree N-l, i.e. writing the rate at maturity i, r (i), as a polynomial function of i:

$$\mathrm{r(i)} = \sum_{k=0}^{N-1} a_k \times i^k \tag{1}$$

After determining the coefficients a_k, the rates r(i) thus modeled represent the expected rate structure. Hedging against their unexpected evolution then amounts to hedging against a_k.

The hedging criterion used is then:

$$\forall k \in \{0;1;\ldots;N-1\}$$

$$\frac{\partial}{\partial a_k}\,[P(T)] = 0 \tag{2}$$

Where P(T) is the market value of the bond portfolio held until maturity T. This optimization criterion is written in the form of N equations, that is to say that it requires N coverage assets denoted F_h and this in proposals X_h

$$\forall k \in \{0;1;\ldots;N-1\}, \frac{\partial}{\partial a_k}\,[P(T) - \textstyle\sum_{h=1}^{N} X_h F_h(T)] = 0 \tag{3}$$

If we set $D_k = \sum \frac{i^k C_i}{(1+r(i))^{i+1}}$ where Ci is the coupon paid on date i by the asset considered, we have: $\forall k \in \{0;1;\ldots;N-1\}$

$$D_k\big(P(T)\big) - \textstyle\sum_{h=1}^{N} X_h D_k(F_h) = 0 \tag{4}$$

This system with N equations, N unknowns gives the composition of the optimal hedging portfolio $(X_h)_{1\le h\le N}$.

Compared to the simple duration, this method has the advantage of neutralizing more complex deformations of the yield curve. However, the polynomial form is completely arbitrary and has little financial significance. In particular, it has the disadvantage of diverging when i tends $+\infty$.

2.1.3. Duration from a Theoretical Model of the Structure of Rates

The principle is the same as that of polynomial duration. It also consists in theoretically modeling the anticipated rate structure for the coverage horizon T, then in considering the risks linked to an unanticipated change in the parameters of the model.

The method based on the Vasicek model [15].

The dynamic process used in this model is as follows:

$$dr(t) = (ar(t) + b)\,dt + v\,dW_t$$

where: r(t) is the short rate at date t; a, b, v are assumed to be constant over the period studied; a is a negative a priori constant representing a restoring force; v is a positive a priori constant representing the volatility of the process; W_t is Brownian motion.

Calculations from this process lead to an interest rate curve of the form:

$$r(i) = R_\infty + \Big(R_\infty - \big(r(t)\big) \times \frac{\alpha_i}{a \times i} - \frac{v^2}{a^2} \times \frac{a \times \alpha_i^2}{4i} \tag{5}$$

Where: $R_\infty = \frac{b}{a} - \frac{v^2}{2a^2}$ is the long bond market rate; $\alpha_i = 1 - e^{a\times i}$ is a constant; r(t) is the short rate at date t.

From there, three sources of risk are taken into account: R_∞, the long rate; $S = R_\infty - r(t)$, the difference between the long rate and the short rate; $\sigma = v^2/a^2$, the variance,

If we consider a portfolio whose expected price is V(T) and which is characterized by a series of flows C_i, we will define three durations, each relating to a source of risk:

$$D^\infty = \frac{\partial V(T)}{\partial R_\infty} = \sum_i \frac{C_i}{[1+r(i)]^{i+1}}$$

$$\frac{1}{a} D^s = \frac{\partial V(T)}{\partial S} = \frac{1}{a} \sum_i \frac{\alpha_i C_i}{[1+r(i)]^{i+1}} \tag{6}$$

$$\frac{a}{4} D^\sigma = \frac{\partial V(T)}{\partial \sigma} = \frac{a}{4} \sum_i \frac{\alpha_i^2 C_i}{[1+r(i)]^{i+1}}$$

We then constitute a cover portfolio with three cover assets F_1, F_2, F_3 in proportions X_1, X_2 and X_3 such as:

$$\forall k = \infty, S, \sigma;$$

$$D^k\big(V(T)\big) = \textstyle\sum_{h=1}^{3} X_h D^k(F_h) \tag{7}$$

Solving the system gives the composition of the optimal hedging portfolio, $X = (X_1; X_2; X_3)$.

This model has some limitations: interest rates are not necessarily positive; the long-term rate remains constant, which is a questionable approximation; the coefficients are assumed to be constant while we seek to hedge against variations in these coefficients

The other models

Other authors have tried to improve the model by introducing a new explanatory variable in addition to the short rate, they generally use a long rate or a difference between the short rate and the long rate. This is the case with the model of Brennan and Schwartz [5], with two state variables: the infinitely short maturity rate and the rate of return on a perpetual annuity that would continuously pay out one euro per year. Subsequently, other authors have developed more sophisticated models, considering multiple state variables. This is how Ho and Lee [10] designed a discrete-time model, where the initial yield curve was given and each zero-coupon bond price followed a binomial process. Then, Heath, Jarrow and Morton [9] proposed a model where each forward rate was considered as a state variable. The first, simpler models do not always offer sufficient coverage, either because they do not take into account enough explanatory factors, or the factors taken into account are not the most explanatory. As for the more recent models [4, 8, 11], they are very complete and very satisfactory from a theoretical point of view, but they are very difficult to set up in practice.

In relation to all these theoretical considerations, we realize the need for empirical work to identify what are the common factors that determine the rate structure. The objective is therefore to identify which factors are necessary and sufficient to model the evolution of the yield curve as accurately and as realistically as possible. Thus, principal component analysis can meet this need.

2.2. Radioscopy of Work Already Carried out in the Field

2.2.1. The Work of R. Litterman and J. Scheinkman [13] on American Data

The authors propose to explain the variation in the returns on fixed income securities by three factors determining the yield curve: the overall level of rates, the slope of the yield curve rates and its curvature. This three-factor approach then allows them to set up hedging strategies.

Implicit zero-coupon curve

Since valuation errors would arise from peculiarities of the zero-coupon treasury bond market, the authors evaluate the zero-coupons that best explain observed coupon bond prices by viewing coupon bonds as a linear combination of zero-coupon bonds. They associate these zero-coupon prices with what they call adjusted interest rates and thus obtain an adjusted rate curve.

Principal Component Analysis (PCA)

In their study, the authors seek to determine the common factors affecting the returns on Treasury bills and bonds. A principal component analysis of the daily returns of securities, in excess of a risk-free rate (they chose the *"overnight"* rate), shows that a three-factor model explains at least 96% of the variance of returns.

A variation of the first factor causes a rate change of equivalent amplitude for each zero-coupon rate, regardless of its maturity, i.e. it causes an upward shift in the curve rate. The authors therefore qualify this factor as the general level of rates. A variation of the factor 2 causes a drop in the rates for zero-coupons of less than five years and an increase in the rates for the others, this increase increasing with maturity. As this factor contributes to increasing the slope of the zero-coupon yield curve, the authors call it the "slope" factor. The third factor, which the authors call "curvature" increases the curvature or concavity of the yield curve for zero-coupons of less than 20 years (more exactly increase of the concavity between the 0 and the 12 years, of the convexity between 12 and 20 years). This impact on the yield curve is analogous to that of a change in the volatility of interest rates.

