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Some Upper Bounds of Maximum *E*-Eigenvalues of Uniform Hypergraphs

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Abstract: A hypergraphs, as a generalization of a general graph, is often used as an effective tool to describe complex structures in discrete mathematics, computer science and other fields. Hypergraph theory and related parameters of hypergraph are important research topics in hypergraph theory. In particular, the problem of spectral extremum of graphs has been widely concerned. This problem originates from the problem proposed by Brualdi and Solheid in 1986. That is to find the upper and lower bounds of spectral radius of a given graph class and characterize the polar graph that reaches the upper and lower bounds. Let *H* be a uniform hypergraph. Let *A*(*H*) be the adjacency tensor of *H*. In this work, by using Perron-Frobenius theorem, Hölder's inequality and inequality of arithmetic and geometric means, we establish some upper bounds for the maximum *E*-eigenvalue of a uniform hypergraph instead of the degrees of vertices and edge number of hypergraph H. In addition, we characterize the extremal hypergraphs that reach the upper bounds.

Keywords: Uniform Hypergraphs, Adjacency Tensor, Maximal *E*-Eigenvalue, Degree

1. Introduction

Denote the set $\{1,2,\cdots,n\}$ by $[n]$. Let $H=(V(H),E(H))$ be a hypergraph with n vertexes and m edges, where $V(H)=\{v_1,v_2,\cdots,v_n\}$ and $E(H)=\{e_1,e_2,\cdots,e_m\}$ represent the vertex set and edge set of the hypergraph H, respectively. If every edge $e_i=\{v_{i_1},v_{i_2},\cdots,v_{i_l}\}$ in a hypergraph H satisfies $|e_i|=l=r$, then a hypergraph H is called r-uniform hypergraph, where $i_j\in[n], j=1,2,\cdots,l$ and $i=1,2,\cdots,m$. Let d_i be the degree of the vertex v_i. If $d_i=d$ is satisfied for any $i\in[n]$, then the hypergraph H is called a d-regular hypergraph. Let Δ and δ be the minimum degree and the maximum degree of hypergraph H, respectively. Tensor's definition is as follows.

Definition 1 [10]. Given any positive integers r,n, a tensor $\mathcal{T}$ of order r and dimension n is defined by a multidimensional array of element $t_{i_1i_2\cdots i_r}$, where $i_1,i_2,\cdots,i_r\in[n]$, $t_{i_1i_2\cdots i_r}\in C$. Obviously, a tensor $\mathcal{T}$ of order r and dimension n has n^r elements.

The definition of tensor product is given, which is a generalization of matrix product [8].

Definition 2 [10, 4]. Let $\mathcal{A}\in C^{n_1\times n_2\times\cdots\times n_m}$, $\mathcal{B}\in C^{n_2\times n_3\times\cdots\times n_{r+1}}$ be two tensors of orders m and r, respectively. Then the product $\mathcal{C}=\mathcal{A}\mathcal{B}$ of tensors $\mathcal{A}$ and $\mathcal{B}$ is a tensor of order $(m-1)(r-1)+1$ with the $(i,\alpha_1,\alpha_2,\text{L},\alpha_{m-1})$-th entry is

$$\mathcal{C}_{i\alpha_1\alpha_2\cdots\alpha_{m-1}}=\sum_{i_2,i_3,\cdots,i_m\in[n_2]}\mathcal{A}_{ii_2i_3\cdots i_m}\mathcal{B}_{i_2\alpha_1}\cdots\mathcal{B}_{i_m\alpha_{m-1}},$$

where $i\in[n_1],\alpha_1,\alpha_2,\cdots,\alpha_{m-1}\in[n_3]\times[n_4]\times\cdots\times[n_{r+1}]$.

