
New Iterative Method Based Jacobian Matrix and λ_{opt} to Solve Power Flow Equation for Islanded MGs

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Abstract: Power flow analysis and calculation play an essential role in analysis of electrical power system. Solving the load flow equations of an electrical power system are very sensitive to the value of the right-hand side constant vector or to the value of the coefficients of the Jacobian matrix, the equations of the electrical power system are called to be ill-conditioned. In an ill-conditioned case study, the determinant of the Jacobian matrix is close to singular or singular. Computing the set of load flow equations of an electrical power system may sometimes lead to the incidence of the ill-conditioned equations. The ill-conditioning mood in MGs power system is due to several reasons such as the situation of the reference bus, high value of r/x ratio of power system's lines, connections of very low and very high impedance power system's lines at a bus and heavy loading condition of an electrical power system. In an islanded MG, the use of traditional PFA is not effective as the voltage of the swing or slack bus and the frequency of the MG are assumed to be constant. This paper proposes a simple and effective iterative method based on the Jacobian matrix λ_{opt} to solve the PF equation for islanded MGs. The new proposed technique prepares a simple, straightforward for implementation, and precise technique to calculate the PF equations for MGs. The new suggested technique is exerted to the 38-bus case study electrical power system. The outcomes are compared against simulation outcomes that accredit the efficiency of the new technique.

Keywords: Power Flow Analysis, Microgrids, Iterative Method

1. Introduction

In an electrical power system, Microgrid (MG) is a small part of an electrical network consisting of energy storage systems distributed generators (DGs), and diffused loads, which can act in both islanded mode and connection mode. DG systems are appropriate for providing highly reliable and safe electric power [1]. Several types of energy resources, such as photovoltaic panels, solar thermal panels, fuel cells, and micro-turbines are currently using. In the connection status of MGs, the frequency and voltage are determinate by the main grid while in an islanded mode, control units of DGs along with administering reactive and active powers are responsible for voltage and frequency adjustment [2]. A control strategy for MGs might be designed in different forms such as distributed, decentralized, and centralized.

In power system, power flow (PF) studies have been a momentous topic for research since the early 1950s [3]. PF analysis tries to computing the amount of PF through transmission lines and calculation in obtaining the steady-state condition of voltage and powers at all buses of the network. These studies are of the utmost importance and frequently provide the starting conditions for other PFA such as fault analysis, transient stability, and contingency analysis, also PFA is used always in the planning of power system networks and PFA is essential to optimize the operation of existing power systems [4]. In connect style of MG, the frequency and voltage in case studies are held by the chief network, but in an island style they are not fixed. The PFA in an island mode of MG cannot be calculated by traditional methods like Newton Raphson techniques (NRM) also the topology of MG power systems weakly mesh or radial that a great amount of (R/X)

ratio of lines so which may lead to ill-conditioned problems in the NRM PFA techniques [5-7].

Backward/forward sweep techniques, is the most generic method to solve PFA in distribution systems. The first version of Backward/forward sweep techniques was presented in 1967, in this version only PQ buses were considered in a radial network [8] also Since 1967, some advances are presented to calculate feebly structure case study [9, 10], systems with a distributed generation [11], distribution systems with voltage-dependent loads [12], and, distribution networks in three-phase mode [13, 14]. A method was advanced in the study of Augugliaro [15] to calculate radial and meshed cases study with only a backward sweep technique.

Comparison and review of backward/forward sweep techniques have existed in [16]. Some techniques named fixed-point kind approaches using Gauss-Seidel and Gauss-like algorithms for computing distribution case study in three-phase mode are available in [17, 18]. A method that is based on the triangular factorization and optimal ordering scheme of matrix Y-bus, also this method is insensitive to the topology of the network has been provide in [17]. A technique developed based on the loop frame of reference, rather than the conventional node frame of reference has been proposed in [18]. Injection current and basic graph theory technique are also applied in the proposed algorithm.

