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Log Transformation of Modified Ratio Estimator in the Presence of Non-Response Error

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Abstract: In this paper log transformation of modified ratio estimator of population mean when non-response error exists on both study variable and auxiliary variable was proposed. Using sub-sampling method of treating unit non-response, the properties of the proposed estimator as well as optimality conditions up to first order approximation were obtained. Theoretical and empirical comparison of the proposed estimator were carried out, comparing it with some existing estimators. The result of the theoretical comparison shows that the proposed estimator under optimum condition is more efficient than classical ratio estimator and Hansen and Hurwitz unbiased estimator. Furthermore, the empirical analysis on two different datasets revealed that the mean squared error of the proposed estimator increases as the value of λ increases. Also the percentage relative efficiency increases with the increase in the value of λ. The theoretical results are in consonant with the empirical results hence the proposed estimator is considered more efficient than classical ratio and Hansen and Hurwitz unbiased estimators in terms having lower mean squared error and more gain in efficiency under optimality condition in estimating population mean in the presence of non-response error and can be used in real life survey.

Keywords: Log Transformation, Modified Ratio Estimator, Optimality Conditions, Sub-sampling, Unit Non-response

1. Introduction

An essential feature of the sampling methods is that a probability sampling provides both estimates of the population parameters and a measure of its precision calculable from the sample based on the assumption that resulting observations from the sample are free of non-sampling errors. In most practical situations, such assumption does not hold and it is generally believed that observations are contaminated with sampling and non-sampling errors. One such non-sampling errors which has attracted the interest of main researchers is non-response errors which frequently occur in human sampling. Non-response error is inability or failure to collect information from all selected sampling units. The reasons for non-response includes absent of the respondent at home as at the time of the survey, refusal of the respondent to answer the questions put forward due to sensitivity of the question or enumerator's attitude.

Some authors who have contributed to the literature on non-response after the pioneer work of Hansen M. H. and Hurwitz W. N. [8] on non-response include Chaudhary M. K. and Kumar A. [3], Unal C. and Kadiliar C. [4-6], Okafor F. C. [14], Onyeka A. C. et al [15], Rao P. S. R. S. [19], Zakir W. H. et al [23], Riaz S. [20], Kumar S. and Bhougal S. [13]. Others include Dansawad N. [7], Khare B. B. and Srivastava S. [11], Kumar S. [12], Rajesh Singh and Sakshi Rai [18]. To achieve a good estimate of population parameters of interest, there should be such techniques and estimators that will minimize the effect of non-sampling errors particularly non-response on the estimates. Thus many authors like Rajesh Singh et al [16], Rajesh Singh et al [17], and Rajesh Singh and Sakshi Rai [18] have considered logarithmic type estimators to achieve this objective. For ease of use, log transformation is one of the most popular transformation used in various fields of physical sciences, engineering, medical sciences and social sciences. The use of log transformation is believed to reduce variability of data, especially when an outlying is present in the observation [1]. In a situation where the conditions that favoured the use of exponential type estimators over customary estimators are violated, logarithmic type estimators are considered most appropriate [9].

Motivated by these considerations and to improve the performance of the modified ratio estimator in the presence of non-response error, log transformation of the modified ratio estimator was considered.

2. Description of Population and Notation

For simple random sampling, let's consider a finite population $U = (U_1, U_2, \dots U_N)$ of size *N*. Let every element of the population U possess a real-valued attribute of interest *y* for study variable *Y* and real-valued attribute of interest *x* for auxiliary variable *X*.

Let

$$\left.\begin{aligned}\vec{Y} &= (Y_1, Y_2, \dots, Y_N)\\ \vec{X} &= (X_1, X_2, \dots, X_N)\end{aligned}\right\} \tag{1}$$

be the vectors of the y-values and x-values respectively, of the units defined over the population U. Our concern is to estimate population mean that is a function of the vectors $\vec{Y}$ and $\vec{X}$ by selecting from the population U, a sample of size n without replacement. Assuming the population is divided into two groups: respondent group N_1 and non-respondent group N_2 such that $N = N_1 + N_2$. Let n_1 units out of n units sampled be the number of respondent in the sample, and $n_2 = n - n_1$ units be the number of non-respondents in the sample. The respondents n_1 and non-respondents n_2 are regarded as random samples from the respondents group N_1 and non-respondent group N_2 respectively. Following Hansen M. H. and Hurwitz W. N. [8] procedure, a sub-sample of $r = {n_2}/{\lambda}$ units, (with $\lambda \geq 1$ or $0 < \frac{1}{\lambda} < 1$) are selected from non-respondent group n_2 by srswor. Where λ is the inverse sampling rate.