Definition 3 [10, 14]. Let $\mathcal{A}\in C^{n_1\times n_2\times\cdots\times n_m}$,

$\mathcal{B} \in C^{n_2 \times n_3 \times \cdots \times n_{r+1}}$

be two tensors of orders m and r, respectively. $\mathcal{T}x^{r-1}$ is a n -dimension complex vector with the i -th entry:

$$\left(\mathcal{T}x^{r-1}\right)_i = \sum_{i_2,i_3,\cdots,i_r=1}^{r} t_{ii_2\cdots i_r} x_{i_2} x_{i_3} \cdots x_{i_r}, i \in [n].$$

$\mathcal{T}x^r$ is a complex number:

$$\mathcal{T}x^r = \sum_{i_1,i_2,\cdots,i_r=1}^{r} t_{i_1i_2\cdots i_r} x_{i_1} x_{i_2} \cdots x_{i_r}, i \in [n].$$

In 2005, Qi and Lim proposed the definition of tensor eigenvalues, respectively [2, 3]. If there exists $\lambda \in C$ and a non-zero vector $x \in C^n$ such that

$$\mathcal{T}x^{r-1} = \lambda x^{[r-1]},$$

then λ is the eigenvalue of tensor $\mathcal{T}$, and x is the corresponding eigenvector related with λ of $\mathcal{T}$, where

$$x^{[r-1]} = \left(x_1^{r-1}, x_2^{r-1}, \cdots, x_n^{r-1}\right)^T.$$

In 2012, Cooper and Dutle defined the adjacency tensor $\mathcal{A}(H)$ of a r-uniform hypergraph H [1].

Definition 4 [1]. Let $H = (V(H), E(H))$ be a uniform hypergraph with n vertexes. The adjacency tensor $\mathcal{A}(H)$ of hypergraph H is a tensor of order r and dimension n, whose $(i_1, i_2, \cdots, i_r)$ -th entry is

$$a_{i_1i_2\cdots i_r} = \begin{cases} \dfrac{1}{(r-1)!}, & if \{i_1, i_2, \cdots, i_r\} \in E(H) \\ 0, & otherwise \end{cases}$$

Qi et al. gave the definition of E -eigenvalue of a uniform hypergraph [11].

Definition 5 [11]. Given a tensor $\mathcal{A}$ of order r and dimension n. Let $\lambda \in C$ and $x \in C^n$, if

$$\lambda x = \mathcal{A}x^{r-1}, xx^T = 1,$$

then x is the E -eigenvector of tensor $\mathcal{A}$ related with its eigenvalue λ.

Lemma 1 [6, 7, 13]. (Perron-Frobenius theorem) Let $\mathcal{A}$ be a non-negative weakly irreducible tensor of order r and dimension n, where $r, n \geq 2$. Let λ is the maximum eigenvalue of $\mathcal{A}$. Then λ is the only eigenvalue that satisfies the corresponding eigenvector is positive.

In 2009, Bulo and Pelillo published the first paper on the spectrum of a hypergraph, and used the concept of H-eigenvalue of a tensor introduced by Qi in 2005 [12]. By the upper bound of clique numbers of graphs derived from Sós and Straus, Motzkin-Strau theorem is extended to specific hypergraphs [15]. In particular, Bulo and Pelillo extended and improved the spectral boundary introduced by Wilf, and established a link between the cluster number and the spectral hypergraph theory [12]. In 2012, Cooper and Dutle systematically studied hypergraph theory through adjacency tensor, and extended the basic results of graph theory to hypergraphs [1]. Kang and Liu et al. gave some upper bounds on the compactness of the eigenvalues of hypergraphs as follows [5].

$$\rho(H) \leq \max_{1\leq i\leq n} \left(\frac{1}{r-1} \sum_{\{v_{i_1}, v_{i_2} \mathrm{L}, v_{i_r}\} \in E(H)} \left(d_{i_2}^{\frac{1}{r-1}} + d_{i_3}^{\frac{1}{r-1}} + \mathrm{L} + d_{i_r}^{\frac{1}{r-1}} \right) \right)^{\frac{r-1}{r}}.$$

Equality holds if and only if the hypergraph H is a regular hypergraph.

Besides, Yi gave a upper bound of H -eigenvalue of a hypergraph [9].

$$\rho(H) \leq \max_{\{v_{i_1}, v_{i_2}, \cdots, v_{i_r}\} \in E(H)} \sqrt[2(r-1)]{d_{i_1}^{r-1} d_{i_2} d_{i_3} \cdots d_{i_r}}.$$

In this work, several upper bounds of the maximal E -eigenvalues of uniform hypergraphs are established in terms of their degrees by using Perron-Frobenius theorem and Holder inequality. In addition, the extremal hypergraphs are characterized.

2. Main Results

Let $E(H(v_1))$ be all edges containing vertex v_1. Write

$$\sum_{\{v_1, v_{i_2} \mathrm{L}, v_{i_r}\} e \in E(H(v_1))} x_{i_2} x_{i_3} \mathrm{L}\, x_{i_r} = \sum_{e \in E(H(v_1))} x^e$$

for short.