Faisal Mumtaz and et. all have been presented a new method-based NRM that proposes a method that is effective modifications to the traditional techniques to solve the PFA for MGs [19]. Three algorithms were branched from the NRM have presented in [20]. These techniques were modified for distribution systems with voltage-dependent loads based on the nodal current injection [21]. PFA for an islanded MG by consideration of constant frequency in an islanded MG has presented in [3]. The traditional technique that used DGs instead of slack, PV, and PQ buses has presented in [22]. A several number of PFA techniques are produced to model MG whit islanded mode based on droop controllers to overcome and improve the convergence of the traditional algorithms [23]. The application and formulation of PFA for DC and AC MG in the low voltage grid. In the low voltage AC MGs, with a partly high R/X ratio of lines, virtual impedance (Z_V) is usually adopted to modify the efficiency of droop control characteristics applied to DG [7]. In both DC and AC MG, by considering the virtual impedance (Z_V), obtain more accurate computation results for the PFA.

In This literature we have presented a new iterative technique to compute PF equations for islanded MGs using droop control DG. This method is iterative and the order of convergence is biquadratic. When used method with biquadratic convergence, the computational, CPU time, and the number of iterations to reach convergence are decreased. To accredit the efficiency of the new suggested PF technique, the outcomes are compared with results achieved from a benchmark method.

This work is organized as following: modeling of equipment and system presented in section 2. PF formulation

in an islanded MG is introduced in section 3. Section 4 introduced mathematical iterative method based λ_{opt} . In section 5 new PF algorithm for islanded MG is proposed. The outcomes of simulations from the new suggested technique are exerted given in section 6 to exhibit the efficiency of the new suggested technique. Finally, the observation-based conclusion is carried out in section 7.

2. Modeling of Equipment and System

In the MG, mathematical modeling of each component is an essential and considerable element to solve a PFA of MGs. This section presented three maim models of MGs following subsections:

DG model

Y-bus model

Load model

For the islanded mode of MG, the modeling of DG is limited to the steady-state behavior that is, voltage and current harmonics are neglected also all currents and voltages are represented by a phasor [24, 25]. The significance of the DGs in an islanded mode of MG has a power inverter interface consist of the filter. Figure 1 shown the droop bus (DB). DB bus is presented in an island mode of MG operation, which does not be in typical PFA. At DR-bus i , the bus voltage magnitude V_i and frequency of the MG f are controlled with reactive Q_{Gi} and real power P_{Gi} generations that Q_{Gi} and P_{Gi} are given by equations (1) and (2), respectively [4]. In these equations, m_p and n_q are the constant reactive and real power droop gains or are the frequency and voltage droop coefficients, respectively. The V_0 and f_0 are the reference amount of the controlled parameters in MG. In the DBi, the unknown variables are V_i and δ_i .

$$P_{Gi} = \frac{2\pi}{m_p} (f_{0i} - f) \quad (1)$$

$$Q_{Gi} = \frac{2\pi}{n_q} (V_{0i} - f) \quad (2)$$

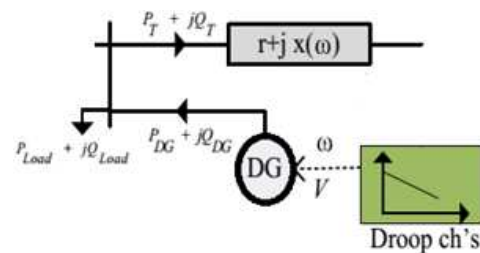


Figure 1. Modeling of droop (DR) bus in PFA.

In the study of MGs, for a static load model, the powers relationship to voltage and frequency is an exponential equation, demand of reactive and active powers is assumed to be constant therefore load models are assumed to be voltage and frequency independent. equations (3) and (4) shows the relationship load model:

$$P_L = P_0(V)^\alpha (1 + 2\pi * K_{fp} \Delta f) \quad (3)$$

$$Q_L = Q_0(V)^\beta (1 + 2\pi * K_{fq} \Delta f) \quad (4)$$

In equations (3) and (4), P_L and Q_L are active and reactive power at operating voltage, P_0 and Q_0 are the active and reactive power at nominal voltages magnitude, V is the voltage magnitude at each bus, in equations (3) and (4) used frequency sensitivity parameters K_{fp} and K_{fq} for modeling of load. K_{fp} ranges from 0 to 3 and K_{fq} ranges from -2 to 0 depending on geographical regions and load type [19]. and α and β are parameters of the mode. The voltage- dependent load models for residential, industrial, and commercial loads. But in the operative condition, loads are not explicitly industrial, residential, and commercial. Table 1 shows the amount of α and β for several loads [26].