To obtain the estimate of the population mean of the study variable and auxiliary variable, the two independent estimates obtained from respondent group and non-respondent group are weighted and summed up. Thus the resulting non-response estimators of population mean for study variable and auxiliary variable are given as:

$$\bar{y}^* = \frac{n_1\bar{y}_1 + n_2\bar{y}_r}{n} \tag{2}$$

$$\bar{x}^* = \frac{n_1\bar{x}_1 + n_2\bar{x}_r}{n} \tag{3}$$

$\bar{y}_1$ and $\bar{x}_1$ are the sample mean of the study and auxiliary variables respectively based on the n_1 response units and $\bar{y}_r$ and $\bar{x}_r$ are the sample mean of the study and auxiliary variables respectively based on the r sub-sample non-response units. The estimators in (2) and (3) are unbiased estimators for population mean $\bar{Y}$ of study variable and population mean $\bar{X}$ of auxiliary variable respectively. Following Hansen M. H. and Hurwitz W. N. [8] their variances were given as

$$V(\bar{y}^*) = \bar{Y}^2\left[\theta_0 C_y^2 + \theta_1 C_{2y}^2\right] \tag{4}$$

$$V(\bar{x}^*) = \bar{X}^2\left[\theta_0 C_x^2 + \theta_1 C_{2x}^2\right] \tag{5}$$

The covariance of study variable and auxiliary variable in the presence of non-response is giving by

$$Cov(\bar{y}^*, x^*) = \bar{Y}^2\bar{X}^2\left[\theta_0 C_{yx}^2 + \theta_1 C_{2yx}^2\right] \tag{6}$$

Where,

$$\left.\begin{aligned}\theta_0 &= \frac{1}{n} - \frac{1}{N}\\ \theta_1 &= \left(\frac{\lambda-1}{n}\right) W_2\end{aligned}\right\} \tag{7}$$

$C_y = \frac{S_y}{\bar{Y}}$ Coefficient of variation of study variable.

$C_x = \frac{S_x}{\bar{X}}$ Coefficient of variation of auxiliary variable.

$C_{yx} = \frac{S_{yx}}{\bar{Y}\bar{X}}$ Coefficient of covariance.

$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$ Population Variance of study variable.

$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$ Population Variance of auxiliary variable.

$S_{yx} = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})(X_i - \bar{X})$ Population covariance.

$C_{2y} = \frac{S_{2y}}{\bar{Y}}$ Coefficient of variation of non-response group in study variable.

$C_x = \frac{S_x}{\bar{X}}$ Coefficient of variation of non-response group in auxiliary variable.

$C_{2yx} = \frac{S_{yx}}{\bar{Y}\bar{X}}$ Coefficient of covariance of non-response group.

$S_{2y}^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$ Variance of non-response group in study variable.

$S_{2x}^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$ Variance of non-response group in auxiliary variable.

$S_{2yx} = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})(X_i - \bar{X})$ Covariance of non-response group.

3. Adapted Estimator

Hansen M. H. and Hurwitz W. N. [8] unbiased estimator of population mean in simple random sampling is given by

$$\bar{y}^* = \frac{n_1\bar{y}_1 + n_2\bar{y}_r}{n} \tag{8}$$

and the variance is given by

$$V(\bar{y}^*) = \bar{Y}^2\left[\theta_0 C_y^2 + \theta_1 C_{2y}^2\right] \tag{9}$$