Theorem 1. Suppose H is a r uniform hypergraph with n vertices. Then

$$\lambda \leq \Delta r^{-\frac{r(r-2)}{4(r-1)}}.$$

Equality holds if and only if the hypergraph H has only one hyperedge, which contain all vertices.

Proof According to Lemma 1,

$$x = (x_1, x_2, \mathrm{L}, x_n)^T, x_i > 0, i \in [n].$$

Let the entry x_1 corresponding to vertex v_1 be the largest entry in the eigenvector x. Suppose edge $e_1 = \{v_1, v_2, \mathrm{L}, v_r\}$ is the edge with the largest product of eigenvector entries. According to the characteristic equation, we have

$$\lambda x_1 = \sum_{e \in E(H(v_1))} x^e \le d_1 x_2 x_3 \cdots x_r, \tag{1}$$

$$\begin{cases} \lambda x_2 = \sum_{e \in E(H(v_2))} x^e \le d_2 x_1^{r-1}, \\ \lambda x_3 = \sum_{e \in E(H(v_3))} x^e \le d_3 x_1^{r-1}, \\ \quad \vdots \\ \lambda x_r = \sum_{e \in E(H(v_r))} x^e \le d_r x_1^{r-1}. \end{cases} \tag{2}$$

Multiply the corresponding $r-1$ inequalities (1), then

$$\lambda^{r-1}(x_2 x_3 \cdots x_r) \le (d_2 d_3 \cdots d_r) x_1^{(r-1)^2}. \tag{3}$$

If the both sides of inequality (2) are $r-1$ power at the same time, then

$$\lambda^{r-1} x_1^{r-1} \le d_1^{r-1} (x_2 x_3 \cdots x_r)^{r-1}. \tag{4}$$

Multiply the two sides of inequality (3) and (4), respectively, then

$$\lambda^{2(r-1)} x_1^{r-1} (x_2 x_3 \cdots x_r) \le d_1^{r-1} (d_2 d_3 \cdots d_r) x_1^{(r-1)^2} (x_2 x_3 \cdots x_r)^{r-1}. \tag{5}$$

By $d_i \le \Delta, i \in [n]$ and (5), we obtain

$$\lambda^{2(r-1)} \le \Delta^{2(r-1)} x_1^{(r-2)^2} \left(x_1^2 x_2^2 \cdots x_r^2\right)^{\frac{r-2}{2}}. \tag{6}$$

From the mean inequality and $\boldsymbol{x}\boldsymbol{x}^T = 1$, it arrives

$$\lambda^{2(r-1)} \le \Delta^{2(r-1)} \left(\frac{1}{r}\right)^{\frac{r(r-2)}{2}} x_1^{(r-2)^2}. \tag{7}$$

Since $\boldsymbol{x}\boldsymbol{x}^T = 1$, $x_1^2 \le 1$, we get

$$\lambda^{2(r-1)} \le \Delta^{2(r-1)} \left(\frac{1}{r}\right)^{\frac{r(r-2)}{2}}. \tag{8}$$

Therefore,

$$\lambda \le \Delta r^{-\frac{r(r-2)}{4(r-1)}}. \tag{9}$$

□

Let $f(r) = r^{-\frac{r(r-2)}{4(r-1)}}$ be a function on r. When $r \ge 3$, $\frac{df}{dr} < 0$. That is, $f(r)$ is a decreasing function with r. When r reaches the maximum value, the corresponding hypergraph is only one hyperedge with n vertexes (see Figure 1). Meanwhile, $x_1 = x_2 = \cdots = x_n$. If $r = 2$, then the hypergraph H is a general graph satisfying $\lambda \le \Delta$.

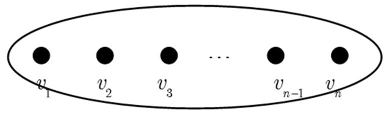

Figure 1. *Hypergraph* H_0.

Theorem 2. Let H be a r-uniform hypergraph with n vertices. Then

Equation Section (Next) $\lambda \le m r^{1-\frac{r}{2}}$.

