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The Utilization of the Radon Transform for the Extraction of the Orientation of Linear Features in Binary Images

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Abstract: This paper considers the idea of using the Radon transform to extract the orientation features of lines in binary images. The Radon transform sends a line at a particular orientation in image space to a point in feature space. The ensuing set of points in feature space is called a sinogram. This computation is usually performed for a large group of angles (over the interval [0,179] degrees taken in integer increments in this paper). Therefore, linear features at specific orientations will be mapped to points having maximum value at particular angles. For angular spacing of at least 5 degrees, the peaks of the sinogram at the angles corresponding to the orientations of the lines will be clearly visible (in a bar plot of sinogram peaks) above sinogram values at other angles. The mapping from image space to feature space accomplished by the Radon transform which maps rectangular coordinates (x,y) to coordinates (range, angle) provides for the garnering of the orientation of the linear features in binary images. In particular, the coordinates (range, angle) in the sinogram allow for distinguishing between lines oriented at one angle versus lines oriented at another angle or angles. This particular property of the sinogram allows for the extraction of the orientation features of lines in an image.

Keywords: Angles, Lines, Orientation, Radon Transform, Sinogram

1. Introduction

Before discussing the Radon transform, let us first take up the topic of inverse problems. The reason for doing this is that the method of solution for many inverse problems involves the Radon transform. Inverse problems start with observations associated with a particular object and then attempt to determine the value of a parameter (a cause) associated with that object. Therefore, the concepts inherent to inverse problems give rise to an underlying theory for remote sensing and non-destructive testing. Inverse problems in the areas of computerized geophysical tomography (CGT), computed axial tomography (CAT), barcode scanners, electron microscopy, and seismology involve the Radon transform.

In two-dimensional space, the Radon transform is the integral transformation which takes a function g defined on the Cartesian plane to a function Rg defined on the set of lines (in two dimensions) in the plane. The value of the transform on a specific line is given by the line integral of the function along that particular line. The Radon transform represents a mapping from coordinates x and y on a rectangular grid to coordinates ρ and θ on another rectangular grid. In two dimensions, the Radon transform can be written using a delta function in two dimensions as

$$R(\rho,\theta) = \int_{R^2} g(x,y)\delta(\rho - x\cos\theta - y\sin\theta)dxdy$$

in which

$$\rho = x\cos\theta + y\sin\theta$$

represents the line over which g is integrated to produce the line integral which is R(ρ,θ). The equation above defining the Radon transform R(ρ,θ) also defines the projection, $P_\theta(\rho)$.

Now, the projection along the line $L_{\rho,\theta}$ parameterized by ρ and θ is given by

$$
\begin{aligned}
P_\theta(\rho) &= \int_{L_{\rho,\theta}} g(x,y)ds \\
&= \int_R g(\rho\cos\theta - s\sin\theta, \rho\sin\theta + s\cos\theta)ds \\
&= \int_{R^2} f(x,y)\delta(\rho - x\cos\theta - y\sin\theta)dxdy = R(\rho,\theta)
\end{aligned}
$$

Therefore, Radon transform of a function is also a projection.

Given a two-dimensional image intensity function, g(x,y), computation of its Radon transform produces projections over the image at different angles, θ, and distances (from the first row of the image) ρ.

The Radon transform sends lines in image space to points in feature space [1]. This property of the Radon transform will be exploited later to extract the orientation features of lines in an image. Whether they are bright or dim, image lines are mapped by the Radon transform to points in feature space. At every orientation θ and every distance ρ, the intensity of a target through which a half-line orthogonal to the ρ axis travels is summed up at $P_\theta(\rho)$. Figure 1 shows the Cartesian plane with a line over which the intensity of an object is added to yield the Radon transform at a particular value of ρ and θ. A collection of several projections computed in this manner at various angles comprises a sinogram.