On all the zero-coupons observed: the factor 1 explains 89.5% of the total variance; factor 2 explains 8.5% of the total variance; the factor 3 explains 2% of the total variance.

After carrying out this analysis, the authors proposed a taxonomy of portfolios simulating each of the factors. These are portfolios of securities that are sensitive to only one factor, while being as diversified as possible.

The variation in profitability of these portfolios therefore simulated that of the factor considered. Then, the authors put together a portfolio that is only sensitive to rate volatility. The correlations of the profitability of this portfolio with the other factors are as follows: 0 for factor 1; 0.2 for factor 2; 0.9 for factor 3. The "curvature" factor is therefore very close to the "volatility" factor.

Application of the model

The authors apply their model to a portfolio of bonds held from February 5 to March 5, 1986. The holding of this portfolio results in a loss of $ 676,200 in total which the three-factor model explains except for $ 11,800. This error is smaller than the one made using the simple duration method.

2.2.2. Principal Component Analyzes Carried out on the French Bond Market

Artus, Belhomme, Elalouf and Minczeles [3] carry out a PCA on yield curves estimated at the end of the month, based on the prices of government bonds and OATs. This study, carried out on the basis of Fininfo, is a principal components analysis on the prices of zero coupons, from 1 year to 9 years, estimated at the end of the month, from 1988 to 1991, from OAT and OAT quotations government bonds, by multiple linear regressions. It highlights a factor corresponding to the general level of rates, explaining 82% of the deformations of the yield curve and a factor corresponding to an indicator of the slope of the curve, explaining 15% of its deformations.

Very recently, other applications work by Litterman and Scheinkman [13] was carried out on the French market using factor analysis over the period 2019-2020 [2, 14]. The yield curves were obtained by interpolations carried out on the daily observations of the prices of Treasury bills. Two factors are highlighted by this analysis, the first explaining 93.7% of the total variance and the second, 6.1%. The first factor is the general level of rates and the second is the spread between the short-term rate and the long-term rate. The second factor systematically cancels out around the seven-month maturity. It is then possible to consider the first factor linked to the seven-month rate and the second as linked to the first derivative of the yield curve for the seven-month maturity.

This approach was used by Jacquillat and Laguiche [12] to analyze the variations in the margins of four variable rate bond issues. Their study, although applied to different bond products, yields similar results to those cited above. Two common factors explain 86% of the variance in margins, the first being a movement of all margins, the second being the margin difference between short borrowings and long borrowings.

The zero -coupons price estimation methodology used in this contribution is inspired by that used by Artus, Belhomme, Elalouf and Minczeles [3]. This methodology will be explained in the second part of this work.

2.3. Interest of a Principal Component Analysis on the Yield Curve

The principle of principal component analysis is the diagonalisation of a variance-covariance matrix of the rates on the maturity scale. If we consider the results of previous studies, carried out in the French context, two factors are sufficient to explain the evolution of the yield curve, each of these factors being associated with an eigenvalue of the diagonalised matrix.

For the date t, initially proposed by EL Karoui and Lacoste [7], is:

$$B_{t;t+\theta} = F_1(\theta)\frac{\mu_1}{\mu}\, N_t^1 + F_2(\theta)\frac{\mu_2}{\mu}\, N_t^2 + \bar{B}(\theta) \qquad (8)$$

Where $F_1(\theta)$ and $F_2(\theta)$ are the coordinates of the zero-coupon maturity θ respectively on the first and second eigenvectors of the diagonalised matrix; μ_1 and μ_2 are the eigenvalues associated with the first and the second factor; μ is the sum of all the eigenvalues or the number of maturities covered by the analysis.

The associated eigenvalues μ_1 and μ_2 can be interpreted as weighting factors, N_t^1 and N_t^2 are random following a reduced centered normal distribution; $\bar{B}(\theta)$ is the empirical average of the price of the zero coupon of maturity θ.

It follow that the zero coupon rate of maturity θ is:

$$Y_{t;t+\theta} = -\frac{1}{\theta}\, 1n\left(B_{t;t+\theta}\right) \qquad (9)$$

Thus, the factors highlighted by the principal component analysis give information on the type of distortion undergone by the yield curve, as we will see later, and define the way in which each rate varies according to its maturity.

3. The Methodology

3.1. Data Selected for the Study

The data used are taken from the AFFI-SBF bond database. This base includes the prices, accrued coupons and transaction volumes for around 4,000 loans from the beginning of January 2007 to the end of December 2020.

For short-term maturities (less than one year), the Euribor rates on the Euro, maturity date are used. one month, two months, three months, six months, nine months, from January 2018 to December 2020. For the price estimate of zero-long-term coupons (one to nine years), were selected, to achieve I ' ACP, loans with the following characteristics: the guarantee is that of the State; the nominal rate is a fixed rate; the depreciation method must be in fine; there must not be any particular option attached to the loan (early repayment, possibility of extension, exchange option, etc.).

Indeed, the zero-coupon rate curves are estimated from the prices of the loans. State corresponding to the criteria mentioned above. In order not to bias the yield curves thus obtained, the securities used must have the same characteristics in order to be able to be explained by a common valuation model.

They must therefore come from issuers benefiting from the same issuance conditions, in order to avoid the rate differential linked to the signature.

They must not be subject to special clauses which can be analyzed as options liable to modify the prices. Usually, the presence of an option affects the price of the loan. Likewise, amortization in equal series can be analyzed as a prepayment option at the option of the issuer. Bond loans with repayment by equal series therefore systematically include a discount, not attributable to the zero-coupon prices explaining the prices of government loans. This explains why only the loans in fine are selected.

3.2. The Determination of Zero-coupon Pyramids

Weekly rate curves are calculated from January 2018 to December 2020.

3.2.1. Calculation of the Prices of Zero Coupons at Less Than One Year

The comparison of the twelve-month Euribor and the one-year rate deducted from the prices of government bonds, over the study period, shows that on average, the one-year Euribor is 40 basis points above the one-year rate deducted from government bonds. This difference has two main explanations: the Euribor is an offered rate while the one-year rate deducted from government loans is an average rate between the offered rate and the demanded rate; Euribor is associated with an interbank signature risk, which can be considered greater than the risk of government signature.

In order to obtain homogeneous yield curves between short and long rates, 40 basis points are systematically deducted from the Euribor rates. The use of Euribor rates thus “adjusted” avoids artificially creating a gap between short and long rates, in the estimation of the yield curves.

The zero-coupon prices are then calculated, from the adjusted Euribor rates, as follows:

$$P_{imonth} = \frac{1}{1 + \frac{imonth}{12}\, T_{imonth}}$$

Where P_{imonth} is the price of the zero-coupon maturity i month; T_{imonth} is the adjusted Euribor rate due i month.