Table 1. Exponent values of different load.

Load type	Parameter, α	Parameter, β
Constant	0	0
Industrial	0.18	6.00
Residential	0.92	4.04

For modeling of Y_{bus} in islanded MG with droop-based control of DGs, the frequency of the power system cannot be applied as a constant variable. In this condition, the frequency of the power system changed the amount of the reactance ($jX(\omega)$) of the line. The line impedance in this model can be calculated by, $Z = R + jX(\omega)$.

3. PF Formulation in Islanded MG

For calculating the PFA in the power case study one of the main steps is to recognize the kinds of buses of the power system. Basically, PFA techniques consider four main parameters in the network, voltage magnitude (V), voltage angle (δ), active power (P) and reactive power (Q) for each bus exist in electrical power system. For LFA, equations are nonlinear and can be divided above equations into real and imaginary parts:

$$P_i = V_i \sum_{k=i}^n V_k (G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik}) \quad (5)$$

$$Q_i = V_i \sum_{k=i}^n V_k (G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik}) \quad (6)$$

Where G_{ik} is the conductance, B_{ik} is the susceptance, V_i and V_k are the voltage magnitudes of each bus. While θ_{ik} is the angle of Y_{ik} . Where P_i and Q_i are the calculated real and reactive power supplied to busk, respectively. Newton PFA uses the NRM to solve PFA problems. Conventionally, a direct solver is used to solve for the linear system of equations that arises in each iteration of the NRM. In order to use the NRM, the PF equations have to be written in the form $F(\delta, V) = 0$. PFA procedure leads to a function $F(\delta, V)$ called the power mismatch function. The power mismatch function contains the active and reactive power schedule and injected active power P_i and reactive power Q_i at each bus. Another element needed for the Newton-like technique, is J .

Tabloid, the voltage angle and magnitude for all buses for the $(k + 1)$ iteration can be calculated by Equation (7), (8).

$$[\delta^T \quad |V|^T]^T = \Lambda \quad (7)$$

$$\Lambda^{k+1} = \Lambda^k + J^{-1} * F(\delta, V) \quad (8)$$

The good division buses for the traditional PFA are the load bus (PQ), the voltage control bus (PV), and the swing or reference bus, when solving the PFA using the traditional algorithms. In an islanded MG, there is no single DG exists or is capable to act similarly to the slack bus. Such techniques specifically consider the frequency constant that these techniques apply to large-scale electrical power systems. In islanded MGs, the frequency of the network cannot be considered constant therefore, these techniques are not applicable to the islanded MGs. Table 2 shows all known and unknown variables in DB, PV, and PQ buses.

Table 2. Known and unknown variable in different buses.

Load type	Bus DB	Bus PV	Bus PQ
Known	---	V, P	P, Q
Unknown	V, δ, P, Q	δ, Q	V, δ

Newton Raphson methods can't use for islanded MG because:

In an island mode there is no reference bus.

DR bus in the MG required to be formulated and defined.

The frequency of the power system in an island mode is not fixed.

In order, the active P_{Gi} and reactive Q_{Gi} powers of the DR bus "i" buses for the $(k + 1)$ iteration can be calculated by equations (9) and (10).

$$P_{Gi}^{k+1} = \frac{2\pi}{m_p} (f_{0i} - f_i^k) \quad (9)$$

$$Q_{Gi}^{k+1} = \frac{2\pi}{n_q} (V_{0i} - V_i^k) \quad (10)$$

For the S quantity of DR buses in the grid, the summation of reactive power and active power can be calculated by equations (11) and (12):

$$P_S = \sum_{i=1}^S \frac{2\pi}{m_p} (f_{0i} - f_i) \quad (11)$$

$$Q_S = \sum_{i=1}^S \frac{2\pi}{n_q} (V_{0i} - V_i) \quad (12)$$

4. Mathematical Iterative Method Based λ_{opt}

The Levenberg-Marquardt technique uses a search direction that is a solution of the linear set of Equation (13):

$$(J(\varphi_k)^T J(\varphi_k) + \lambda_k I) d_k = -J(\varphi_k)^T F(x_k) \quad (13)$$

or, optionally, of the Equation (14):

$$\left((J(\varphi_k)^T J(\varphi_k) + \lambda_k \text{diag}(J(\varphi_k)^T J(\varphi_k))) \right) d_k = -J(\varphi_k)^T F(\varphi_k) \quad (14)$$