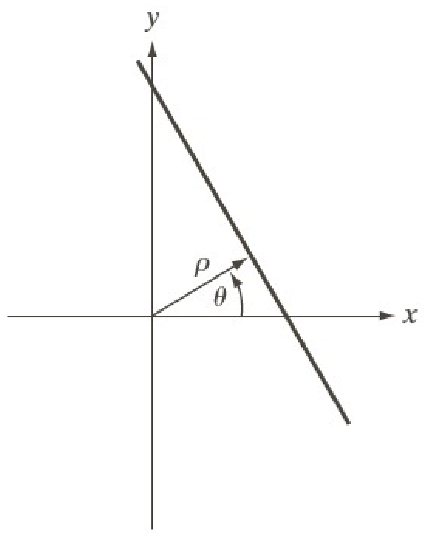

Figure 1. *Typical Geometry of an Intensity Summing Line in Image*

2. A Brief Review of the Relevant Literature

The paper by Jafari-Khouzani and Soltanian-Zadeh describes a technique for approaching the problem of rotation invariant texture classification [2]. For directional textures, the wavelet properties need to be computed at a specific orientation. In their work, the primary texture direction is obtained by computing the Radon transform. The texture is then rotated so that its primary direction is at zero degrees. However, the authors don't consider the case of texture with multiple orientations in a single image.

The paper by Aggarwal and Karl observes that the extraction of the position and angle information associated with lines in images is very imporant in areas such as computer vision and image processing [3]. The Hough transform (a special case of the Radon transform) has been used to resolve this issue for binary images. These researchers view the problem of finding lines in images as an inverse problem. They utilize the inverse Radon operator which establishes a correspondence between variables associated with the line position and angle and the image which has been degraded by noise. This puts the problem in a regularization setting and gives rise to a better capability for Hough-based line detection via the utilization of prior regularization data.

In another paper, the authors show that a convolutional network can pick up particular properties to estimate the angle information of an image [4]. While this technique can be applied to a video stream, it doesn't appear that this approach is used to find the orientations of multiple images simultaneously.

In a different paper, the researchers propose a technique for determining the camera orientation in a vehicle by obtaining three orthogonal vanishing points [5]. The vanishing point along the driving direction is first estimated. Then this vanishing point is obtained by using the Hough Transform. The remaining vanishing points are chosen from the circular histogram, and they are orthogonal to the vanishing point along the driving direction. Finally, the angle information is estimated using the three orthogonal vanishing points.

In another paper addressing this problem, the author proposes a technique for determining the orientation angle of text using the Radon transform [6]. However, this author considers only the case of an image with all of the lines of text at one particular angle and not multiple angles.

In another paper exploring orientation extraction, the authors discuss a method to obtain the orientation of text in a document image as well as placing the image at the proper orientation [7]. This is based on pattern recognition where a particular pattern has been discovered by extensive studies of different scripts.

In another paper utilizing the Radon transform, the authors discuss a technique for detecting automobiles using a template-based directional chamfer matching approach and automobile direction determination using a filtered segmentation and then a new method for finding maximum values after computing a

Radon transform [8].

The paper by Rajput, Som, and Kar utilizes the Radon Transform to ascertain the angle information of license plates [9]. In that work, each image contains a license plate positioned at a particular orientation. However, the authors don't consider the idea of more than one license plate at varying angles in an image. The researchers give a short description of other techniques which attempt to consider multiple orientations and their limitations.

The authors of another work employ a discrete wavelet transform (DWT) to detect the edges in an image. They follow this with the Radon transform of the edge-detected scene to determine the orientation information of the resulting texture [10]. They don't however consider the orientation of multiple textures.

The authors of another work propose a method for determining the orientation of an image using information contained in the image and expressed by local binary patterns (LBP) [11]. The orientation determination occurs through the use of logistic regression.

Other authors of research in the area of orientation extraction use the Radon transform to ascertain the angle information of fingerprints in connection with a system for recognizing fingerprints [12]. They observe that the angle of the peak and the trough associated with a fingerprint is of great importance in the ability to identify fingerprints. The Radon transform is used to determine the angle information. They compute the Radon transform at seven distinct angles to obtain the projections for a particular fingerprint image but do not take up the idea of an image with more than one fingerprint. They do not attempt to address the problem of orientation determination of multiple fingerprints in a single image.

In another article, the authors present an algorithm involving a two-level classification method and two groups of properties that allow for obtaining the inherent textual information [13].

