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What the Teacher Must Master to Direct the Learning Process

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Abstract: Determining what the Mathematics teacher must master is a fundamental aspect to specify what the correct preparation of said teacher should be, which is a widely debated topic, on which there are various proposals for action, in each of which can be find positive aspects, despite the different approaches found in the specialized literature, at least there is consensus that such preparation must be composed of mathematical knowledge and didactic knowledge, which is supported by research, experiences and proposals aimed at achieving a good preparation of these professionals; several authors have specified the notion of Pedagogical Content Knowledge for the teaching of Mathematics, understanding as such the mastery of the mathematical content to be explained together with the appropriate didactic procedures to explain said content, other authors refer to the Specific Knowledge of the Content, as the “mathematical knowledge to teach”. The objective of this work is to scientifically argue the domain of epistemological and ontological characteristics of Mathematics, which can allow the teacher to establish the appropriate link between mathematical knowledge and didactic knowledge.

Keywords: Ontology, Epistemology, Mathematics, Teacher

1. Introduction

In the specialized bibliography, in recent years a unity of criteria has been appreciated regarding the dependence of the didactics of Mathematics on mathematical knowledge, it is evident that one cannot even think that whoever does not master the concept of limit can achieve that their students understand it; the study of the didactics of Mathematics, and even more research in this field, requires a strong base interrelation on the competence in Mathematics by the person who carries it out Fandiño-Pinilla [14] D'Amore, and Fandiño-Pinilla [6].

There are two extreme positions in this regard, on the one hand when Mathematics content is taught to teachers without referring to how to teach it, although as stated before, the Mathematics teacher must have a good command of this science; on the other, general didactic courses are offered to Mathematics teachers taught by people without mathematical knowledge, here it is also true that every teacher needs a certain mastery of general didactics, but both things separately leave a fundamental gap in the preparation of both the teacher in training and active teacher.

Indeed, the teaching of Mathematics is multifaceted, which is why it requires that the teacher possess a set of skills that allow him to guide his students so that they can achieve good results, such as skills related to the specific knowledge of Mathematics and also the knowledge and skills to bring mathematical knowledge to a teaching practice that promotes student learning Hill, Rowan and Ball [18] and Ottmar and Rimm-Kaufman [22].

On the other hand, the mathematical content that a teacher must possess is related to the school level where the teacher will exercise his professional activity, but even specifying the school level, there is still room to discuss the breadth and depth of the required knowledge, in addition to according to Even and Ball [13], knowledge about content derived from what is explained at a given school level is also included; Therefore, the objective of this work focuses specifically on the domain of epistemological and ontological characteristics of Mathematics that the teacher must master at any school level that taught this science.

In this regard, the Mathematics Teaching Study Group, over several years of work, with international collaboration, has been able to identify ontological and epistemological characteristics of Mathematics, which should be the domain of the Mathematics teacher, regardless of the level or content that explains, the knowledge of these characteristics has been used and validated in several defended doctoral theses, the results of the aforementioned group have been recognized and endorsed by three prizes from the Cuban Academy of Sciences. In addition, these results are the systemic concretion of results that appear in the specialized bibliography and recognized by different authors, which includes an empirical support from Pérez [23].

2. Developing

Considering the didactics of Mathematics as an applied branch of science Mathematics D'Amore and Fandiño-Pinilla [6], it is essential to analyze the epistemological and ontological characteristics of Mathematics, taking into account that epistemology or theory of knowledge studies the foundations scope and validity of scientific knowledge of a science and ontology studies the nature of things, in this case the nature of mathematical knowledge. The ontological basis of a theory can be described as the answer to the following questions: what classes of objects does it refer to and what are the basic theoretical assumptions that the theory makes about those objects; ontology deals with referential and theoretical aspects Girnat [15].

In the present work it is not intended to give a definition of the epistemology and ontology of Mathematics, only characteristics of both aspects of Mathematics Byas and Blanco [3] will be described and argued, these characteristics are listed below, which will be analyzed in themselves, in their interrelationships and in their relationship with the teaching-learning process of Mathematics.

3. Ontological and Epistemological Characteristics of Mathematical Knowledge

3.1. Ontological Characteristic of Mathematical Knowledge

1. Non-ostensive character of mathematical objects.
2. It is medium and object in itself.
3. Relative nature of mathematical truth.
4. Own language.
5. Systemic structure.
6. Intrinsic development logic.