3.2.2. Calculation of the Prices of Zero-coupons at One Year and More

Presentation of the method

The estimation of the prices of zero-coupons, best explaining the prices of the loans selected, results from the following valuation model:

$$CP_i = \sum_{t=0}^{T_i} C_i \times P^{\theta_i + t} + 100 \times P^{\theta_i + T_i} \tag{10}$$

Where CP_i is the full price coupon of security i; C_i is the nominal rate of security i; T_i is the entire part of the maturity remaining for security i (in number of years); θ_i is the fraction of maturity remaining to run for security i exceeding T_i; and $P^{\theta_i + t}$ is the discounted zero-coupon price of one payable on the date t + θ_i.

The zero coupons rate at maturity t + θ_i, noted r(t + θ_i), verifies:

$$P^{t+\theta_i} = \left[\frac{1}{1 + r(t + \theta_i)}\right]^{t + \theta_i}$$

Materializes the fact that, for each loan, the date of detachment of coupons is different. However, on each estimate date of the yield curve, the estimated zero-coupon prices must be common to all the securities and correspond to full maturities. To remedy this problem, we is a convex linear combination of the prices of zero-coupons with respective maturity t and t + l.

$$P^{t+\theta_i} = (1 - \theta_i)P^t + \theta_i P^{t+1} \tag{11}$$

By replacing, in the equation (10), $P^{\theta_i + t}$ by its value in equation (11), we obtain, for T_i strictly greater than 1:

$$CP_i = \sum_{t=0}^{T_i} C_i \times [(1 - \theta_i)P^t + \theta_i P^{t+1}] + 100 \times [(1 - \theta_i)P^{T_i} + \theta_i P^{T_i + 1}] \tag{12}$$

$$\underbrace{CP_i - C_i \times (1 - \theta_i)}_{PP_i} = C_i[\textstyle\sum_{t=1}^{T_i - 1} P^t] + [C_i + 100(1 - \theta_i)] \times P^{T_i} + \theta_i (C_i + 100) P^{T_i + 1} \tag{13}$$

Where PP_i is the foot-of-coupon price of title i.

For loans with a remaining maturity of less than one year, equation (13) becomes:

$$\begin{gathered} CP_i = C_i[(1 - \theta_i) + \theta_i P^1] + 100 \times [(1 - \theta_i) + \theta_i P^1] \\ CP_i - C_i(1 - \theta_i) = 100(1 - \theta_i) + (C_i + 100)\theta_i P^1 \\ PP_i \text{ - } 100(1 - \theta_i) = (C_i + 100)\theta_i P^1 \end{gathered} \tag{14}$$

This allows the following model to be written:

$$\begin{gathered} PP_1 \text{ - } 100(1 - \theta_1) = (C_1 + 100)\theta_1 P^1 \\ \vdots \\ PP_1 = C_i P^1 + \dots + C_i P^n + \dots + [C_i + 100(1 - \theta_i)]P^{T_i} + \theta_i (C_i + 100) P^{T_i + 1} \\ \vdots \\ PP_s = C_s P^1 + \dots + C_s P^n + \dots + [C_s + 100(1 - \theta_s)]P^{T_s} + \theta_s (C_s + 100) P^{T_s + 1} \end{gathered} \tag{15}$$

The system (15) can be written in the form PP_t = M_t. P_t + u_t where u_t is the random vector, P_t is the vector estimated zero coupon prices, PP_t is the vector of the footing prices of coupons for the date t. The vectors P_t, PP_t and the matrix M_t can be expressed as follows:

$$PP_i = \begin{bmatrix} PP_i - 100(1 - \theta_i) \\ PP_i \\ PP_s \end{bmatrix} \quad P_t = \begin{bmatrix} P^1 \\ P^{15} \end{bmatrix}$$

$$M_t = \begin{bmatrix} (C_1 + 100)\theta_1 & 0 & \dots\dots & \dots\dots & \dots 0 \\ \dots. & \dots & \dots\dots & \dots\dots & \dots \\ C_i & \dots C_i & \dots C_i + (1 - \theta_i)100 & \theta_i(C_i + 100)\dots & \dots 0 \\ \dots & \dots & \dots\dots & \dots\dots & \dots \\ C_{20} & \dots\dots & C_{20}\dots.. & C_{20}+(1 - \theta_{20})100 & \theta_{20}\ (C_{20} + 100) \end{bmatrix}$$

Depending on the dates, the maturities range from one to thirteen years, from one to fourteen years or from one to fifteen years. Beyond these maturities, the absence of repayment of securities for certain maturities and the insufficient number of loans make it problematic to estimate the prices of zero coupons.

The results of the regressions

The coefficients representing the prices of zero - Coupons could only be calculated, by regression, from one to ten years because the vectors of coupons corresponding to longer maturities were linear combinations of each other due to the low number of loans available on the market. In addition, the anomalies obtained on the ten-year zero-coupon do not allow this maturity to be taken into account in the analysis.

On the other hand, for other maturities, the conditions for non-arbitrage are respected (inequalities 16), and the zero-coupon rate values obtained are consistent.

$$\forall i, 0 > P_i > 1 \; et \; \forall i, \forall j, P_i > P_j \; si \; j > i \tag{16}$$

The regressions make it possible to constitute a sample of 133 zero-coupon price pyramids, with maturities ranging from one to nine years. The correlation coefficients obtained are all in the neighborhood of one, which confirms that the evaluation model used is fully explanatory.

The calculated Student's t are also all very high. The valuation error on a security is on the order of a penny. The implicit zero-coupon yield curves thus obtained are ascending from January 2018 to June 2019, then inverted from July 2019 to June 2020 and again ascending from July 2020 to December 2020.

3.3. Principal Component Analyzes

The PCA in the space of the variables considered, consists in projecting each reduced-centered price variable P, on the k factors or orthogonal axes (initially unobservable) that best explain the inertia of the cloud of variables.

Three PCA are carried out over the entire study period: the first on the fourteen calculated zero-coupon prices, the second on the short-term zero-coupon prices (one month to one year) and the third on the zero-coupon prices long-term coupon (one to nine years). The study is supplemented by an analysis of the results over three annual sub-periods 2018, 2019 and 2020.

4. The Results Obtained

4.1. Presentation of the Factors Obtained

4.1.1. Results Obtained Over the Entire Period

Over the period studied, 98.36% of the variance is explained by the first three factors: the first factor explains 89.96% of the deformation of the yield curve, the second factor in explains 5.84% and the third explains 2.56%. The latter can therefore almost be neglected. The coordinates of the 14 price variables are therefore only indicated on the first three factors, the coordinates on the other seven factors being very close to zero. These coordinates appear in table 1.

The projection of these fourteen price variables on the plane defined by the first two factors is found in appendix 2, we notice on this figure that the points projected on the plane are all very close to the center circle zero and radius one, which attests to the fact that they are all very well represented by the two factors. Whatever the zero-coupon price considered, the percentage of price variance explained by the first two factors is always greater than 89.78%. The representation of the deformations of the implicit interest rate curve by two factors is therefore relatively satisfactory. Taking into account a third factor brings the minimum percentage of variance explained for each zero-coupon to 94.16% and improves the results mainly for maturities.