3. Extraction of Orientation Features

The orientation feature extraction algorithm calculates the Radon transform or sinogram of a binary image containing lines at different orientations. The maximum sinogram value is then computed at each angle and is displayed as a bar plot showing the maximum value at each angle. For angles that have at least five degrees of separation from neighboring angles, the peaks of the sinogram at the angles corresponding to the orientation of the lines will be clearly visible (in a bar plot of sinogram peaks) above sinogram values (the clutter) at other angles. The image clutter occurs as a result of the many lines at different orientations lying next to neighboring lines as they emanate from a point. Observation of the bar plot of the maximum values of the sinogram reveals the level of image clutter.

When many angles obscure the line orientation in a bar plot, the two-dimensional sinogram plot will show the orientations of the lines. The mapping from image space to feature space accomplished by the Radon transform which maps rectangular coordinates (x,y) to coordinates (ρ,θ) provides for the garnering of the angles of lines in binary images. In particular, the coordinates (ρ,θ) in the sinogram allow for distinguishing between lines oriented at one angle versus lines oriented at another angle or angles. This discrimination property of the sinogram allows for the extraction of the orientation features of lines in an image.

The novelty of this technique is due to the fact that this approach utilizes the Radon transform to obtain the orientation of a line in an image. Other authors have employed the Radon transform to obtain the angle information of texture, fingerprints, text, and license plates in an image but not lines. Moreover, this technique uses the Radon transform to detect multiple lines at different angles in a single image.

4. Results

This technique was verified using an image which contained lines at 14 different angles and with an image which contained lines at 35 different angles and was reported in [14], and [15]. For the case of the image with 14 lines at 14 different angles, a bar plot of the maximum sinogram values clearly showed 14 peaks corresponding to the 14 different lines. For the case of the 35 lines at 35 different angles, it was only possible to discern 31 peaks corresponding to 31 of the 35 lines from a bar plot of the maximum sinogram values. However, a sinogram plot of the lines showed 35 peaks corresponding to the 35 lines.

In addition, this algorithm was verified using an image which contained 10 lines at an angle of 25 degrees and 5 lines at an angles of 60 degrees, on an image with 8 lines at 8 distinct angles, and finally using an image with 7 lines at 25 degrees, 3 lines at 40 degrees and 5 lines at 60 degrees.

Consider the following input image in figure 2 of 8 lines at angles of 50, 55, 60, 65, 70, 75, 80, and 85 degrees. Figure 3 is the corresponding sinogram.

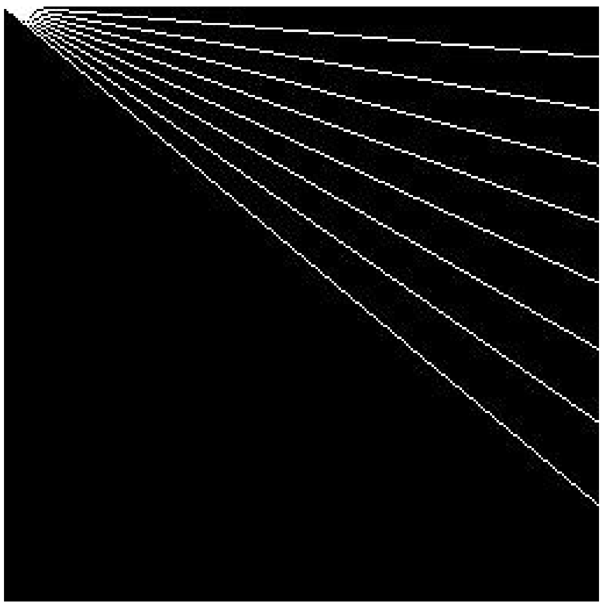

Figure 2. *Input Image Containing Lines at Eight Distinct Angles*

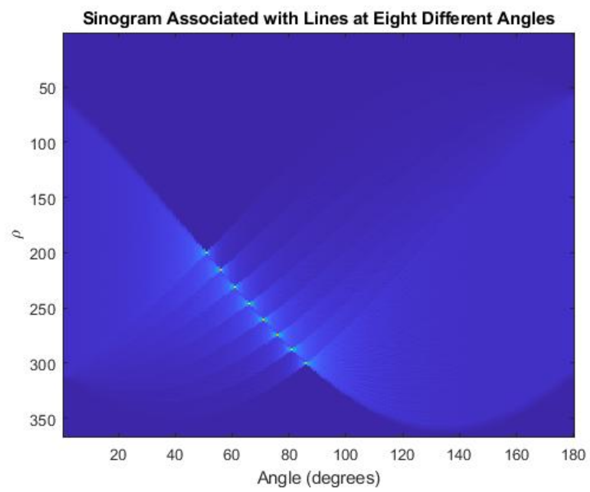

Figure 3. *Sinogram Corresponding to Image Containing Lines at Eight Distinct Angles*