3.2. Epistemological Characteristics of Mathematical Knowledge

1. Concepts are formed through their representation in different semiotic registers.
2. Interrelation: procedural activity–conceptual formation.
3. Mathematical knowledge is produced when activities are carried out through which it is developed.
4. Individual and systemic character of the mathematical object.
5. General singular character of the mathematical model.

3.3. Analysis of the Ontological and Epistemological Characteristics of Mathematical Knowledge

Both the non-ostensive character of mathematical objects and the conceptual formation through different registers of semiotic representation characterize both Mathematics itself and the development process of mathematical knowledge, which is abundantly treated in the specialized bibliography Winsløw [30], D'Amore [5], Moreno-Armella and Sriraman [21] and Radford [26] as well as the close relationship between them.

Mathematical objects, unlike the objects of other sciences, materialize conceptually, that is, they do not have ostensive representation, which gives rise to mathematical objects being expressed conceptually and the mathematical object being identified by the concept that characterizes it. Therefore, the non-ostensive character of mathematical objects is in the very nature of Mathematics, what determines that the development of the knowledge of this science, that is, the conceptual appropriation is closely linked to the need for the representation of the mathematical object in different registers of semiotic representation.

From this relationship derives the paradox raised by R. Duval and taken up in the specialized literature by many authors, which, paraphrased by the authors of this work, expresses that since access to mathematical objects is through representations of these, which we ourselves do, it is natural to confuse the object with one of its representations Duval [9], therefore, to prevent the student from identifying the object with one of its representations, he has to work with different representations of it. In addition, it must be taken into account that each representation highlights certain characteristics of the object, which is why the use of different representations is required so that the student manages to integrate all the elements that characterize the object Duval [10].

The problem that implies the non-ostensive character of mathematical objects and the use of changes in semiotic registers in the learning of Mathematics is widely treated, the referenced bibliography on the subject is a small sample of the variety of work on the matter; The intention here is to understand the importance that the Mathematics teacher, whatever the school level in which he works, internalizes its importance in the teaching-learning process of Mathematics and is documented about them, since the character not ostensive of mathematical objects implies their access to them only through their semiotic representations, in addition the changes of representation are pertinent not only for the understanding of the concept, in many occasions the resolution of a problem is simplified using an adequate representation, since be it graphic, analytical or even literal object. in different registers of semiotic representation.

Unlike other sciences whose development requires knowledge outside the science in which they work, the ontological development of Mathematics is intrinsic to itself, no other science intervenes in the development of Mathematics, this does not mean that in their teaching process, problems and tasks where Mathematics is applied to other sciences are ignored, on the contrary, especially from the motivational point of view, these examples are very useful. Although it must be said, that sciences such as Physics have influenced the development of Mathematics given the demand for mathematical resources that they have raised throughout its development. But undoubtedly the development of Mathematics is produced from Mathematics itself.

This characteristic of Mathematics determines the need for prior knowledge to learn new concepts and procedures, although it can be said that this is a truism, unfortunately many teachers avoid setting tasks for their students where prior knowledge is required, with the intention of obtaining better results from the point of view of passing and failing, but the harmful effects on the education of their students do not compensate for the better promotion rates that they can obtain in this way.

On the contrary, it is essential to confront students with tasks where they need prior knowledge, particularly in obtaining new results that they will have to use later, with which students internalize that learning Mathematics is not only to pass an exam. Many times previous knowledge is wasted in mathematical work, for example when solving an equation such as: $e^{2x+2} = e^{x^2}$ students are told that the method of resolution is to equal the arguments of the function, in this case the exponents, but in the case of equations such as: $\sin(2x) = \sin(x^2 +1)$ they are told that in this case they cannot equate the arguments, instead of using the students' prior knowledge and instructing them that in the first case the arguments can be equated because the function is injective.