However, Hansen M. H. and Hurwitz W. N. [8] estimator does not utilize the auxiliary information on X. One of the sampling methods that utilizes auxiliary information is the classical ratio estimator. Assume that $\bar{X}$ is known, to estimate population mean of the study variable $\bar{Y}$ in the presence of non-response error, the estimator proposed by Unal C. and Kadiliar C. [4] is given as:

$$t_R = y^* \frac{\bar{X}}{\bar{x}^*} \tag{10}$$

while the mean squared error obtained by them is given as

$$MSE(t_R) = \bar{Y}^2\left[\theta_0\left(C_y^2 + C_x^2 - 2C_{yx}\right) + \theta_1\left(C_{2y}^2 + C_{2x}^2 - 2C_{2yx}\right)\right] \tag{11}$$

4. Proposed Estimator

Singh H. and Tailor R. [22] utilizing knowledge of correlation coefficient ρ, proposed a modified ratio estimator of population mean given as

$$t_0 = y\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right) \tag{12}$$

Boniface I. O. et al [2] studied the modified ratio estimator of population mean in the presence of correlated measurement errors. The suggested estimator is given as

$$t_1 = y\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right)^{\alpha} \tag{13}$$

This estimator is a biased estimator for the population mean $\bar{Y}$ of the study variable. The properties (bias and mean squared error) of this estimator up to first order approximation was obtained by them.

The objective of this study was to apply log transformation on Boniface I. O. et al [2] estimator in the presence of non-response in simple random sampling. If the population mean $\bar{X}$ of the auxiliary variable is known, the proposed estimator of population mean $\bar{Y}$ when non-response error occurs in both study and auxiliary variables is given as:

$$t_{pro} = y^*\left(\frac{\bar{X}+\rho}{\bar{x}^*+\rho}\right)^{\alpha} \tag{14}$$

5. Properties of the Proposed Estimators

Theorem 5.1. In the presence of non-response error in both study and auxiliary variables, the estimator

$$t_{pro} = y^* log\left(\frac{\bar{X}+\rho}{\bar{x}^*+\rho}\right)^{\alpha} \tag{15}$$

is a biased estimator of $\bar{Y}$ and up to first order approximation the bias is given as

$$Bias\left(t_{pro}\right) = -\bar{Y}\alpha\beta\left[\theta_0\left(\frac{1}{2}C_x^2 + C_{yx}\right) + \theta_1 C_x^2 C_{2yx}\right] \tag{16}$$

While the mean squared error is given as

$$MES\left(t_{pro}\right) = \bar{Y}^2\left[\theta_0\left(C_y^2 + \alpha^2\beta^2 C_x^2 - 2\alpha\beta C_{yx}\right) + \theta_1\left(C_{2y}^2 + \alpha^2\beta^2 C_{2x}^2 - 2\alpha\beta C_{2yx}\right)\right] \tag{17}$$

where, θ_0 and θ_1 are as defined in (7).

Proof: To prove the theorem, we use the following defined error terms and their expectations.

$$\left.\begin{aligned} \delta_0^* &= \frac{\bar{y}^*-\bar{Y}}{\bar{Y}} \\ \delta_0^* &= \frac{\mathrm{x}^*-\bar{X}}{\bar{X}} \end{aligned}\right\} \tag{18}$$

This gives

$$\left.\begin{aligned} \bar{y}^* &= \bar{Y}(1+\delta_0^*) \\ \bar{x}^* &= \bar{X}(1+\delta_1^*) \end{aligned}\right\} \tag{19}$$

such that

$$E(\delta_0^*) = E(\delta_1^*) = 0 \tag{20}$$

$$E(\delta_0^{*2}) = \theta_0 C_y^2 + \theta_1 C_{2y}^2 \tag{21}$$

$$E(\delta_1^{*2}) = \theta_0 C_x^2 + \theta_1 C_{2x}^2 \tag{22}$$

$$E(\delta_0^*\delta_1^*) = \theta_0 C_{yx} + \theta_1 C_{2yx} \tag{23}$$

(14) can be rewritten as

$$t_{pro} = \bar{y}^*\left(1 - \alpha\beta log(x^* - \bar{X})\right) \tag{24}$$

where,

$$\beta = \frac{1}{log(\bar{X}+\rho)}, log(\bar{X}+\rho) \neq 0$$

Rewriting (24) in terms of $\delta_i^*, i = 0, 1$

$$t_{pro} = \bar{Y}(1+\delta_0^*)\left(1 - \alpha\beta log(1+\delta_1^*)\right) \tag{25}$$

Assuming that $|\delta_1^*| < 1$, the expression $log(1+\delta_1^*)$ can be expanded to a convergent infinite series using binomial expansion.