Proof By Lemma 1, we have

$$\boldsymbol{x} = (x_1, x_2, \cdots, x_n)^T, x_i > 0, i \in [n].$$

According to the characteristic equation, then

$$\lambda x_i = \sum_{\{v_i, v_{i_2}, v_{i_3} \cdots, v_{i_r}\} \in e_i \subseteq E(H)} x_{i_2} x_{i_3} \cdots x_{i_r}. \tag{10}$$

From Holder inequality, we get

$$\begin{aligned} \lambda x_i &= \sum_{\{v_i, v_{i_2}, v_{i_3} \cdots, v_{i_r}\} \in e_i \subseteq E(H)} 1 \cdot x_{i_2} x_{i_3} \cdots x_{i_r} \\ &\le d_i^{\frac{1}{r}} \left(\sum_{\{v_i, v_{i_2}, v_{i_3} \cdots, v_{i_r}\} \in e_i \subseteq E(H)} \left(x_{i_2} x_{i_3} \cdots x_{i_r}\right)^{\frac{r}{r-1}} \right)^{\frac{r-1}{r}} \\ &\le d_i^{\frac{1}{r}} x_i^{-1} \left(\sum_{\{v_i, v_{i_2}, v_{i_3} \cdots, v_{i_r}\} \in e_i \subseteq E(H)} \left(x_i^2 x_{i_2}^2 x_{i_3}^2 \cdots x_{i_r}^2\right)^{\frac{r}{2(r-1)}} \right)^{\frac{r-1}{r}}. \end{aligned} \tag{11}$$

Then,

$$\lambda x_i^2 \le d_i^{\frac{1}{r}} \left(\sum_{\{v_i, v_{i_2}, v_{i_3} \cdots, v_{i_r}\} \in e_i \subseteq E(H)} \left(x_i^2 x_{i_2}^2 x_{i_3}^2 \cdots x_{i_r}^2\right)^{\frac{r}{2(r-1)}} \right)^{\frac{r-1}{r}}. \tag{12}$$

As $\boldsymbol{x}\boldsymbol{x}^T = 1$, $x_1^2 + x_2^2 + \cdots + x_n^2 = 1$. For any r elements $x_{i_1}, x_{i_2}, \cdots, x_{i_r}$, we have

$$x_{i_1}^2 + x_{i_2}^2 + \cdots + x_{i_r}^2 \le x_1^2 + x_2^2 + \cdots + x_n^2 = 1. \tag{13}$$

According to the mean inequality, we obtain

$$\lambda x_i^2 \le d_i^{\frac{1}{r}} \left(\sum_{\{v_i, v_{i_2}, v_{i_3} \cdots, v_{i_r}\} \in e_i \subseteq E(H)} \left(\frac{1}{r}\right)^{\frac{r^2}{2(r-1)}} \right)^{\frac{r-1}{r}}. \tag{14}$$

Then

$$\lambda x_i^2 \le d_i \left(\frac{1}{r}\right)^{\frac{r}{2}}. \tag{15}$$

Summing over i from 1 to n for (15), it arrives

$$\lambda \le rm \times r^{-\frac{r}{2}} = mr^{1-\frac{r}{2}}. \tag{16}$$

□

Theorem 3. Let H be a r-uniform hypergraph with n vertices. Then

$$\lambda \le \max_{1\le i\le n}\left(\frac{1}{r-1}\sum_{\{v_{i_1},v_{i_2},\cdots,v_{i_r}\}\in E(H)}\left(d_{i_2}^{\frac{1}{r-1}}+d_{i_3}^{\frac{1}{r-1}}+\cdots+d_{i_r}^{\frac{1}{r-1}}\right)\right)^{\frac{1}{r-1}}.$$

Equality holds if and only if the hypergraph H is regular. Equation Section (Next).

Proof According to the characteristic equation, we have

$$\lambda x_i = \sum_{\{v_i,v_{i_2},v_{i_3},\cdots,v_{i_r}\}\in e_i\subseteq E(H)} x_{i_2}x_{i_3}\cdots x_{i_r}. \tag{17}$$

By Holder inequality, we get

$$\begin{aligned}\lambda x_i &= \sum_{\{v_i,v_{i_2},v_{i_3},\cdots,v_{i_r}\}\in E(H)} 1\cdot x_{i_2}x_{i_3}\cdots x_{i_r}\\ &\le d_i^{\frac{1}{r}}\left(\sum_{\{v_i,v_{i_2},v_{i_3},\cdots,v_{i_r}\}\in E(H)}\left(x_{i_2}x_{i_3}\cdots x_{i_r}\right)^{\frac{r}{r-1}}\right)^{\frac{r-1}{r}}.\end{aligned} \tag{18}$$