Naturally, the ontology and epistemology of Mathematics are closely related, the fact that Mathematics is a means and an object in itself is inextricably linked to the fact that mathematical knowledge is produced when activities are carried out through which mathematical knowledge is developed itself, in other words, for the student to acquire mathematical knowledge, he must perform tasks where he has to induce, graph, prove, demonstrate and in general do the tasks inherent to mathematical work. When speaking here of proving, it is not exclusively referring to proving the properties and theorems that correspond to the content of a subject, since it is not always possible to prove all the theorems and properties that correspond to the content of a subject, but in all mathematical content there are tasks of interesting demonstration and especially those that require the use of previous content, which leads to the recurring approach that expresses that "knowing mathematics" is "doing mathematics" Vilanova [28] and Medina [20].

In the ontology of mathematical knowledge it is necessary to take into account the relative nature of mathematical truth, it is the teacher's obligation to ensure that students take into account the hypotheses under which the assigned task is developed, since in colloquial language expressions are used like: the order of the factors does not alter the product, or that is as true as 2 + 2 is 4, for example the product of matrices is not commutative, 2 + 2 is 4 only in number systems with a base greater than or equal to 5.

In the case of geometry tasks where auxiliary constructions are required, it is usual for the student to make the construction without taking into account whether said construction satisfies the requirements for its existence. Taking into account the hypotheses of the problem in question is fundamental in mathematical work, but many times the students are not aware of its importance, which can be said that it is not the fault of the students, but of the way in which it is developed the teaching process. In different Mathematics texts, it is proposed at the beginning that work will be done with a real variable and, of course, throughout the text no more references are made to working with a real variable, so it is not a concern for students to pay attention to the numerical field in which you work.

The fact that mathematical truth is relative to hypotheses is at the very foundation of Mathematics, it is one of the reasons why in Mathematics a result has never had to be rectified, as is the case of Euclidean Geometry with its more than twenty centuries of existence; the results that are obtained in non-Euclidean geometries do not negate the results of the Euclidean, because they are simply results that are obtained under different hypotheses.

If the teacher is not aware of the relative nature of mathematical truth, regarding the hypotheses, he will not worry about getting his students to take into account the hypotheses under which the task they perform is fulfilled, which has negative implications not only in the resolution of specific tasks, but in the training of students.

Finally, it is opportune to add that although mathematical truth is relative to the hypotheses, it is absolute from the social point of view, that is, each mathematical result is true wherever it is on the planet where it is worked, which provides a character of universality to this science.

The relative nature of Mathematical truth is also linked to the interrelation: procedural activity-conceptual formation, since obviously both in procedural activity and in conceptual formation it is necessary to take into account the conditions or hypotheses under which one works that guarantee the veracity of the obtained result. The process-concept relationship is a subject widely treated in the specialized bibliography, although it is not recognized as an epistemological characteristic of mathematical knowledge, but from the fact that mathematical objects materialize conceptually, it follows that the development of mathematical knowledge is through conceptual domain, on the other hand, taking into account Vygotsky's approach where he states that the concept cannot be put in a finished form in the mind of the student Vygotsky [29], it can be seen that the process-concept relationship characterizes the development of mathematical knowledge.

According to Vygotsky's previous approach, conceptual appropriation starts from elementary forms of the concept, called in the literature in different ways, among them: pre-concept, image concept, pseudo-concepts; initial forms of the concept with which one works until arriving at what is called the concept definition or scientific concept, where the means to achieve the transition from the elementary forms or pre-concepts to the finished form is the work with different processes that require the application of these initial forms, which will be referred to in this work as pre-concept Elia and Spyrou [11] and Cunningham and Roberts [12].

The teacher must understand that the goal of the procedural activity is not limited to students learning to solve certain tasks, this activity must lead students to conceptual appropriation, for which the processes with which they work must contain in different moments all the characteristics of the concept to be formed; here we can refer to the classic example that students do not identify the height of a triangle when it is outside the triangle, which is a result of the fact that in the tasks assigned to the students they never worked with the height outside the triangle. In general, when in the processes with which the student has worked for the formation of a concept, there is a characteristic of the concept that was not used, the concept acquired by the student will lack said characteristic; Therefore, it is essential that the teacher keep in mind what has been proposed for the orientation of the work of their students, regardless of the school level where they work.

Here it is necessary to return to the semiotic representation of mathematical objects, since, in Mathematics, as has been explained, ostensive references are not possible; every mathematical concept is forced to use representations, since there are no "objects" to exhibit in its name or in its evocation D'Amore (2001); In Mathematics, the sign, understood as any semiotic representation of the mathematical object, mediates two processes: the development of the mathematical concept in the student and his individual interaction in the socially established mathematical activity Radford [24].