Figure 4 exhibits an image containing 10 lines at an angle of 25 degrees and 5 lines at an angle of 60 degrees, where the angles are measured with respect to the vertical edge of the image.

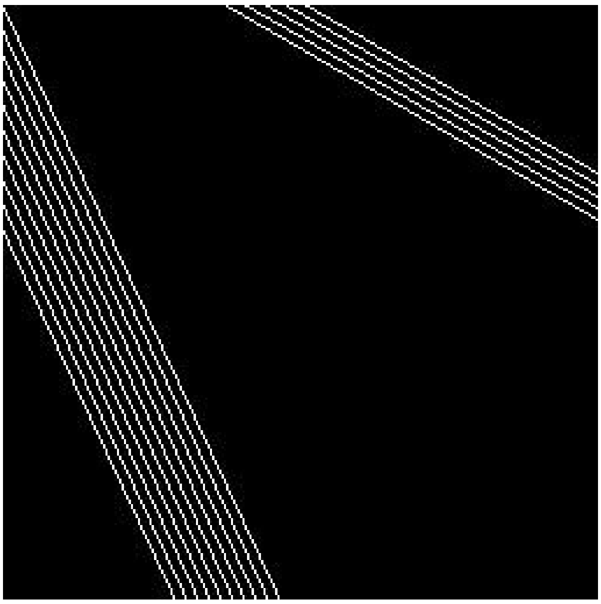

Figure 4. *Image with 10 Lines at 25 degrees and 5 lines at 60 degrees, with angles measured with respect to the vertical*

Recall, from the Introduction, that lines in image space are sent by the Radon transform to points in feature space. Therefore, if two or more lines in an image are parallel and thus oriented at the same angle, they should map to different distances but the same angle in feature space which is denoted by coordinates (ρ,θ). It follows that when the Radon transform processed the image in figure 4, it should produce peaks at the two angles, equal to the number of lines at each angle. The resulting sinogram shown in figure 5 shows the peaks at the angles 25 degrees and 60 degrees, exactly what the theory predicts. In figure 5, the vertical axis denoted by ρ in the figure gives the distance in pixels from the top row of the image (considered as an array).

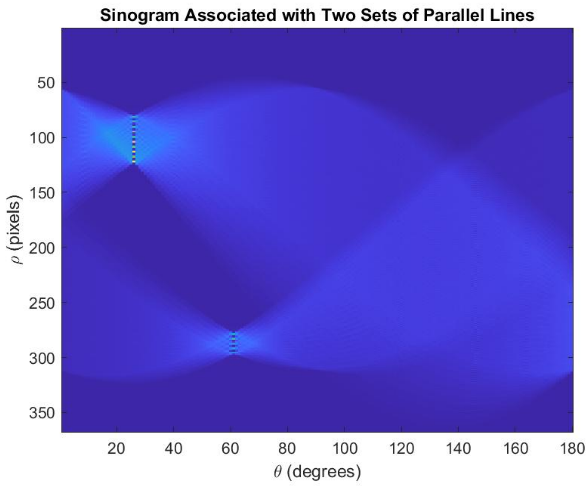

Figure 5. *Sinogram Showing Peaks at Two Different Angles*

Figure 6 shows an image with 7 lines at an angle of 25 degrees, 3 lines at an angle of 40 degrees, and 5 lines at an angle of 60 degrees, where the angles are measured with respect to the vertical edge of the image.