In equations (13) and (14) λ_k is damping factor that controls both the value and direction of d_k . Set option scale problem to 'none' to select equation (13), and set scale problem to 'Jacobian' to select equation (14). When amount of damping factor is zero $\lambda_k = 0$, the direction d_k is identical to that of

the Gauss-Newton method. As amount of damping factor joins to infinity ($\lambda_k \rightarrow \infty$), magnitude of d_k close to zero. This mentions that for some enough great λ_k , the term $F(\varphi_k + d_k) < F(\varphi_k)$ maintains true. The term λ_k can consequently be controlled to certify descent even when second-order terms, which restrict the effectiveness of the Gauss-Newton technique, are confronted. The Levenberg-Marquardt techniques were essentially used for nonlinear parameter approximation problems however have also been applied beneficially for the solution of ill-conditioned linear problems [27, 28].

5. New PF Technique for Islanded MG

Step 1: Input data (line and bus data)

Step 2: Build Y_{bus}

Step 3: Calculated P_L and Q_L with equations (3) and (4)

Step 4: According to equations (5) and (6), calculated P_i and Q_i .

Step 5: According to equations (11) and (12), calculated P_S and Q_S .

Step 6: Calculated P_{LOSS} and Q_{LOSS} with equations (15) and (16)

$$P_{LOSS} = \frac{1}{2} \sum_{k=1}^N \sum_{n=1}^N \Re \{Y_{kn}(V_k^* V_n + V_n^* V_k)\} \quad (15)$$

$$Q_{LOSS} = -\frac{1}{2} \sum_{k=1}^N \sum_{n=1}^N \Im \{Y_{kn}(V_k^* V_n + V_n^* V_k)\} \quad (16)$$

Step 7: Calculated P_t and Q_t with equations (17) and (18)

$$P_t = P_L + P_{LOSS} \quad (17)$$

$$Q_t = Q_L + Q_{LOSS} \quad (18)$$

Step 8: Calculated power residual by equation (19):

$$\Delta = \begin{bmatrix} P_{sch} - P_i \\ Q_{sch} - Q_i \\ P_t - P_S \\ Q_t - Q_S \end{bmatrix} \quad (19)$$

Step 9: Calculated Jacobian matrix (J) by equation (20):

$$J = \begin{bmatrix} J_1 & J_2 & J_{13} & J_{14} \\ J_3 & J_4 & J_{23} & J_{24} \\ J_{31} & J_{32} & J_{33} & J_{34} \\ J_{41} & J_{42} & J_{43} & J_{44} \end{bmatrix} \quad (20)$$

Step 10: In this step, unknown parameters ($|V|$, δ and f) of next iteration number are computed by equations (21) or (22):

$$\Lambda^{k+1} = \Lambda^k + (J^T J + \lambda_k \text{diag}(J^T J))^{-1} J^T \Delta \quad (21)$$

$$\Lambda^{k+1} = \Lambda^k + (J^T J + \lambda_k I)^{-1} J^T \Delta \quad (22)$$

6. Case Study and Simulation Result

The new suggested technique has been examined on 38-bus MG [22]. In order, the new suggested PFA technique is examined on the 38 bus power case study that is illustrated in

Figure 2. The case study consists of five DGs located on buses 34- 38. The DGs do in the Q-V and P- ω droop modes.

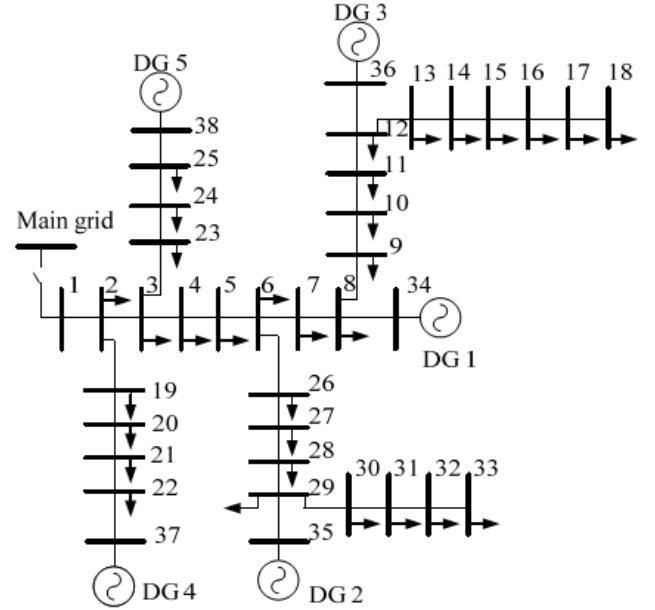


Figure 2. The topology of 38-bus AC system.