Recall,

$$(1+\delta_1^*) = 1 + \delta_1^* + \frac{1}{2}\delta_1^{*2} + \cdots \tag{26}$$

By integrating (26) the binomial expansion of $log(1+\delta_1^*)$ up to first order approximation becomes

$$log(1+\delta_1^*) = \delta_1^* + \frac{1}{2}\delta_1^{*2} \tag{27}$$

Hence (25) becomes

$$t_{pro} = \bar{Y}(1+\delta_0^*)\left(1 - \alpha\beta\left(\delta_1^* + \frac{1}{2}\delta_1^{*2}\right)\right) \tag{28}$$

$$t_{pro} = \bar{Y} + \bar{Y}\left[\delta_0^* - \alpha\beta\left(\delta_1^* + \frac{1}{2}\delta_1^{*2} + \delta_0^*\delta_1^*\right)\right] \tag{29}$$

$$t_{pro} - \bar{Y} = \bar{Y}\left[\delta_0^* - \alpha\beta\left(\delta_1^* + \frac{1}{2}\delta_1^{*2} + \delta_0^*\delta_1^*\right)\right] \tag{30}$$

To obtain the bias of the proposed estimator, the expectation of both side of (30) was taken and using (20), (21), (22) and (23) to make necessary substitutions. This yield the bias as:

$$Bias\left(t_{pro}\right) = E\left(t_{pro} - \bar{Y}\right)$$

$$= \bar{Y}\left[E(\delta_0^*) - \alpha\beta\left(E(\delta_1^*) + \frac{1}{2}E(\delta_1^{*2}) + E(\delta_0^*\delta_1^*)\right)\right]$$

$$Bias(t_{pro}) = -\bar{Y}\alpha\beta\left[\theta_0\left(\frac{1}{2}C_x^2 + C_{yx}\right) + \theta_1 C_x^2 C_{2yx}\right] \quad (31)$$

as stated in the theorem.

Squaring both side of (30) and ignoring powers greater than 2, yields

$$\left(t_{pro} - \bar{Y}\right)^2 = \bar{Y}^2\left[\delta_0^* - \alpha\beta\left(\delta_1^* + \frac{1}{2}\delta_1^{*2} + \delta_0^*\delta_1^*\right)\right]^2$$

$$\left(t_{pro} - \bar{Y}\right)^2 = \bar{Y}^2[\delta_0^{*2} + \alpha^2\beta^2\delta_1^{*2} - 2\alpha\beta\delta_0^*\delta_1^*] \quad (32)$$

Taking expectation of (32) up to first order approximation and making necessary substitutions using (21), (22), and (23) the mean squared error of the proposed estimator becomes:

$$MES(t_{pro}) = \bar{Y}^2\left[\theta_0\left(C_y^2 + \alpha^2\beta^2C_x^2 - 2\alpha\beta C_{yx}\right) + \theta_1\left(C_{2y}^2 + \alpha^2\beta^2C_{2x}^2 - 2\alpha\beta C_{2yx}\right)\right] \quad (33)$$

as stated in the theorem.

The proposed estimator will be as efficient as the classical ratio estimator if $(\alpha, \beta) = (1, 1)$. Similarly, the proposed estimator will be as efficient as the classical product estimator if $(\alpha, \beta) = (-1, 1)$ and the proposed estimator will be as efficient as the Hansen M. H. and Hurwitz W. N. [8] estimator if $(\alpha, \beta) = (0, 1)$.