The interpretation of the first two factors corresponds to the classic results of PCA linked to the yield curve. According to the figure in appendix 2, we see that factor one groups together all the variables in the right part of the circle since the coordinates of the prices of zero-coupons on factor one are very similar, while factor two opposes the maturities short (lower part of the circle) to long maturities (upper part of the circle). The factor three, for its part, according to appendix 3, opposes maturities of two years to 5 years (medium term) to extreme maturities: it is an indicator of curvature.

Table 1. *Price coordinates of zero- coupons on the first three factors.*

Price form 1 month to 9 years	Coordinate on the first factor	Coordinate on the second factor	Coordinate on the third factor
$P_{1\ month}$	0,95005	-0,29888	0,0374
$P_{2\ months}$	0,95283	-0,29156	0,06574
$P_{3\ months}$	0,95744	-0,27209	0,08839
$P_{6\ months}$	0,96824	-0,21455	0,11936
$P_{9\ months}$	0,97622	-0,16103	0,13443
$P_{1\ year}$	0,97873	-0,11598	0,15600
$P_{2\ years}$	0,97354	-0,00053	-0,20481
$P_{3\ years}$	0,96873	-0,01099	-0,23151
$P_{4\ years}$	0,96969	0,0931	-0,20215
$P_{5\ years}$	0,95221	0,02862	-0,28903
$P_{6\ years}$	0,94296	0,27615	-0,03398
$P_{7\ years}$	0,95319	0,23655	0,03689
$P_{8\ years}$	0,85283	0,45149	0,14865
$P_{9\ years}$	0,85159	0,37096	0,20938

The factor one therefore represents the general level of rates: the projection of the point corresponding to the zero-coupon one year being almost located on the axis representing the first factor (95.79% of the variance of the price of zero - one-year coupon is explained by the factor one while the factor two explains only 1.35%), we can assimilate the first factor not only to the general level of rates but also essentially to the one-year rate.

The second factor is an indicator of the slope of the yield curve. Indeed, the coordinates of the price variables on this factor are characterized by negative values (from one month to three years) then positive (from four to nine years) a shock on this factor therefore leads to opposite movements on long maturities and on short maturities. The factor two therefore represents the difference between short and long rates. As this factor particularly opposes the one-month zero-coupon to the eight-year zero-coupon, the second factor will be assimilated to the difference between these last two rates.

As regards the third factor, it opposes intermediate maturities (two to five years) at extreme maturities. The former have negative coordinates on the factor three while the latter have positive coordinates on this factor. Thus, a positive shock on the factor three will increase the maturity rates from two to five years and decrease the others. The third factor therefore represents the concavity of the yield curve.

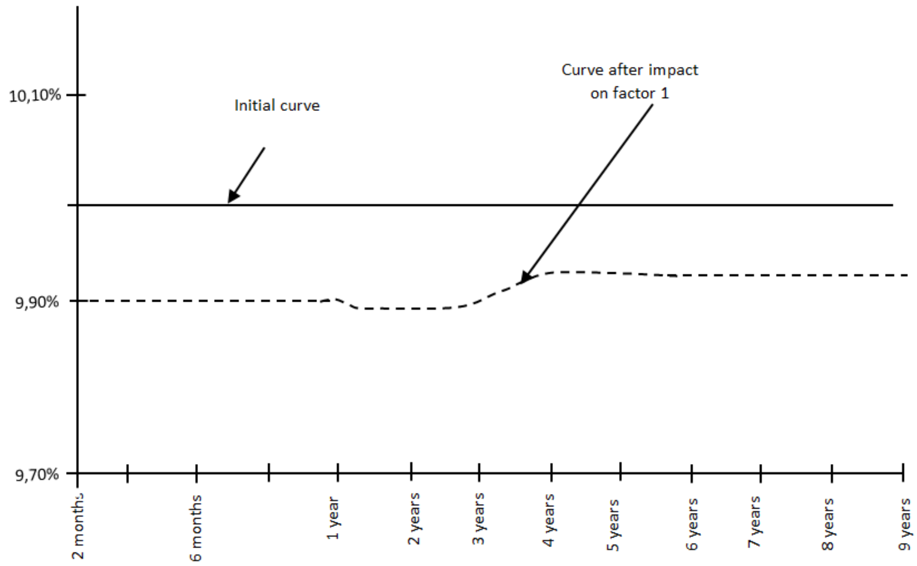

Figure 1. *Effect of a 10% increase in factor 1 on the yield curve.*

The simulations of shocks on each factor and the observation of their effects on the yield curve confirm these conclusions. We simulate, on the zero-coupon price Pi, the effect of a shock on factor j as follows:

$$\Delta\left[\frac{P_i - \bar{P_i}}{\sigma_{P_i}}\right] = \frac{\mu_j}{\mu} \times C_j^i \times \Delta F_j \tag{17}$$

Where C_j^i is the coordinate of P_i with the factor j, ΔF_j is the simulated variation of the factor j, μ_j is the eigenvalue associated with the factor j and μ is the sum of all the eigenvalues or the number of variables, $\frac{\mu_j}{\mu}$ is therefore the percentage of variance explained by the factor j and $\Delta\left[\frac{P_i - \bar{P_i}}{\sigma_{P_i}}\right]$ is the variation of the price of zero-coupon maturity i, centered reduced. By arranging the terms of relation 17, we obtain:

$$\Delta P_i = \sigma_{pi} \times \frac{\mu_j}{\mu} \times C_j^i \times \Delta F_j \tag{18}$$

The simulation of the effect of a variation of 10% of the first factor on a fictitious rate curve flat at 10% is represented in figure. Figure 2 represents the consequences of a shock of the same magnitude on factors two and three.

We again see that a positive shock on the first factor lowers rates as a whole, while a positive shock on the second factor increases rates for maturities less than three years and decreases rates for maturities greater than three years. Finally, a positive shock on the third factor accentuates the concavity of the curve.

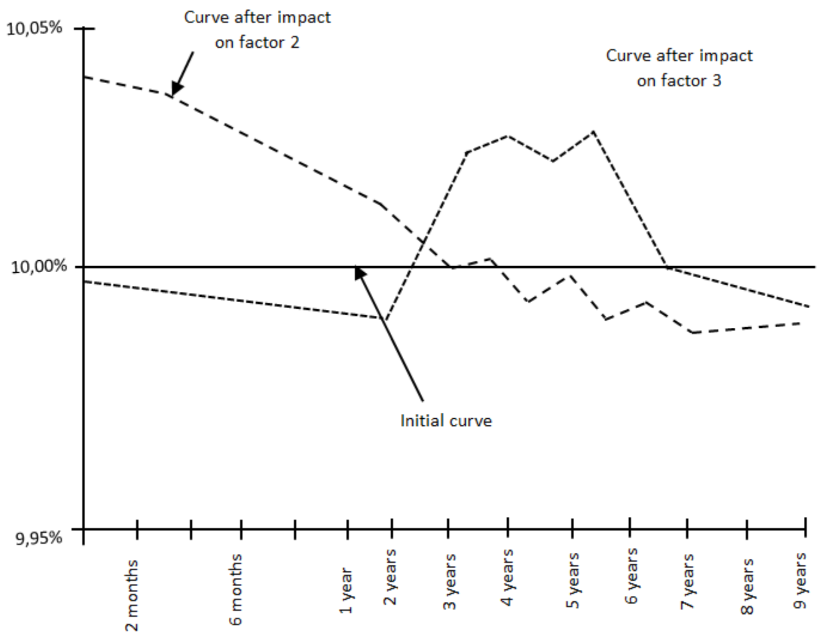

Figure 2. *Respective effects of a 10% increase in factors 2 and 3 on the yield curve.*

The results of the PCA on the six short-term zero-coupon prices.