According to the mean inequality, then

$$\lambda x_i \le \left(\frac{1}{r-1}\right)^{\frac{r-1}{r}} d_i^{\frac{1}{r}}\left(\sum_{\{v_i,v_{i_2},v_{i_3},\cdots,v_{i_r}\}\in E(H)}\left(x_{i_2}^r+x_{i_3}^r+\cdots+x_{i_r}^r\right)\right)^{\frac{r-1}{r}}. \tag{19}$$

If both sides are $\frac{r}{r-1}$ power, then

$$\left(\lambda x_i\right)^{\frac{r}{r-1}} \le \frac{1}{r-1} d_i^{\frac{1}{r-1}}\sum_{\{v_i,v_{i_2},v_{i_3},\cdots,v_{i_r}\}\in E(H)}\left(x_{i_2}^r+x_{i_3}^r+\cdots+x_{i_r}^r\right). \tag{20}$$

Then

$$\lambda^{\frac{r}{r-1}} \le \frac{1}{r-1} d_i^{\frac{1}{r-1}} x_i^{\frac{r-1}{r}}\sum_{\{v_i,v_{i_2},v_{i_3},\cdots,v_{i_r}\}\in E(H)}\left(x_{i_2}^r+x_{i_3}^r+\cdots+x_{i_r}^r\right)\cdot \tag{21}$$

Since $\boldsymbol{x}\boldsymbol{x}^T=1$, $x_1^2\le 1$ and $x_i^{\frac{r-1}{r}}\le x_i^{-2}$,

$$\lambda^{\frac{r}{r-1}} x_i^2 \le \frac{1}{r-1} d_i^{\frac{1}{r-1}}\sum_{\{v_i,v_{i_2},v_{i_3},\cdots,v_{i_r}\}\in E(H)}\left(x_{i_2}^r+x_{i_3}^r+\cdots+x_{i_r}^r\right)\cdot \tag{22}$$

Due to $\boldsymbol{x}\boldsymbol{x}^T=1$, we have $x_1^2+x_2^2+\cdots+x_n^2=1$.

Summing over i from 1 to n for (22), we obtain

$$\begin{aligned}\lambda^{\frac{r}{r-1}} &\le \frac{1}{r-1}\sum_{i=1}^{n}\left(d_i^{\frac{1}{r-1}}\sum_{\{v_i,v_{i_2},v_{i_3},\cdots,v_{i_r}\}\in E(G)}\left(x_{i_2}^r+x_{i_3}^r+\cdots+x_{i_r}^r\right)\right)\\ &= \frac{1}{r-1}\sum_{i=1}^{n}\left(x_i^{\frac{1}{r-1}}\sum_{\{v_i,v_{i_2},v_{i_3},\cdots,v_{i_r}\}\in E(G)}\left(d_{i_2}^{\frac{1}{r-1}}+d_{i_3}^{\frac{1}{r-1}}+\cdots+d_{i_r}^{\frac{1}{r-1}}\right)\right)\\ &\le \frac{1}{r-1}\max_{1\le i\le n}\left(\sum_{\{v_i,v_{i_2},v_{i_3},\cdots,v_{i_r}\}\in E(G)}\left(d_{i_2}^{\frac{1}{r-1}}+d_{i_3}^{\frac{1}{r-1}}+\cdots+d_{i_r}^{\frac{1}{r-1}}\right)\right).\end{aligned} \tag{23}$$

Then

$$\lambda \le \max_{1\le i\le n}\left(\frac{1}{r-1}\sum_{\{v_{i_1},v_{i_2},\cdots,v_{i_r}\}\in E(H)}\left(d_{i_2}^{\frac{1}{r-1}}+d_{i_3}^{\frac{1}{r-1}}+\cdots+d_{i_r}^{\frac{1}{r-1}}\right)\right)^{\frac{1}{r-1}}. \tag{24}$$

For the above results, equality holds if and only if

$$x_1 = x_2 = \cdots = x_n .$$

Obviously, equality holds if and only if the hypergraph H is regular. □
A regular hypergraph H_1 is shown in Figure 2.