The transition from the pre-concept to the scientific concept must necessarily pass through representation registers, so the procedural activity implies the use of different registers of semiotic representation, given that each representation provides different characteristics of the concept. Here it is essential that the student sees the same mathematical object in its different representations, since if the student sees the expression $f(x)= x^2$ and the graph of this function as two different mathematical objects and not as the representation of the same object, it is not achieved that in the transition from the process to the concept it incorporates its different characteristics.

According to the analysis carried out, the procedural activity implies the use of different registers representing the same mathematical object, but paying special attention to the fact that the student sees the same object in each of its representations, so that it can incorporate the characteristics that each representation contributes to the object of study, so that the student can appropriate the scientific concept, that is, the concept with each of its properties.

The procedural activity-conceptual formation make up a dialectical interrelation, that is, the conceptual formation depends on the procedural activity but this in turn depends on the conceptual formation, although starting in the first instance from pre-concepts from which the scientific concept through the correctly oriented procedural activity, as has already been stated, so that in the tasks that are oriented to the students, each one of the characteristics of the concept that is desired to be formed in the students appears, to achieve the passage from concepts as instruments to concepts as objects.

In Mathematics, as in all human activity, language is present, but it is used in a new way to build mathematical knowledge; mathematical language is characterized by condensing technically reliable information with a certain level of abstraction De Olivera and Cheng [8], colloquial language words acquire new meanings in mathematical language such as: similarity, group, integral, etc; others acquire a more precise meaning, such as: limit, monotony, convergence, etc; it is a language in which there are no understatements or ambiguities, which, being highly specialized, is subject to a very rigorous syntax.

The language of Mathematics is part of its ontology since it is intrinsic to it and plays a fundamental role in its development; language is intrinsically and constitutively present in both praxis and logos; the logos finds its reason for being in praxis and praxis develops and is governed by the logos. Language becomes the means by which not only the non-ostensive ones are expressed, but it is also an instrument for their constitution and development, which is why it must be considered as the ostensive facet of mathematical objects Godino [16], it is also a resource essential for social interaction.

In order to express mathematical relationships between objects and phenomena, it is essential to have a mathematical language, through which to carry out said actions; such language, as already explained, is in the ontology of Mathematics, but students generally do not see that this science has a specific language, which in many aspects differs from colloquial language, therefore, the student does not become aware of his lack of knowledge. Therefore, it is the task of the teacher to ensure that their students understand the need for mathematical language in order to express their ideas clearly and precisely.

It is the case that when students work with the Mean Value Theorem, which has among its hypotheses that the function is continuous in [a,b] and differentiable in (a,b), they only take into account one of these hypotheses, In this case, the error is not in not taking into account the hypotheses, but in considering the conjunction "and" as disjunctive and not as inclusive, which shows the need for students to achieve an adequate command of mathematical language.

Regarding the use of mathematical language, a fundamental aspect is the handling of functions, since these are the mathematical tool to establish relationships between their objects and phenomena. The difficulties that students manifest in the use of mathematical language with the function as its fundamental element, are manifested when the student cannot establish the relationships between the variables of a rate of change problem or an optimization problem; For a student to be efficient in his mathematical work, he needs to master and handle mathematical language appropriately.

Undoubtedly all science is systemic, but in the case of Mathematics, its "systemic structure" characterizes it in a special way, each specific knowledge is a link in the mathematical theory, all the branches of Mathematics are intertwined, a categorical example to the respect is the equation $e^{i\pi} = -1$, in which, each one of the constants present in the equation, appeared in Mathematics at different moments of its historical development and in different contexts; the logarithm function was developed with the specific goal of simplifying arithmetic operations with very large numbers, but it allows its systemic structure to be revealed in equality $i\pi = \ln(-1)$. Another relationship that highlights the systemic structure of Mathematics is: $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, which, moreover, is the Fourier series development of the function f(x) = x^2, elements that could be thought to have no relation to each other.