The same three factors are highlighted. However, on maturities of less than one year, the first factor, ie the general level of rates, is more than enough to explain all of the movements of the curve since it explains 99.03% of the variance.

The results of the PCA on the nine long-term zero-coupon prices (from one to nine years).

Here again the same factors are identifiable but the first two factors, the rate level factor and the indicator factor slope, are necessary and sufficient to explain the deformations, since they both explain 96.62% of the deformations of the curve of maturity rates greater than or equal to one year. The results are detailed in Table 2.

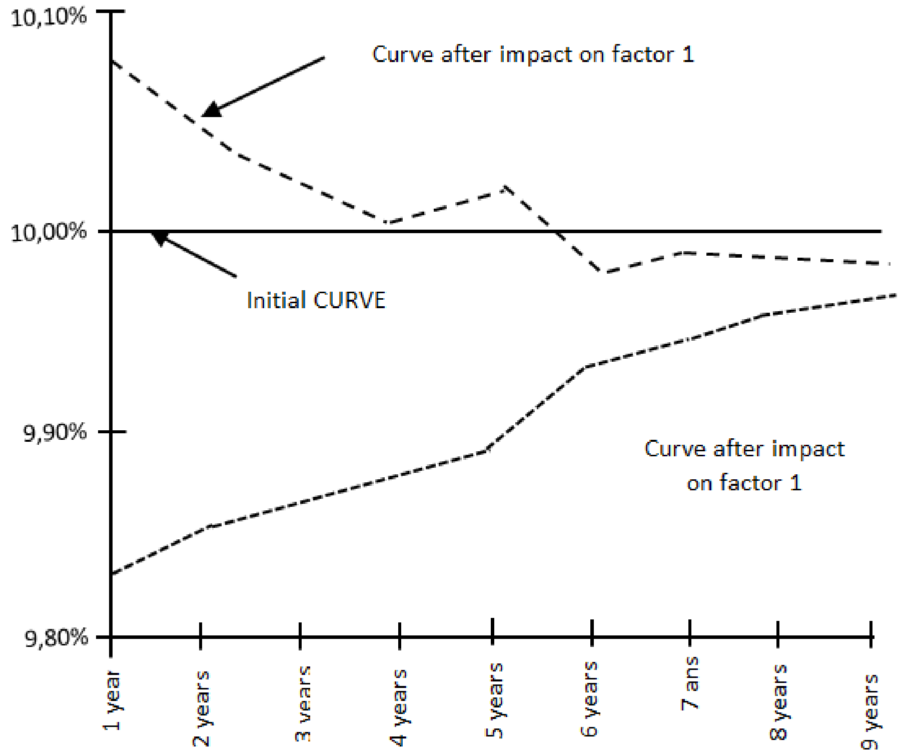

Figure 3. *Effect of a 10% increase in factor I on the long-term yield curve.*

Table 2. *Long-term zero-coupon price coordinates on the first two factors.*

zero-coupons prices	Coordinates on the	
	Factor 1	Factor 2
$P_{1\ year}$	0,89652	-0,39256
$P_{2\ years}$	0,95957	-0,25094
$P_{3\ years}$	0,97831	-0,19112
$P_{4\ years}$	0,98562	-0,07015
$P_{5\ years}$	0,97024	-0,18114
$P_{6\ years}$	0,97083	0,14899
$P_{7\ years}$	0,96291	0,19168
$P_{8\ years}$	0,88619	0,39954
$P_{9\ years}$	0,88816	0,3843

Note: The variance explained by the first factor is 89.31%, the second factor explains 7.31% additional.

With a few differences, the first two factors can be interpreted in the same way as in previous PCAs. Factor one groups the zero-coupon prices from one to nine years: it therefore represents the general level of rates. Moreover, it is easy to notice that the projection of the zero-coupon price due four years is almost on the first factor in the plan defined by factors one and two; its coordinates on the second factor are close to zero. This means that the first factor can be assimilated to the level of the four-year rate. Indeed, 91.16% of the variance of the four-year zero-coupon is explained by the first factor while only 0.39% of its variance is explained by the factor two and 0.6% by the factor three.

The second factor once again opposes short maturities to long maturities: it particularly opposes one year to eight years. The second factor can therefore be interpreted as the difference between these two rates. The effects of a change in each of these two factors on the yield curve are shown in Chart 3.

Table 3. *Explanatory power of the first factor or rate level factor.*

	2018	2019	2020	Total period
Global curve	71,31%	79,04%	62,78%	89,96%
Short-term curve	93,48%	98,11%	97,73%	99,03%
Long-term curve	80,71%	77,33%	64,17%	89,31%

4.1.2. Evolution of Factors over Time

The same analyzes were carried out over three sub-periods, each corresponding to one year of the study period. The results obtained confirm that the rate level factor is very stable and still remains the predominant explanatory factor for movements in the rate curve. Its explanatory power is shown in Table 3.

Table 4. *Explanatory power of the second factor or slope factor of the rate curve.*

	2018	2019	2020	Total period
Global curve	20,26%	8,65%	26,27%	5,84%
Short-term curve	5,78%	1,71%	1,86%	0,87%
Long-term curve	14,73%	13,51%	19,02%	7,31%

The slope factor turns out to be less stable in the time although it is always an explanatory factor to be taken into account, whatever the period considered. However, it should be noted that the direction in which it acts on the slope of the curve may be reversed over certain periods, it will help to increase short rates and decrease long rates (decrease in the slope of the curve), on others, it will act in the opposite direction on the slope of the curve, regardless of the maturity scale considered. The evolution of its explanatory power is detailed in Table 4. This factor therefore mainly helps to explain maturities greater than one year and relatively little short-term maturities.