Using least square method that minimizes sum of squared errors, we obtain the optimum value of α that minimizes the mean squared error of the proposed estimator as

$$\alpha = \alpha_{opt} = \frac{\theta_0C_{yx}+\theta_1C_{2yx}}{\beta(\theta_0C_x^2+\theta_1C_{2x}^2)} = \frac{A}{\beta} \quad (34)$$

Where,

$$A = \frac{\theta_0C_{yx}+\theta_1C_{2yx}}{(\theta_0C_x^2+\theta_1C_{2x}^2)}$$

Thus under optimality condition, the minimum mean squared error of the proposed estimator was obtained as

$$MSE_{min}(t_{pro}) = \bar{Y}^2\left[\theta_0\left(C_y^2 + A^2C_x^2 - 2AC_{yx}\right) + \theta_1\left(C_{2y}^2 + A^2C_{2x}^2 - 2AC_{2yx}\right)\right]$$

6. Theoretical Efficiency Comparison

To determine the efficiency of the proposed estimator, relative to the Hansen M. H. and Hurwitz W. N. [8] estimator as well as the classical ratio estimator, theoretical and empirical comparison were carried out.

6.1. Theoretical Efficiency Comparison

In this subsection, theoretical comparison was carried out and condition under which the proposed estimator will have small mean squared error when compared with the Hansen M. H. and Hurwitz W. N. [8] estimator as well as the classical ratio estimator was obtained. The results shows that the proposed estimator will perform better than Hansen M. H. and Hurwitz W. N. [8] estimator if

$$V(\bar{y}^*) - MSE_{min}(t_{pro}) > 0 \quad (35)$$

That is if

$$1 > \frac{\theta_1(AC_{2x}^2-2C_{2yx})}{\theta_0(AC_x^2-2C_{yx})} \quad (36)$$

Which is always true, therefore under condition (36) the proposed estimator will perform better than Hansen M. H. and Hurwitz W. N. [8] estimator.

Similarly, the proposed estimator will perform better than the classical ratio estimator if

$$MSE(t_R) - MSE_{min}(t_{pro}) > 0 \quad (37)$$

That is if

$$1 > \frac{\theta_0(A^2C_x^2-2AC_{yx})+\theta_1(A^2C_{2x}^2-2AC_{2yx})}{\theta_0(C_x^2-2C_{yx})+\theta_1(C_{2x}^2-2C_{2yx})} \quad (38)$$

This is always true, therefore under condition (38) the proposed estimator is more efficient than the classical ratio estimator.

6.2. Empirical Efficiency Comparison

In order to buttress the results of theoretical analysis and further ascertain the effectiveness of the proposed estimator, empirical analysis was carried out using data from two different populations.

In population I, dataset from Satici E. and Kadiliar C. [21] cited in Unal C. and Kadiliar C. [4] was adopted, while in population II, dataset from Khare B. B. and Sinha R. R. [10] cited in Unal C. and Kadiliar C. [6] was adopted. The table 1 gave the parameter values for population I.

Table 1. *Parameter Values for Population I.*

N	N	$\bar{X}$	$\bar{Y}$	C_x	C_y	C_{yx}	C_{2x}	C_{2y}	C_{2yx}	W_2
96	40	144.87	137.92	0.81	1.32	0.823	0.94	2.08	1.408	0.25

The mean squared errors of the proposed estimator, classical ratio estimator and Hansen M. H. and Hurwitz W. N. [8] estimator obtained from population I dataset for different values of λ were shown in table 2.

Table 2. *Mean Squared Errors of Proposed estimator and some Existing Estimators for Different Values of λ.*

λ	A	t_{pro}	t_R	$\bar{y}^*$
2	1.378477	452.2425	493.3614	997.7
3	1.436079	703.4129	777.977	1512.053
4	1.469339	953.081	1062.593	2026.406
5	1.490994	1202.033	1347.208	2540.759
6	1.506217	1450.588	1631.824	3055.112
7	1.517502	1698.9	1916.439	3569.464
8	1.526203	1947.052	2201.055	4083.817
9	1.533115	2195.094	2485.67	4598.17
10	1.53874	2443.056	2770.286	5112.523

The percentage relative efficiency of the proposed estimator for different value of λ over classical ratio estimator and over Hansen M. H. and Hurwitz W. N. [8] estimator were obtained as defined below.

$$PRE = \frac{MSE(t_{pro})}{MSE(*)} \times 100 \quad (39)$$

Table 3 shows the result of the relative efficiency for population I obtained.