This ontological characteristic has important implications in how mathematical knowledge is developed, that is, in its epistemology, since it determines the need to study the mathematical object in its individual character and in its systemic character, that is, when studying the equation: $ax^2 + bx + c = 0$, so that students learn to solve it in its different forms and by different methods, they work with said equation in its individual form, as a specific object of Mathematics, but mathematical knowledge does not end with efficient learning, but specific to it, This is followed by an aspect of singular importance, which is its integration into the systemic structure of Mathematics, when it becomes a tool for mathematical work, which is the basis for Mathematics to be a means and an object in itself.

The same happens with each mathematical object, in the first instance it has a singular character, when its specific study is particularized, but the learning of mathematical objects cannot end in their learning disconnected from the rest of the mathematical objects, it has to be integrated into the systemic structure of Mathematics, which allows the student to advance in the development of their mathematical knowledge. Fundamental aspects, that the teacher must take into account for the orientation of tasks to their students.

The systemic structure of Mathematics also implies the particular general character of the mathematical model. Understanding as a model a set of symbols and mathematical relationships, which somehow represent a phenomenon or situation. In this way, the resulting model, as a solution, facilitates not only achieving a particular output but also becoming a means for other applications or theories. In the practical order, said model is related to any area of Mathematics, especially with the essential instruments of its applications, according to Biembengut and Hein [2].

This concept of mathematical model expresses in a concrete way that the model is not limited to the solution of a specific problem, it is also a means for other applications; which can be exemplified with the function: which represents the model for the resolution of a considerable variety of problems, from finding the rectangle of maximum area with constant perimeter, or to determine the dimensions of a rectangular window at the bottom and a semicircle at the top, in a way that allows the entry of the greatest possible amount of light, with a constant perimeter, just to give a few examples.

When the model of a given problem is sought and the resulting model is known, the resolution of the problem is simplified, which is undoubtedly an advantage of the general nature of the mathematical model, but when in the teaching process the student identifies the way of solving a problem associating it with a known model means that a mechanistic teaching process has been developed, where memory prevails over reasoning; so it is necessary for the teacher to understand these two aspects of the mathematical model and take advantage of them properly in guiding their students.

The systemic structure of Mathematics has already been analyzed, in systems theory it is stated that every system has its own development logic, and indeed among the aspects that characterize the ontology of Mathematics is its intrinsic development logic, which is not the mathematical logic, although of course it is closely related to the latter. The development logic describes basic aspects of the development of mathematical knowledge, we do not have a specific definition of it, but we can relate the aspects that characterize it, which are the following:

1. The absolute character of the general in Mathematics.
2. Lack of exceptions in mathematical theory.
3. The rigidity and specificity of the equality relationship.
4. Precise determination of the field of action of each property or law.
5. Its axiomatic construction.

These aspects are related to each other and to the elements that characterize the ontology and epistemology of Mathematics, as will be explained below.

Generalization in Mathematics is closely related to conceptual formation and, as in the case of concepts, there is an elementary or empirical generalization and a theoretical generalization Radford [25], elementary generalization begins by abstracting common aspects that the subject appreciates in a given context set of objects, which may or may not be essential, later the subject manages to separate the essential aspects from the non-essential ones, until abstracting only the essential ones and arriving at the theoretical generalization Davydov [7].

Theoretical generalization is achieved through procedural activity, and just as in the case of concepts, when an essential characteristic is absent in said activity, the generalization will lack said characteristic; in the same way, if there is always a non-essential characteristic in the procedural activity, it will be included as essential in the generalization that is achieved; Therefore, it is an aspect that the teacher must take into account.

The theoretical generalization has an absolute character, since when in Mathematics it is stated that a property is fulfilled in a general way under certain hypotheses, there cannot be a case under the same hypotheses where said property is not fulfilled; there is a notable difference between the general in everyday language and the general in mathematical language, since in everyday language the general is identified with almost all or the majority; for example, in everyday language and in medical sciences, the general has a certain degree of certainty, but admits cases in which that generality does not hold. This is a notable difference between colloquial language and mathematical language.

This absolute character of the general implies another characteristic of the intrinsic logic of development of Mathematics, which is the lack of exceptions in mathematical theory, if a property is fulfilled for real numbers, it can be guaranteed that there cannot be a real number that does not comply with said property; this implies a method of mathematical work, which is the possibility of refuting an approach by means of a counter example, which would not be possible if there were exceptions in mathematical theory.