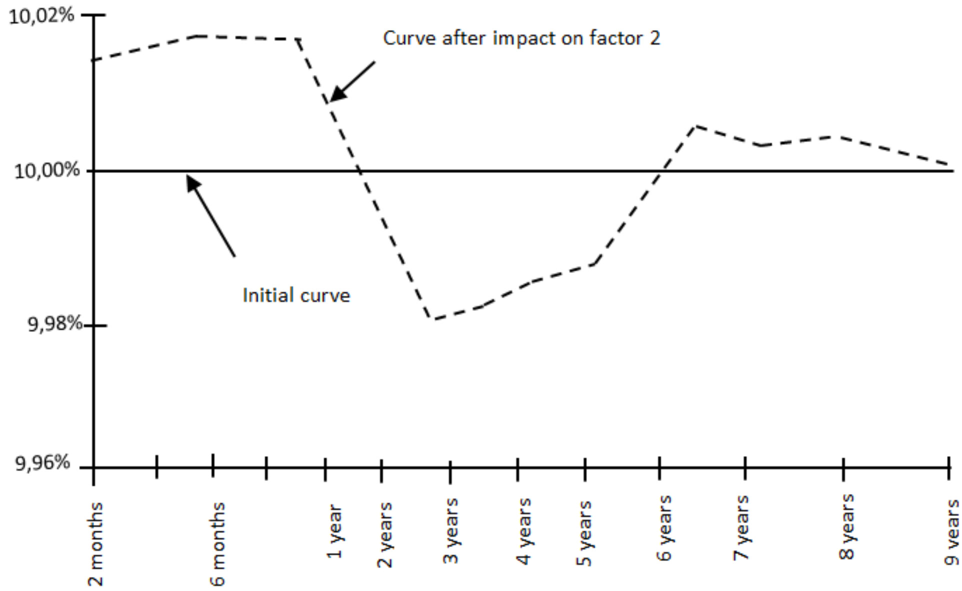

Figure 4. *Effect on the overall yield curve of an increase of 10% of the "convexity" factor which became factor 2 during the year 2019.*

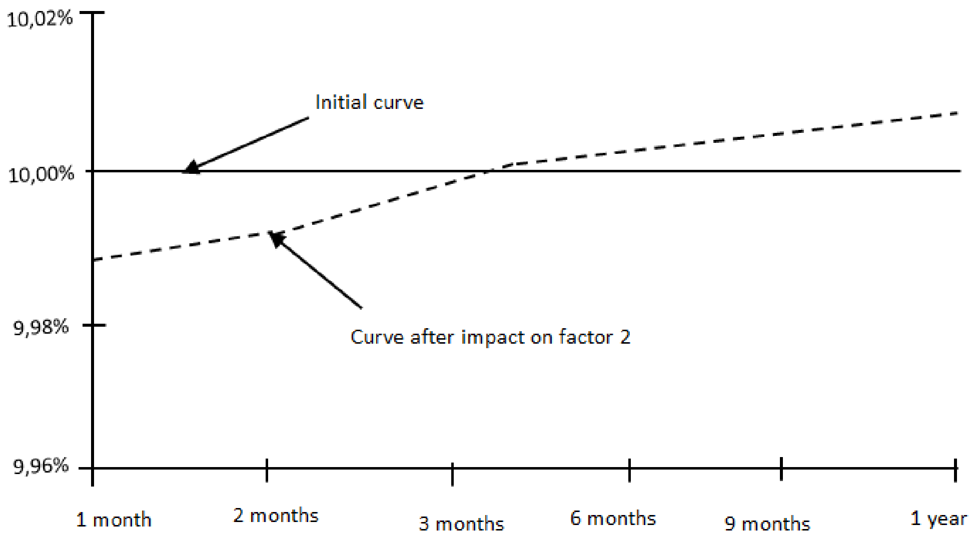

Figure 5. *Effect of a 10% increase of the factor 2 during the year 2019, on short-term rates.*

The PCAs give particular results for the year 2019. In 2019, on the overall yield curve (one month to nine years), the curvature factor became the second explanatory factor, its explanatory power rising to 9.61%. If we observe the PCAs carried out separately on short-term maturities and on long-term maturities, we see that the factor representing the maturity differential (factor two), for each of the two parts of the yield curve, acts in the opposite direction on the slope of this one: it decreases the slope of the curve of the short-term rates and it increases that of the long-term rates (see figure 4 and 5). This is reflected on the overall curve by a greater convexity factor over this period (see figure 6). This is just a symptom of different behaviors of the short term curve and the long term curve. This phenomenon corresponds to a period of soaring key interest rates at the Banque de France.

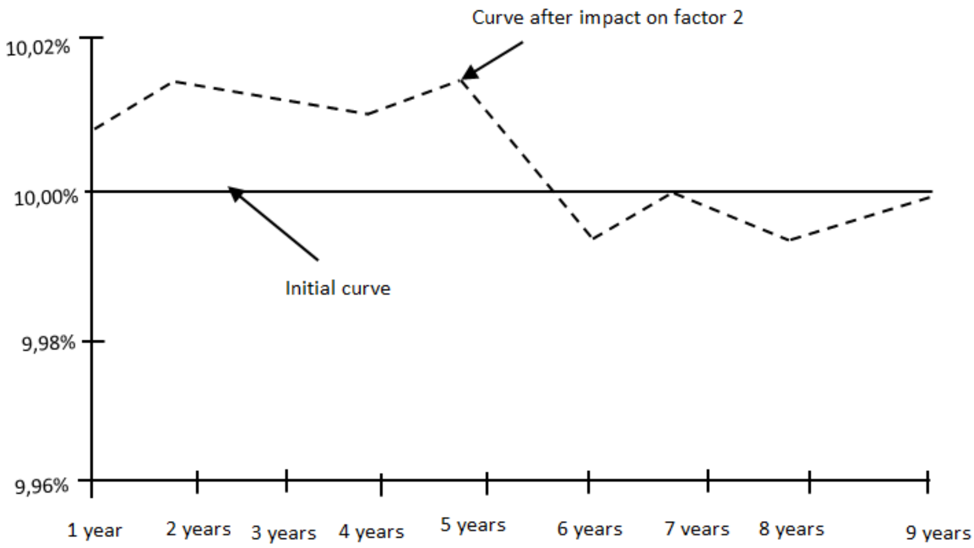

Figure 6. *Effect of a 10% increase in factor 2 during 2019, on long-term rates.*

4.2. The Reliability of the Results and the Possible Improvements

The results of the regressions were discussed in section 2. As for the principal component analyzes, on the whole, whatever the PCA considered, the variables are very well represented in the subspace of the results factors selected, since the percentage of variance explained by the factors considered, for each price variable, all PCA combined, oscillates between 82% and 98.5%. Overall, the results obtained can therefore be used reliably. Their quality can only be questioned for the fact that they are obtained from data (zero-coupon prices) whose estimation can be improved.

4.3. Interpretation of Results and Extensions

4.3.1. Interpretation

Following the identification of the factors, it seems necessary to push the economic interpretation further. The difference between the results obtained on the short-term part of the curve and on its long-term part corroborates a hypothesis according to which the markets are compartmentalized. Thus, agent's investment strategies are most certainly different depending on the investment horizon: short term (less than one year) or long term (greater than one year). Moreover, in most financial institutions, the management of short-term interest rate instruments is separate from the management of long-term interest rate instruments. Perhaps these findings are a confirmation of the theory of habitat. Preferred On short-term maturities (less than one year), one factor is sufficient to explain the movements of the yield curve, and that is the level of the six-month rate.