Table 3. *Percentage Relative Efficiency of the Proposed Estimator over some Existing Estimators for Different Values of λ.*

λ	A	PRE of t_{pro} over t_R		PRE of t_{pro} over $\bar{y}^*$	
		t_{pro}	t_R	t_{pro}	$\bar{y}^*$
2	1.378477	109.0922	100	220.6117	100
3	1.436079	110.6003	100	214.9595	100
4	1.469339	111.4903	100	212.6163	100
5	1.490994	112.0775	100	211.3718	100
6	1.506217	112.494	100	210.612	100
7	1.517502	112.8047	100	210.1045	100
8	1.526203	113.0455	100	209.7436	100
9	1.533115	113.2375	100	209.4749	100
10	1.53874	113.3943	100	209.2675	100

The table 4 shows the parameter values for population II.

Table 4. *Parameter Values for Population II.*

N	n	$\bar{X}$	$\bar{Y}$	C_x	C_y	C_{yx}	C_{2x}	C_{2y}	C_{2yx}	W_2
261	90	306.45	222.58	1.76	1.87	3.185	1.23	1.22	1.46	0.25

Mean squared errors of the proposed estimator, classical ratio estimator and Hansen M. H. and Hurwitz W. N. [8] estimator obtained from the population II dataset for different values of λ were shown in table 5.

Table 5. *Mean Squared Errors of Proposed estimator and some.*

λ	A	t_{pro}	t_R	$\bar{y}^*$
2	1.018290	91.7107	92.1540	1465.983
3	1.011060	103.1546	103.3422	1670.811
4	1.005558	114.4766	114.5304	1875.639
5	1.001231	125.7157	125.7186	2080.468
6	0.997739	136.8958	136.9068	2285.296
7	0.994862	148.0326	148.0950	2490.124
8	0.992450	159.1365	159.2832	2694.952
9	0.990399	170.2149	170.4714	2899.780
10	0.988633	181.2732	181.6596	3104.608

Existing Estimators for Different Values of λ

The percentage relative efficiency of the proposed estimator for different value of λ over classical ratio estimator and over Hansen M. H. and Hurwitz W. N. [8] estimator were obtained as defined in (39).

Table 6 shows the result of the relative efficiency for population II.

Table 6. *Percentage Relative Efficiency of the Proposed Estimator over some Existing Estimators for Different Values of λ.*

λ	A	PRE of t_{pro} over t_R		PRE of t_{pro} over $\bar{y}^*$	
		t_{pro}	t_R	t_{pro}	$\bar{y}^*$
2	1.018290	100.4834	100	1598.488	100
3	1.011060	100.1819	100	1619.715	100
4	1.005558	100.0470	100	1638.448	100
5	1.001231	100.0024	100	1654.899	100
6	0.997739	100.0081	100	1669.369	100
7	0.994862	100.0422	100	1682.146	100
8	0.992450	100.0922	100	1693.485	100
9	0.990399	100.1507	100	1703.599	100
10	0.988633	100.2132	100	1712.668	100

7. Discussion

The results obtained from theoretical analysis revealed that the proposed estimator is more efficient than Hansen M. H. and Hurwitz W. N. [8] estimator and classical ratio estimator under the conditions given in (36) and (38) respectively at its optimum value. When $\alpha = 1, \beta = 1,$ the proposed estimator will be as efficient as the classical ratio estimator. Similarly if $\alpha = -1, \beta = 1,$ the proposed estimator will be as efficient as the classical product estimator and if $\alpha = 0, \beta = 1,$ the proposed estimator will be as efficient as the Hansen M. H. and Hurwitz W. N. [8] estimator. The mean squared errors were obtained for different values of λ. It was observed that the mean square error of the proposed estimator as well as classical ratio and Hansen M. H. and Hurwitz W. N. [8] estimators increases with increase in value of λ. For each value of λ, the proposed estimator has more gain in efficiency than the classical ratio and Hansen M. H. and Hurwitz W. N. [8] estimator.

8. Conclusion

Log transformation of modified ratio estimator of population mean was proposed. The properties of the proposed estimator was obtained. The optimum condition under which the proposed estimator will be efficient than some existing estimators was derived. The study concludes that the proposed estimator at its optimum value is more efficient than classical ratio estimator and Hansen M. H. and Hurwitz W. N. [8] estimator must especially, when the value of λ is small and therefore may be used in practice.

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