The rigidity and specificity of the relation of equality is a characteristic of the intrinsic development logic of Mathematics, given that the rigidity of this relation in Mathematics is what always allows one thing to be replaced by its equal in the development of a problem, also implies an absolute character in this relationship, it is what allows a usual work method in Mathematics, which is the change of variables, assuming mathematical equality between the original variable and the one used for its replacement; This feature also implies precision in mathematical language, in colloquial language it is said that identical twins are the same, which is not entirely true, since close relatives can distinguish them, so they are not really equal mathematically speaking, some difference they have if someone can distinguish them.

The specific character of this relation refers to the fact that it can be used to specify equality between particular aspects of objects that are not equal in their totality, as is the case of the equality between the angles of similar triangles, or the equality of the area between two polygons, which may not even have the same number of sides, which is a necessary resource in mathematical work.

The precise determination of the field of action of each property or law is essential in the functioning of Mathematics, given the relative character of truth in Mathematics, which as a whole is sustained and supported by the absolute character of the general in Mathematics.; As explained above, students are not given to considering all the hypotheses under which they are working, so it is the teacher's task to get their students to pay special attention to the hypotheses they are working with, for which they can use a wide variety of hypotheses of examples where the result depends on the hypotheses, for example the fact that polynomial equations have the same number of roots as the degree of the polynomial, is true in the field of Complexes, but not so in the field of Reals.

The axiomatic construction, in addition to characterizing its development, is a fundamental reason why Mathematics is a means and an object in itself; it is also at the base of why in Mathematics it has never been necessary to rectify any of its results. Actually the axiomatic construction of Mathematics is directly or indirectly at the base of the ontology and epistemology of this science, due to this construction in Mathematics there are no circular definitions, it also produces its rigorous systemic structure.

But here it is necessary to take into account that an axiom does not exist independently, but in the set of axioms that support a theory, an axiom is not such but in combination with the other axioms; its character derives from the fact that it is independent of and consistent with the other axioms; in the system of axioms a contradiction cannot be concluded, and no axiom can be derived from the rest. If the axioms satisfy these conditions, then the objects defined through them will be true and exist.

In general, the axiomatic construction goes unnoticed by the students, so it is the teacher's task to bring this idea to the students, for what it implies for the construction of Mathematics, as a powerful theory, without contradictions or inaccuracies, with its own language.

3.4. The Ontology and Epistemology of Knowledge in the Teaching Process

The aspects analyzed here are implicit in the teaching-learning process of Mathematics, so they must be used for the correct orientation of the students' activity, but unfortunately, in general, teachers do not have them incorporated as part of their professional preparation, regardless of the school level where they work, although many times they apply some of these aspects, but without taking into account the close interrelation between them.

The ontological aspects govern the development of mathematical knowledge, that is, its epistemology. Teachers know the need for prior knowledge, but most of the time, without taking into account that Mathematics is a means and an object in itself, they do not see the fundamental importance of assigning tasks to their students where they are obliged to use previous knowledge, also they do not work with the perspective that their students use what they learn today, to continue their future mathematical learning.

The process-concept relationship is not something that most teachers have internalized, so many teachers are only concerned that their students master the processes that will later make up the exams that they apply to their students, without worrying about achieving conceptual training of their students, logically the students thus trained will only be able to solve known tasks and, obviously, they will only be able to solve known tasks and, obviously, they will not be able to apply concepts that do not have mechanized. At present, when most of the mathematical processes can be carried out by means of computation, the conceptual training of students prevails, so that they are capable of building mathematical models, which will be executed by means of computation.

Given the popularity, scientifically speaking, that the transfer of semiotic registers has reached in recent years, as was previously referenced, it is to be assumed that many teachers take into account that concepts are formed through their representation in different semiotic registers, but In general, they do not take into account that these changes are an integral part of the transition from the process to the concept, since in the procedural activity the student has to work with all the components of the concept, and that there cannot be a non-essential element of the concept in training that appears in each of the student-oriented activities, because if so, this non-essential element will be part of the concept formed by the student.

One aspect to which teachers in general do not pay due attention is the fact that mathematical knowledge is produced when activities are carried out through which it is developed; but one view about the meaning and nature of mathematics is to consider it as a social construction that includes conjectures, proofs and refutations; the idea behind this view is that "knowing mathematics" is "doing mathematics". These situations require creative thinking, which allows conjecturing and applying information, discovering, inventing and communicating ideas, as well as testing those ideas through critical reflection and argumentation Medina [20].