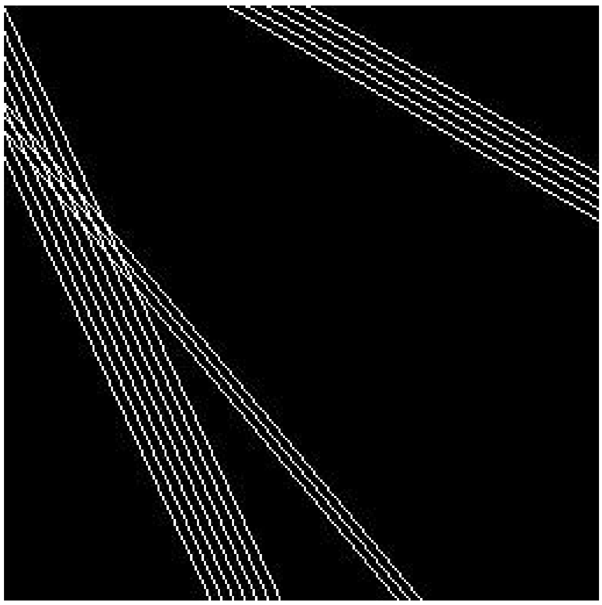

Figure 6. *Image with 7 Lines at 25 degrees, 3 lines at 40 degrees, and 5 lines at 60 degrees, with angles measured with respect to the vertical*

Based upon the discussion above with the two sets of parallel lines at two different angles, it follows that when the Radon transform processed the image in figure 6, it should produce peaks at the three angles, equal to the number of lines at each angle. The resulting sinogram shown in figure 7 shows the peaks at the angles 25 degrees, 40 degrees, and 60 degrees, exactly what the theory predicts. In figure 7, similar to figure 5 above, the vertical axis denoted by ρ gives the distance in pixels from the top of the image (considered as an array).

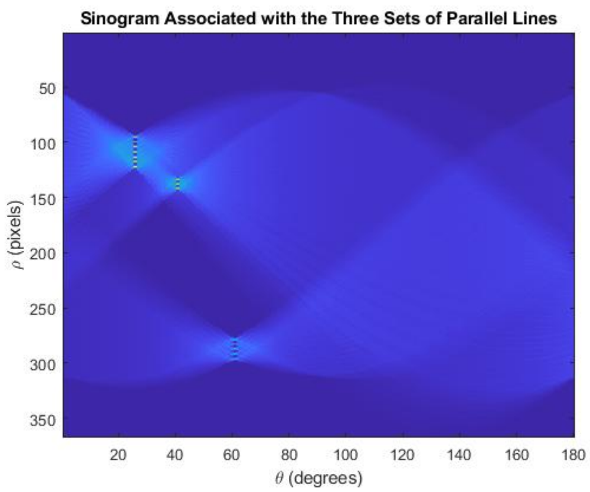

Figure 7. *Sinogram Showing Peaks at Three Different Angles*

5. Conclusions

This work considered the problem of the extraction of orientation features of a line. The utilization of the Radon transform allowed for the recovery of the angles at which lines were oriented in an input image. The peaks denoting Radon space points corresponding to the lines in image space could be clearly seen at the angles which were the same as the angles of the lines in the input image. The only exception to the sinogram not displaying those peaks is when a line does not extend across a sufficient part of the image. The use of different test images, which explored different aspects of the orientation algorithm, illustrated properties of the Radon transform like the mapping of a line to a point and that parallel lines will map to points at the same angle in Radon space but be displaced by the amount of separation between the lines. This was demonstrated for the cases of two sets of parallel lines and three sets of parallel lines. The sinogram which is the Radon transform of an input image clearly displayed peaks in Radon space which were points to which lines in image space at different angles were mapped by the Radon transform (a property of the Radon transform cited previously). The mapping from image space to feature space accomplished by the Radon transform which maps rectangular coordinates (x,y) to coordinates (ρ,θ) provides for the garnering of the angle information of lines in binary images. This was shown in the image with eight lines at eight different angles as well as the images with two and three sets of parallel lines. The Radon transform detects lines whose length is a significant part of one of the image dimensions. Thus, as long as a line extends along most of one of the image dimensions, the orientation algorithm will detect the angle at which that line is oriented.

The practical applications of these algorithms are the following: the extraction of road networks from high resolution images obtained from airborne or spaceborne platforms, detection of text of arbitrary orientation, and in the ability to determine crack orientation.