For long-term maturities (from one to nine years), two factors seem necessary to explain the term structure of rates: the general level of rates represented by the value of the four-year rate and the difference between the one-year rate and the eight-year rate. The first factor represents the current level of rates and reflects the monetary policy in place, the second represents investors' expectations on the level of future rates and therefore reflects their idea of the evolution of monetary policy.

4.3.2. Possible Extensions of the Study

Extending the study to other environments would make it possible to observe the stability of the factors over a greater number of sub-periods. The use of other data analysis techniques could possibly be considered to improve the interpretation of the results. In addition, two theoretical extensions also seem possible to us.

a) The development of a multifactor model of the forward rate structure: this would involve modeling each of the factors identified and then deducing valuation methods for the assets linked to the rates. Another possible use of the results obtained is to model the evolution of each rate or each zero coupon price using the parameters estimated by the PCA and variables following reduced centered normal distributions.
b) Developing portfolio hedging strategies: this would involve using the zero-coupon price coordinates on each factor to calculate the asset sensitivities linked to the rates to each of these factors. Once the sensitivities of the assets have been calculated, hedging consists in constructing a combination of these assets such that the sensitivity (s) of the portfolio constituted to one or more factors is zero. However, the development of such hedging strategies requires a readjustment of the positions according to the evolution of the factors and the evolution of their respective explanatory powers.

5. Conclusion

The principal components analysis thus carried out on curves of zero - coupons from one month to nine years, weekly, from January 2018 to December 2020 highlighted three factors making it possible to explain the distortions of the yield curve up to 98.36%, If the third may seem optional since it explains only 2.56% of the movements of the curve, it can nevertheless take on a non-negligible importance at certain periods. These factors which determine the forward rate structure are, a priori, unobservable and uncorrelated factors.

However, from the coordinates of each zero-coupon on these “factors” and from shock simulations dc 10% on each of these factors, interpretations have been proposed: the first factor represents the general level of rates and is comparable to the one-year rate; the second factor represents the slope of the yield curve and is similar to the difference between the one-month rate and the eight- or nine-year rate; the third factor is an indicator of curvature.

The explanatory power of the first factor is always predominant regardless of the period observed. On the part of the short-term yield curve, the first factor explains almost all of the changes in interest rates. On the long-term part (one to nine years) two factors are necessary: the level of the four-year rate and the difference between the one-year rate and the eight-year rate differences in behavior between short rates (less than one year) and long rates (more than one year), which seems to go in the direction of the theory of the preferred habitat.

The interpretation of these empirical results would lead to modeling the curve by three factors: the level of the 6-month rate for short maturities; the level of the 4-year rate and the spread between the 1-year rate and the 8-year or 9-year rate for long maturities. The real epistemological concern that we are confronted with today is that of knowing what will be the impact of the covid-19 currently in force on the future functioning of the economies? The percentage of variance explained is 99.03% by the first factor, 0.87% by the second and 0.08% by the third.

Appendix

Appendix 1: Short-term Maturities Coordinated on the First Three Factors

Table 5. *Short-term maturities coordinated on the first three factors.*

	Factor 1	Factor 2	Factor 3
P_1 month	0,99177	-0,12125	0,03904
P_2 month	0,99577	-0,08963	-0,00099
P_3 month	0,99817	-0,04545	-0,03644
P_6 month	0,99883	0,03159	-0,03104
P_9 month	0,99593	0,08920	0,00200
P_1 year	0,99027	0,13580	0,2792

Note: The percentage of variance explained is 99.03% by the first factor, 0.87% by the third.

Appendix 2: PCA on Zero-coupons from One Month to Nine Years: Projection in Terms of Factors 1 and 2

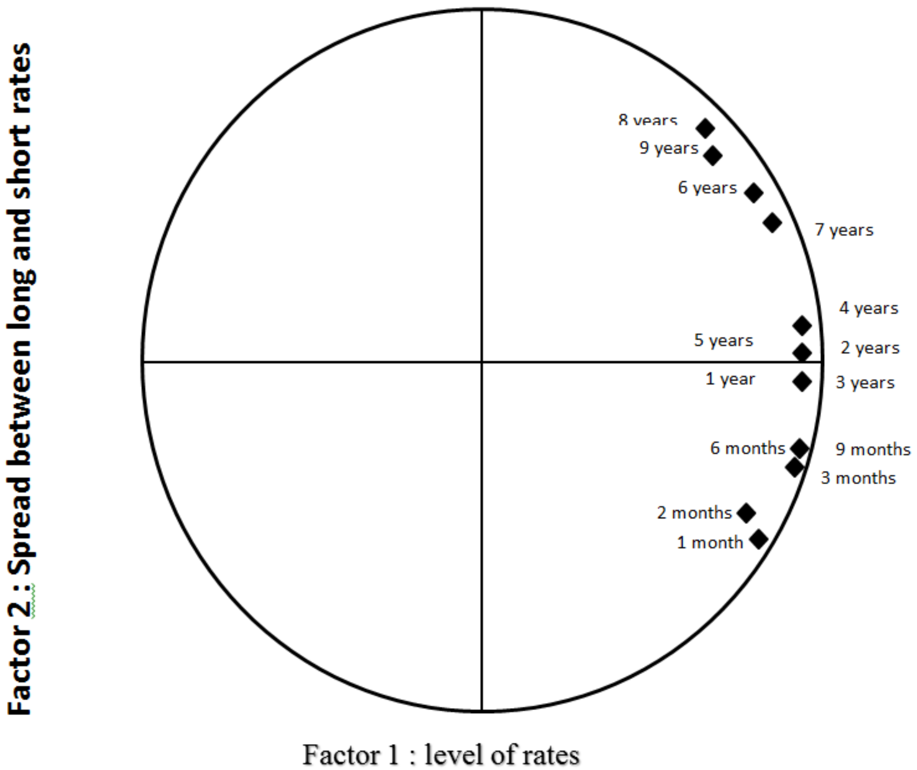

Figure 7. *PCA on zero-coupons from one month to nine years: projection in terms of factors 1 and 2*

Appendix 3: PCA on Zero-coupons from 1 Month to 9 Years: Projection in Terms of Factors 1 and 3.