It is relatively common for teachers to assign tasks to their students of the same type, generally calculation and frequently algorithmic, instead of assigning tasks where students have to conjecture, graph, induce, prove; in short, do the activities through which Mathematics is developed, so that their students effectively learn Mathematics.

Another aspect neglected by teachers is working with hypotheses, that is, assigning tasks to their students where failure to consider one or more of the hypotheses of the problem posed leads to an erroneous solution, since it is essential to ensure that the student is aware that in Mathematics a result is fulfilled for certain hypotheses and not for others, this is the relative character of mathematical truth; It is relatively common for students to assume invalid hypotheses, often because of the appearance of the graph that represents the problem with which they work and others because they are usually fulfilled, as is the case of the commutative law and the associative law of the sum of real numbers, which are true whenever the sum has a finite number of addends.

The good use of the hypotheses is manifested in the resolution of problems with a literal statement that must be transferred to an analytical representation, the correct interpretation of the conditions of the problem, that is, of its hypotheses, determines the analytical representation that enables the resolution of the problem issue. In practically all the texts that contain problems of the referred type, the data that appear in the problems are only those necessary for their analytical representation, but it is necessary to confront the students with problems in whose statement there are superfluous data, so that they are trained in the discrimination of hypotheses that are required to solve the problem.

3.5. Interrelation of the Ontological and Epistemological Characteristics of Mathematics

Although in the teaching-learning process of Mathematics, each one of the ontological and epistemological characteristics of Mathematics can be treated in a specific way at a given moment, it is important to bear in mind that they form a whole in the didactics of Mathematics, given mutual influence between them.

That is, the non-ostensive nature of mathematical objects determines the need for their representation in different semiotic registers, so that the student can effectively appreciate the different properties of the object with which they work and achieve the correct appropriation of the concept that materializes the object. The changes in register must adjust to the hypotheses under which the concept that is formed is valid, given the relative nature of truth in Mathematics.

Register changes are part of the procedural activity through which the concept is arrived at and evidently each concept is a tool to form new concepts, because Mathematics is a means and an object in itself, but the mathematical activity must consist of the activities through which Mathematics is developed, among which the use of the language of Mathematics should be emphasized, as it is one of the forms of semiotic representation of the mathematical object.

The usefulness of the concept is effective when the concept is incorporated into the corresponding conceptual network, given the systemic structure of Mathematics, so it is the teacher's task for the student to appreciate the mathematical object in its individual and systemic form, as well as the character general singularity of the mathematical model, for which it must be based on the logic of intrinsic development of Mathematics, highlighting the absolute character of the general in Mathematics, the lack of exceptions in mathematical theory, the rigidity and specificity of the equality relationship and the precise determination of the field of action of each property or law, all of which is in correspondence with the axiomatic construction of Mathematics.

4. Conclusions

As can be seen, the present work has not focused on a specific school level, nor on a particular content, but on general but fundamental aspects, so that the teacher understands what Mathematics is and how it works, so that he can guide the activity of its students so that it achieves a correct mathematical formation of the same.

Some of the aspects analyzed here are extensively treated in the specialized literature, as is the case of changes in registers of semiotic representation, the use of mathematical language and the process-concept relationship; but to achieve the stated objective, it is necessary to show not only the fundamental aspects of the ontology and epistemology of Mathematics, but also what is fundamental in the relationship between them and how they are manifested in the teaching-learning process.

Unfortunately, while teachers and administrators are satisfied that students learn to do certain tasks repetitively, it will be difficult to get teachers to worry about achieving a true mathematical education in their students and to be interested in documenting what Mathematics is and how it works, which would help to solve, or at least improve many of the difficulties of the students that are reported in the specialized bibliography.

Doing Mathematics is not repeating mechanized algorithms by heart, it is being able to make conceptual applications to solve new problems, that is, doing Mathematics implies reasoning, imagining, discovering, intuiting, testing, generalizing, using techniques, applying skills, estimating, checking results.

The entire work has been focused on arguing scientifically, the domain of epistemological and ontological characteristics of Mathematics, which can allow the teacher to establish the appropriate link between mathematical knowledge and didactic knowledge; in other words, to the fulfillment of the stated objective.

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