Another application of orientation detection is that of detecting text of an arbitrary orientation in an image obtained from a mobile computing device.

A final application of the orientation algorithm is that of line detection and tracking for an advanced driver assistance system.

References

[1] Lesley M. Murphy. “Linear feature detection and enhancement in noisy images via the Radon transform.” In: Pattern Recognition Letters Issue 10, September 1986, pp. 279-284.

[2] Kourosh Jafari-Khouzani and Hamid Soltanian-Zadeh “Radon Transform Orientation Estimation for Rotation Invariant Texture Analysis”. In: IEEE Transactions on Pattern Analysis and Machine Intelligence (June. 2005),27(6) pp. 1004-1008. URL : https://doi.org/10.1109/TPAMI.2005.126

[3] N. Aggarwal and W.C. Karl. “Line Detection in images through regularized hough transform” In: IEEE Transactions on Image Processing (Volume: 15, Issue: 3, March 2006),pp. 582-591. URL : https://doi.org/10.1109/TIP.2005.863021

[4] Philipp Fischer, Alexey Dosovitskiy, and Thomas Brox “Image Orientation Estimation with Convolutional Networks” In: Gall J., Gehler P., Leibe B. (eds) Pattern Recognition. DAGM 2015. Lecture Notes in Computer Science, vol 9358. Springer, Cham URL: https://doi.org/10.1007/978-3-319-24947-6

[5] Youngran Jo, Jinbeum Jang, Minwoo Shin, and Joonki Paik “Camera orientation estimation using voting approach on the Gaussian sphere for in-vehicle camera” In: Optics Express, Vol. 27, No. 19, 16 Sep 2019.

[6] AbdulSattar M. Khidhir “Use of Radon Transform in Orientation Estimation of Printed Text” In: The 5th International Conference on Information Technology (ICIT) 2011.

[7] Lalita Kumari, Swapan Debbarma, and Radhey Shyam “Text Orientation Detection from Document Image of Indian Scripts” In: International Journal of Computer Communication and Information System (IJCCIS) Vol 2. No. 1. July-Dec 2010.

[8] Rengarajan Pelapur, Filiz Bunyak, Kannappan Palaniappan, and Gunasekaran Seetharaman. “Vehicle Detection and Orientation Estimation Using the Radon Transform.” In: Proc. of SPIE vol. 8747, 2013 URL: https:/doi.org/10.1117/12.2016407

[9] Hitesh Rajput, Tanmoy Som, and Soumitra Kar. “Using Radon Transform to Recognize Skewed Images of Vehicular License Plates.” In: Computer Vol 49, Issue 1, January 2016, pp. 59-65 URL: https:/doi.org/10.1109/MC.2016.14

[10] Satyabrata Sahu, Santosh Kumar Nanda, and Tanushree Mohapatra. “Digital Image Texture Classification and Detection Using Radon Transform.” In: International Journal of Image, Graphics and Signal Processing (IJIGSP), vol. 5, No. 12, October 2013.

[11] Gianluigi Ciocca, Claudio Cusano, and Raimondo Schettini. “Image orientation detection using LBP-based features and logistic regression.” In: Multimedia Tools and Applications 74, pp 3013-3034, 2015.

[12] Diwakar Tiwary, K. Srinivasulu, and Anveshraj. “Fingerprint Identification Using Radon Transform.” In: International Refereed Journal of Engineering and Science (IRJES) Vol 3, Issue 10, October 2014, pp. 106-110.

[13] Cong Yao, Xin Zhang, Xiang Bai, Wenyu Liu, Yi Ma, and Zhuowen Tu. “Rotation-Invariant Features for Multi-Oriented Text Detection in Natural Images” In: PLoS ONE 8(8): e70173 URL: https://doi.org/10.1371/journal-pone-0070173

[14] Sean Matz “Orientation Detection of Lines in Binary Images Using the Radon Transform” In: 6th Annual Conference on Computational Science and Computational Intelligence (CSCI) December 05-07, 2019.

[15] Sean Matz “Orientation and Line Thickness Determination in Binary Images” In: The 2020 World Congress in Computer Science, Computer Engineering, and Applied Computing (CSCE) July 27-30, 2020.