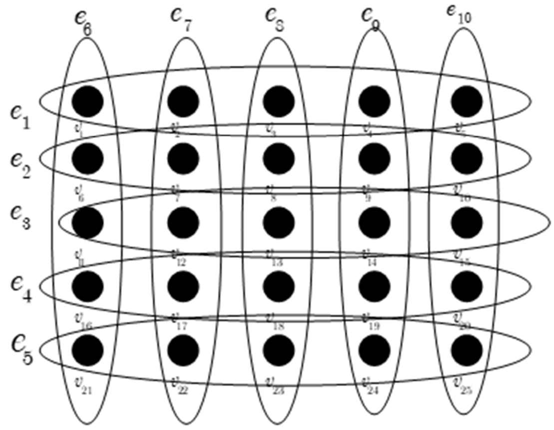

Figure 2. *Hypergraph* H_1.

From theorem 3 and using the maximum and minimum degree of a hypergraph, the following corollary can be obtained.
Corollary 3.1 Let H be a r-uniform hypergraph with n vertices. Then

$$\lambda \le \left(\frac{(rm)^{\frac{1}{r-1}}n^{\frac{r-2}{r-1}}+\left((r-1)\delta^{\frac{1}{r-1}}\Delta-\Delta^{\frac{1}{r-1}}\right)-(n-1)\delta^{\frac{1}{r-1}}}{r-1}\right).$$

Proof According to the Power mean inequality, we have

$$\sum_{i=1}^{n} d_i^{\frac{1}{r-1}} \le n^{\frac{r-2}{r-1}}\left(\sum_{i=1}^{n} d_i\right)^{\frac{1}{r-1}} (rm)^{\frac{1}{r-1}} n^{\frac{r-2}{r-1}}. \tag{25}$$

According to Theorem 3, then

$$(r-1)\lambda^{\frac{r}{r-1}} \le \max_{1\le i\le n}\left(\sum_{\{v_{i_1},v_{i_2},\cdots,v_{i_r}\}\in E(H)}\left(d_{i_2}^{\frac{1}{r-1}}+d_{i_3}^{\frac{1}{r-1}}+\cdots+d_{i_r}^{\frac{1}{r-1}}\right)\right). \tag{26}$$

Substitute (25) into (26), then

$$(r-1)\lambda^{\frac{r}{r-1}} \le \max_{1\le i\le n}\left((rm)^{\frac{1}{r-1}} n^{\frac{r-2}{r-1}} - d_i^{\frac{1}{r-1}} - \left(n-(r-1)d_i-1\right)\delta^{\frac{1}{r-1}}\right). \tag{27}$$

So,

$$(r-1)\lambda^{\frac{r}{r-1}} \le \max_{1\le i\le n}\left((rm)^{\frac{1}{r-1}} n^{\frac{r-2}{r-1}} + \left((r-1)\delta^{\frac{1}{r-1}} d_i - d_i^{\frac{1}{r-1}}\right) - (n-1)\delta^{\frac{1}{r-1}}\right). \tag{28}$$

Let $f(x) = (r-1)\delta^{\frac{1}{r-1}} x - x^{\frac{1}{r-1}}$ be a function of x. When $1 \le \delta \le x \le \Delta$, $\frac{df}{dx} < 0$. That is, $f(x)$ a decreasing function on x as $1 \le \delta \le x \le \Delta$. When $x = \Delta$, $f(x) = (r-1)\delta^{\frac{1}{r-1}}\Delta - \Delta^{\frac{1}{r-1}}$. Therefore,

$$\lambda \le \left(\frac{(rm)^{\frac{1}{r-1}} n^{\frac{r-2}{r-1}} + \left((r-1)\delta^{\frac{1}{r-1}}\Delta - \Delta^{\frac{1}{r-1}}\right) - (n-1)\delta^{\frac{1}{r-1}}}{r-1}\right). \tag{29}$$

□

3. Conclusion

In this work, using Perron-Frobenius theorem and Holder inequality, some upper bounds of the maximum E -eigenvalue of hypergraphs are obtained. In fact, spectral radius can effectively characterize the structure of a hypergraph. For example, spectral radius can be used to study the connectivity, diameter and matching number of hypergraphs. Therefore, in the future research, we consider more properties of hypergraph radius, and establish the relationship between the invariant of a hypergraph such as its girth, diameter, matching number, domination number and hypergraph radius.

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