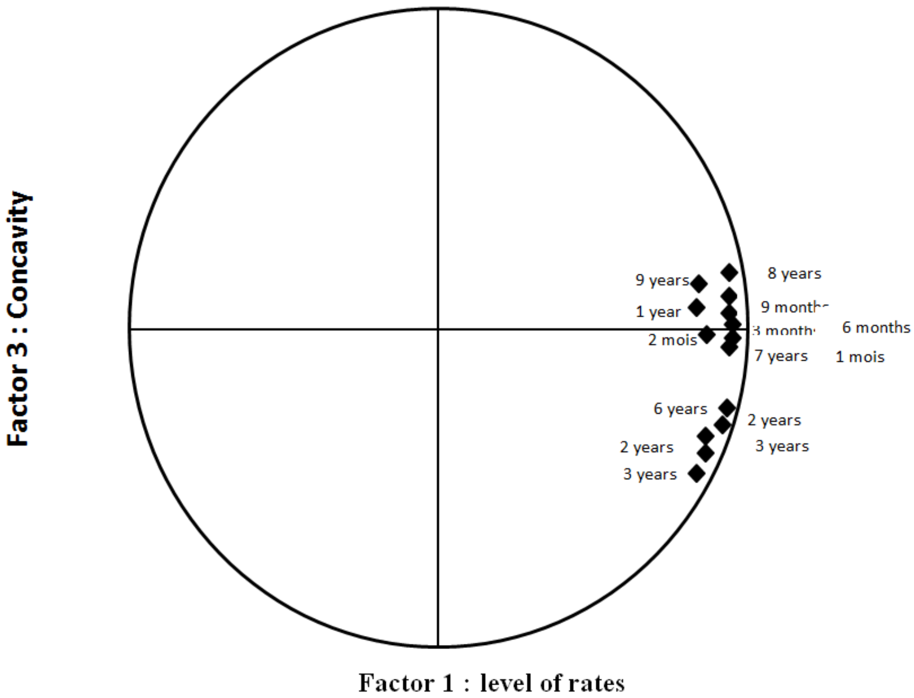

Figure 8. *PCA on zero-coupons from 1 month to 9 years: projection in terms of factors 1 and 3.*

Appendix 4: PCA on Zero Coupons from 1 to 9 Years: Projection in Terms of Factors 1 and 2.

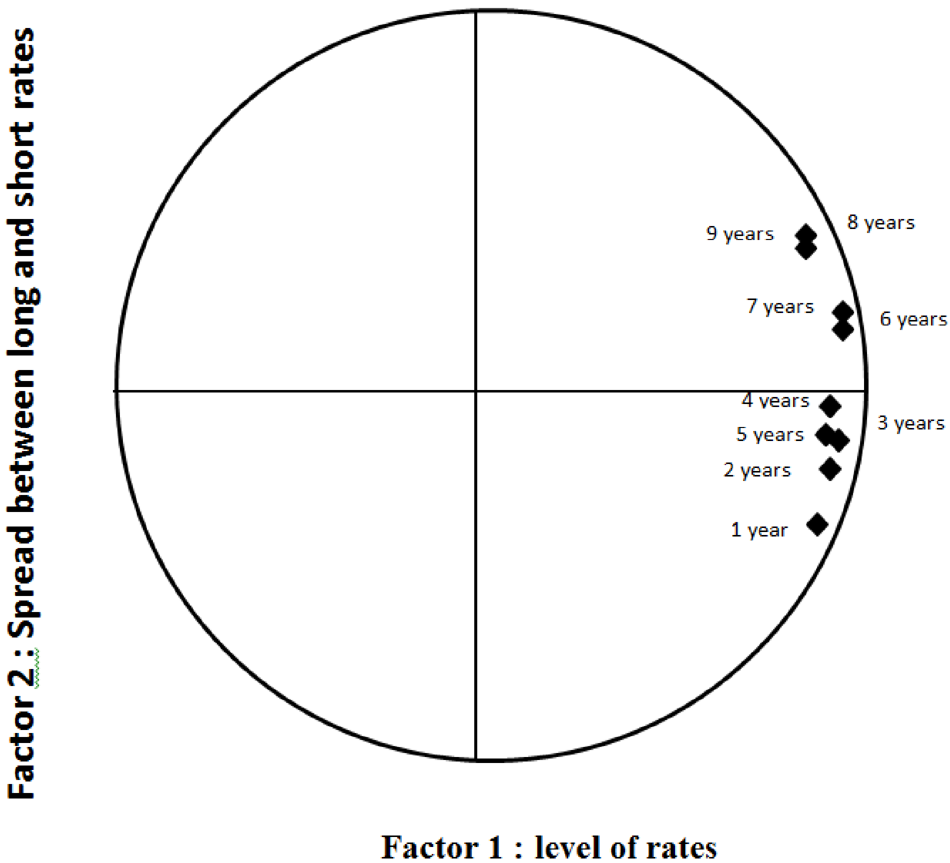

Figure 9. *PCA on zero coupons from 1 to 9 years: projection in terms of factors 1 and 2.*

References

[1] ALPHONSE P., DESMULIERS G., GRANDIN P. et LEVASSEUR M., *Gestion de portefeuille et marchés financiers*, Pearson, 2017.

[2] AMF, Opérations sur titres: l'importance de l'information sur les modalités de réponses possibles, 02 février 2021.

[3] ARTUS P., BELHOMME C., ELALOUF C. et MINCZELES A., 1992, "Les déformations de la structure des taux et la couverture du risque de taux d'intérêt", Caisse des Dépôts et Consignations, Document de travail n°1992-01/F, mai.

[4] BIERWAG G., FOOLADI I., «Duration analysis: An historical perspective», Journal of Applied Finance février 2006, vol. 16, n°2, pp. 144-160.

[5] BRENNAN J. et SCHWARTZ E. S., 1982, "An Equilibrium Model of Bond Pricing and a Test of Market Efficiency", *Journal of Financial and Quantitative Analysis*, 17, September, pp. 301-329.

[6] COBBAUT R., GILLET R. et HUBENER G., La gestion de portefeuille, De Boeck, Coll. «Comptabilité, contrôle et gestion», 2015.

[7] EL KAROUI N. et LACOSTE V., 1992, "Multifactor Models of the Term Structure of Interest Rates", *Actes du Congrès de l'AFFI*, Paris, Juillet.

[8] Gresse C., Marché de taux d'intérêt, Economica, 2017.

[9] HEATH D., JARROW R. et MORTON A., 1990, "Bond Pricing and the Term Structure of Interest rates a Discrete Time Approximation", *Journal of Financial and Quantitative Analysis*, 25, n°4, December, pp. 419-440.

[10] HO T. et LEE S., 1986, "Term Structure Movements and Pricing Interest Rate Contingent Claims", *Journal of Finance*, December, 41, pp. 1011-1029.

[11] JACQUILLAT B., SOLNIK B. et PERIGNON C., *Marchés financiers*, Dunod, 2014.

[12] JACQUILLAT B. ET LAGUICHE S., 1990, "Risque des obligations à taux variable: une étude empirique du marché français", *Conférence Les problèmes non-linéaires appliqués: modèles mathématiques en finance*, CEA-EDF-INRIA, mars.

[13] LITTERMAN R. et SCHEINKMAN J., 1988, "Common Factors Affecting Bond Returns", *Financial Strategies Group*, Goidman Sachs, September.

[14] SIMON Y., Notation et agences de rating, Economica, 2017.

[15] VASICEK O., 1977, "An Equilibrium Characterization of the Term Structure", *Journal of Financial Economics*, 5, pp. 177-188.

[16] VERNIMMEN P., Finance d'entreprise, Dalloz, 2021.