Table 3 illustrates the quantity of iterations and CPU time of the offered techniques and also the traditional method on the 38 bus power system. The decrease of the amount of iterations improves the CPU time, particularly in large-scale cases study.

To evaluate the effectiveness of the parameter λ on the computation time, Table 3. compares the number of iterations and the computation times by changing the converge parameter λ in the IEEE 38-bus system.

According to Table 3, by a suitable setting of the converge parameter λ , one can decrease the number of iterations and improves the CPU time. As shown in this Table, the optimal setting of λ is $\lambda_{opt} \approx 9.7 \times 10^{-0}$. We named this setting as “best convergence parameter” (λ_{opt}). Note that by a very bad set of converging parameters, the PF problem may diverge but by the semi-good setting of λ , a solution will be achieved. In all simulations, the amount of λ calculated by equation (23):

$$\lambda = \mu \|\Delta\|^\xi \quad (23)$$

That: $\mu \in [0.1 \ 100]$ $\xi \in [1 \ 2]$

Table 3. Simulations result in 38 bus case study $\max(\text{error}) = 10^{-12}$.

parameter ξ	parameter μ	CPU Time (s)	Number of Iteration
1.0	0.65	0.153112	2
1.5	0.65	0.145990	2
2.0	0.65	0.162894	2
1.0	0.85	0.182772	2
1.5	0.85	0.169226	2
2.0	0.85	0.129507	2
1.0	1	0.131612	2
1.5	1	0.161897	2
2.0	1	0.137982	2

Figure 3 and Table 4. shown rate of convergence in four simulations by:

Test 1: $\mu = 0.1$
 Test 1: $\mu = 1$
 Test 1: $\mu = 10$
 Test 1: $\mu = 100$

Table 4. Rate of convergence in four test.

Iterations	Test 1	Test 2	Test 3	Test 4
0	10	10	10	10
1	1.86E-11	2.48E-10	2.27E-08	1.77E-06
2	3.03E-13	4.93E-13	3.84E-13	4.36E-13

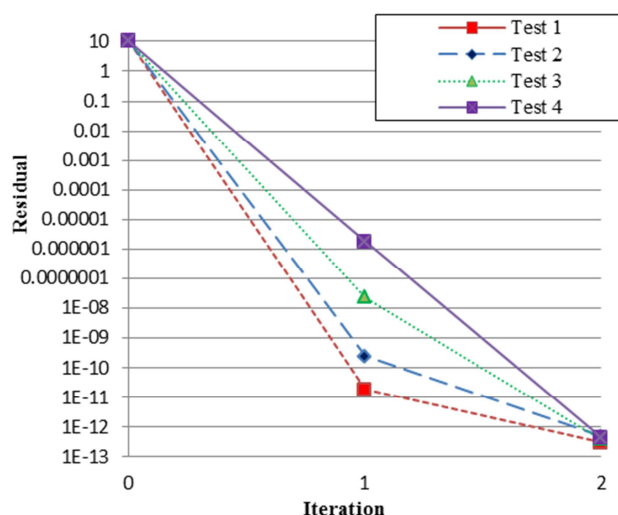


Figure 3. Rate of convergence in 4 test.

7. Conclusions

Newton Raphson methods can't use for islanded MG because:

In an island mode there is no reference bus.

DR bus in the MG required to be formulated and defined.

The frequency of the power system in an island mode is not fixed.

We have proposed a new iterative technique to calculate a set of PF equations in an island mode of MGs with droop control DG. This method is iterative and the order of convergence is biquadratic. When used method with biquadratic convergence, the computational, CPU time, and the number of iterations to reach convergence are decreased. To accredit the efficiency of the new suggested PF technique, the outcomes are compared with results achieved from a benchmark method.

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